1.12 — metody
numeryczne i komputerowe
4.42 — stany graniczne

Krzysztof Doliński

FIRST ORDER APPROXIMATION AND IMPORTANCE SAMPLING IN RELIABILITY OF STRUCTURES UNDER COMBINED RANDOM LOAD SEQUENCES

21/1990

WARSZAWA 1990

ISSN 0208-5658

Praca wpłynęła do Redakcji dnia 22 grudnia 1989 r.

Na prawach rękopisu

Instytut Podstawowych Problemów Techniki PAN

Nakład 100 egz. Ark. wyd. 1,4 Ark.druk. 1,75
Oddano do drukarni w maju 1990 r.
Nr zamówienia 263/90

Warszawska Drukarnia Naukowa, Warszawa,
ul. Śniadeckich 8

Krzysztof Doliński
Dept. of Structural Mechanics
Institute of Fundamental
Technological Research
Warsaw, POLAND

FIRST ORDER APPROXIMATION AND IMPORTANCE SAMPLING IN RELIABILITY OF STRUCTURES UNDER COMBINED RANDOM LOAD SEQUENCES

ABSTRACT

The reliability of structures under combined load sequences of the Ferry Borges-Casthaneta type is considered. The first order approximation is derived by consequent application of the Rosenblatt transformation. It allows us to find the estimate of the failure probability and the design point (β-point) for any reliability problem of structural systems and to include the load combination problem in numerical algorithms which are mostly restricted to time-invariant cases. Since the error of the first order approximation is in general unknown it is important to have a method which allows us to verify the approximate results whenever it is necessary. The importance sampling technique gives such a possibility in time-invariant reliability problems. This technique is applied in the paper for the load combination problems. It is shown that the generalization leads to the effective numerical simulation. In simple examples the results and computational effort of the proposed importance sampling are compared with the crude Monte Carlo simulation.

1. INTRODUCTION

In many problems of structural reliability analysis the quantities which enter the limit conditions vary randomly in time and/or space. One of the simplest models of the random temporal behaviour of actions is the Ferry Borges-Castanheta (FBC) action effect model [1]. Within the model the single action history is represented by a sequence of rectangular pulses of constant deterministic durations following immediately after each other. The pulse amplitudes for the same action history are independent and identically distributed

random variables. Such a sequence may also be described by the Bernoulli sequence of random variables.

An approximate method of reliability calculation for structures subjected to a number of actions modelled by the FBC processes was proposed by R. Rackwitz and B. Fissler in [2]. The authors there subsequently applied the "principle of normal tail approximation" extensively discussed in [3], say. The approach allowed to replace the impracticable, computation of multifold convolution integral which would be necessary in failure probability calculation with the method which is now commonly being used in structural reliability analysis and is called the Fist Order Reliability Method (FORM).

The Rackwitz-Fissler algorithm (RF) appeared to be very useful in the approach proposed recently by Ditlevsen et al. in [4]. The authors combined that approach with the directional Monte Carlo simulation technique and worked out an efficient method to calculate the confidence intervals for the values of the both probability distribution and density functions of a random variable defined as a sum of clipped dependent or independent normal random variables.

The present approximate method of reliability calculation under combined random load sequences is based on the arguments given in [2]. The only difference is that the sequence of the "normal tail approximations" is consequently replaced by an optimization problem. It seems to offer some new possibility to apply this kind of approximation in numerical algorithms in which the reliability calculation is based rather on searching for the minimum of Euclidean distance between the limit surface and the origin in an appropriate space than on the "principle of normal tail approximation".

Finally, it is shown how the importance sampling technique can be applied to verify the approximate results.

II. FIRST ORDER APPROXIMATION

The typical formulation of the reliability problem is to find the probability, P_F, that a function of random variables, $g(\underset{\sim}{X})$, is positive. This probability denotes the failure probability of a structure. The function $g(\underset{\sim}{x})=0$ defines the boundary between the failure domain, $D_F=(\underset{\sim}{x}: g(\underset{\sim}{x})>0)$, and the safe domain, $D_S=(\underset{\sim}{x}: g(\underset{\sim}{x})\leq 0)$, in the space X of basic variables, $\underset{\sim}{X}=[X_1,X_2,\ldots,X_n]$. Since the basic variables are random quantities the structural failure and not failure (survival) are the random events with respective probabilities:

$$P_F = P[g(\underset{\sim}{X}) > 0] \qquad (a)$$

$$P_S = 1 - P_F = P[g(\underset{\sim}{X}) \leq 0] \qquad (b) \qquad (1)$$

The usual way of reliability calculation is to transform the problem into a new space, Y, where the probability measure is defined by independent, standard, Gaussian random variables, Y_i, $i=1,2,\ldots,n$. Such a transformation, $\underset{\sim}{Y}=\underset{\sim}{T}(\underset{\sim}{X})$, taking its origin in [5] is called the Rosenblatt transformation and has the following form:

$$\begin{aligned}
\Phi(y_1) &= F_{X_1}(x_1) \\
\Phi(y_2) &= F_{X_2|X_1}(x_2|x_1) \\
&\cdots\cdots\cdots\cdots\cdots\cdots \\
\Phi(y_n) &= F_{X_n|X_1,X_2,\ldots,X_{n-1}}(x_n|x_1,x_2,\ldots,x_{n-1})
\end{aligned} \qquad (2)$$

where $\Phi(\cdot)$ denotes the standard normal probability distribution

$$\Phi(z) = \int_{-\infty}^{z} \varphi(t)\,dt = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} \exp\left(-\frac{t^2}{2}\right) dt$$

and $F_{X_i|X_1X_2\ldots X_{i-1}}(x_i|x_1,x_2,\ldots,x_{n-1})$ denotes the conditional probability distribution of random variable X_i given X_1

$=x_1$, $X_2=x_2,\ldots,X_{n-1}=x_{n-1}$.

The limit condition, $g(\underset{\sim}{x})=0$, is also transformed into the space Y and the failure probability (1) can be equivalently determined as

$$P_F = P[g(\underset{\sim}{T}^{-1}(\underset{\sim}{Y})) > 0] = P[h(\underset{\sim}{Y}) > 0] \tag{3}$$

where $h(\underset{\sim}{y})=0$ denotes the transformed limit condition in the space Y and $\underset{\sim}{Y}=[Y_1,Y_2,..,Y_n]$ is the random Gaussian vector with standard independent Gaussian components, Y_i, $i=1, 2,..,n$.

The first order approximation of (3) (and of (1) as well) is the probability measure contained in the approximate failure domain $\Delta_F^*=(\underset{\sim}{y}: l^*(\underset{\sim}{y}) > 0)$, where $l^*(\underset{\sim}{y})=\underset{\sim}{\alpha}^{*T}\underset{\sim}{y}-\beta^*=0$ denotes a hyperplane in Y. The hyperplane is tangent to the transformed limit surface $h(\underset{\sim}{y})=0$ in the point $\underset{\sim}{y}^*$ which is the closest one to the origin. $\underset{\sim}{\alpha}^*$ is the vector of directional cosines of the hyperplane in $\underset{\sim}{y}^*$. $|\beta|=\delta^*$ denotes the distance between the surface $h(\underset{\sim}{y})=0$ and the origin. The sign of β depends on the safe set position with respect to the origin, i.e.

$$\underset{\sim}{\alpha}^* = \frac{\underset{\sim}{H}_y^*}{(\underset{\sim}{H}_y^{*T}\cdot\underset{\sim}{H}_y^*)^{1/2}} \tag{4}$$

$$\beta = l\cdot\delta^*$$

where $\underset{\sim}{H}_y^*$ denotes the gradient of the function $h(\underset{\sim}{y})=0$ in $\underset{\sim}{y}=\underset{\sim}{y}^*$

$$\underset{\sim}{H}_y^{*T} = \left[\frac{\partial h}{\partial y_1}, \frac{\partial h}{\partial y_2}, \ldots, \frac{\partial h}{\partial y_n}\right]_{\underset{\sim}{y}=\underset{\sim}{y}^*} \tag{5}$$

$$\delta^* = |\underset{\sim}{\alpha}^{*T}\cdot\underset{\sim}{y}^*|$$

and l determines the sign of β as follows

$$l = -\text{sign}[h(\underset{\sim}{0})] \tag{6}$$

It means that β is positive if the origin belongs to the safety domain. Otherwise, it is negative.

Hence, the first order approximation of the failure

probability is obtained in the following simple form

$$P_F \approx P[l^*(\underset{\sim}{Y}) > 0] = P[\underset{\sim}{\alpha}^{*T} \cdot \underset{\sim}{Y} > \beta] = \Phi(-\beta) \tag{7}$$

In order to find the shortest distance δ^* the following optimization problem with a constrain has to be solved

$$\delta^{*2} = \min\ (\underset{\sim}{\chi}^T \cdot \underset{\sim}{\chi}) \tag{8}$$

provided that $h(\underset{\sim}{\chi})=0$ or equivalently

$$\delta^{*2} = \min\ [\underset{\sim}{T}^T(\underset{\sim}{x}) \cdot \underset{\sim}{T}(\underset{\sim}{x})] \tag{9}$$

provided that $g(\underset{\sim}{x})=0$.

III. RANDOM SEQUENCES AS BASIC VARIABLES

Let us assume that some basic variables represent the time-varying actions which a structure is subjected to. They are modelled by sequences of identically distributed independent random pulses. Let us denote these basic variables by $R_j(t)$, $j=1,2,..,m$. The pulse durations, τ_j, are constant and deterministic for every sequence $R_j(t)$ and the sequences are ordered according to decreasing pulse durations, $\tau_j > \tau_{j+1}$, while τ_i is an integral multiple of τ_j for any $i<j$. The case of groups of sequences of the same pulse durations will be discussed further on.

Let us define the time T, called the structural lifetime, at which the failure probability is to be calculated. Thus, for the given lifetime the pulse sequence time characteristics may be alternatively described by repetition numbers, $r_j=T/\tau_j$, or by relative repetition numbers, $\rho_j = \tau_j/\tau_{j-1}$, while $\tau_0=T$.

Let us apply the Rosenblatt transformation, Eqs.(2), to all basic variables (pulse amplitudes R_j as well), $\underset{\sim}{x} = \underset{\sim}{T}_X^{-1}(\underset{\sim}{Y})$ and $\underset{\sim}{r} = \underset{\sim}{T}_R^{-1}(\underset{\sim}{s})$, $i=1,2,...,n$ and $j=1,2,...,m$. The failure probability (3) takes the form

$$P_F = 1 - P_S = 1 - P\Big[\max_{t\in[0,T]} h(\underset{\sim}{Y},\underset{\sim}{S};t) \le 0\Big] \tag{10}$$

where the argument t indicates the variability of the limit condition in time. Following the first order approximation concept the limit condition, $h(\underset{\sim}{Y},\underset{\sim}{S})=0$, is linearized in a point $(\underset{\sim}{Y},\underset{\sim}{S})=(\underset{\sim}{Y}^o,\underset{\sim}{S}^o)$ and the approximation takes the form

$$\max_{t\in[0,T]} h(\underset{\sim}{Y},\underset{\sim}{S};t) \approx \max_{t\in[0,T]} l^o(\underset{\sim}{Y},\underset{\sim}{S};t) =$$

$$= \underset{\sim}{H}_y^{oT}\cdot\underset{\sim}{Y} - \underset{\sim}{H}_y^{oT}\cdot\underset{\sim}{Y}^o + \max_{t\in[0,T]}\Big[\underset{\sim}{H}_s^{oT}\cdot\underset{\sim}{S}(t) - \underset{\sim}{H}_s^{oT}\cdot\underset{\sim}{S}^o)\Big] \tag{11}$$

$\underset{\sim}{H}_y^o$ and $\underset{\sim}{H}_s^o$ denote the vectors whose components are the first derivatives of the transformed limit condition, $h(\underset{\sim}{Y},\underset{\sim}{S})=0$, in the point $(\underset{\sim}{Y}^o,\underset{\sim}{S}^o)$ with respect to y_i and s_j, $i=1,2,\ldots,n$ and $j=1,2,\ldots,m$, respectively.

Random maximum of the approximation (11) within the time interval $[0,T]$ can be expressed in the form given in [2], i.e.

$$\max_{t\in[0,T]} l^o(Y,S;t) = \underset{\sim}{H}_y^{oT}\cdot\underset{\sim}{Y} - \underset{\sim}{H}_y^{oT}\cdot\underset{\sim}{Y}^o +$$

$$+ \max\Big\{\max_{\rho_1}\cdots\max_{\rho_{m-2}}\Big(\max_{\rho_{m-1}}\big[\max_{\rho_m}(h_m^o s_m) + h_{m-1}^o s_{m-1}\big] + h_{m-2}^o s_{m-2}\Big) + \cdots$$

$$+ \cdots + h_2^o s_2\Big) + h_1^o s_1\Big\} - \underset{\sim}{H}_s^{oT}\cdot\underset{\sim}{S}^o \tag{12}$$

where $\max_{\rho_j}[\cdot + \cdot]$ denotes that the maximum is to be found from ρ_j respective random variables and h_j^o is the j-th component of the vector $\underset{\sim}{H}_s^o$. Eq.(12) can be alternatively written as

$$\max_{t\in[0,T]} l^o(\underset{\sim}{Y},\underset{\sim}{S};t) = \underset{\sim}{H}_y^{oT}\cdot\underset{\sim}{Y} - \underset{\sim}{H}_y^{oT}\cdot\underset{\sim}{Y}^o + s_1^{o+} - \underset{\sim}{H}_s^{oT}\cdot\underset{\sim}{S}^o =$$

$$= l^{o+}(\underset{\sim}{Y},s_1^{o+}) \tag{13}$$

where the random variable s_1^{o+} results from the following sequence of maxima

$$s_m^{o+} = \max_{\rho_m} (h_m^o s_m)$$

$$s_{m-1}^{o+} = \max_{\rho_{m-1}} [s_m^{o+} + h_{m-1}^o s_{m-1}] \qquad (14)$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

$$s_1^{o+} = \max_{\rho_1} [s_2^{o+} + h_1^o s_1]$$

All Y_i in (13), $i=1,2,\ldots,n$, are standard normal independent random variables. Thus, the random variable S_1^{o+} has only to be transformed into the standard normal one, Ξ_1, i.e.

$$\Phi(\xi_1^o) = F_{S_1^{o+}}(s_1^{o+}) \qquad (16)$$

where $F_{S_1^{o+}}(\cdot)$ denotes the probability distribution of S_1^{o+}. In order to calculate the first order approximation of the load combination problem (10) it is necessary to find the best linearization point $(\underset{\sim}{\chi}^*, \underset{\sim}{\xi}^*)$ of the function $h(\underset{\sim}{\chi}, \underset{\sim}{\xi}) = 0$. For this linearization point the distance between the counterpart, $(\underset{\sim}{\chi}^*, \xi_1^*)$, of the point $(\underset{\sim}{\chi}^*, \underset{\sim}{\xi}^*)$ in the reduced transformed coordinate system $(\underset{\sim}{\chi}, \xi_1)$ and the origin of this system is the shortest one, where

$$\xi_1^* = \Phi^{-1}\left[F_{S_1^{*+}}(s_1^{*+})\right] \qquad (17)$$

$$s_1^{*+} = \sum_{j=1}^{m} h_j^* s_j^* \qquad (18)$$

Now, the usual optimization problem has to be solved

$$(\delta^*)^2 = \sum_{i=1}^{n} (y_i^*)^2 + (\xi_1^*)^2 = \min_{(K)} \left[\sum_{i=1}^{n} (y_i^{(K)})^2 + (\xi_1^{(K)})^2\right] \qquad (19)$$

provided that $h(\underset{\sim}{\chi}^{(K)}, \underset{\sim}{\xi}^{(K)}) = 0$, where the superscript (K) denotes the (K)-th iteration step of the optimization procedure. The solution gives an approximation of the failure probability P_F

$$P_F \approx \Phi(-\beta^*) \quad \text{with } \beta^* = -\operatorname{sign}\left\{1^{**}[\underset{\sim}{0}, F_{S_1^*}^{-1}(\tfrac{1}{2})]\right\} \cdot \delta^* \qquad (20)$$

In order to make the algorithm efficient it is necessary to determine the probability distribution of $S_f^{(K)+}$ for every iteration step. In the next chapter the solution of this problem is proposed in terms of subsequent application of the Rosenblatt transformation instead of the normal tail approximation approach which has been proposed in [2].

IV. PROBABILITY DISTRIBUTION OF THE MAXIMUM S_1^{o+}

It is easily seen that the rigorous determination of the probability distribution of the random maximum $S_f^{(K)+}$ of a sum of random sequences $S_j(t)$, $j=1,2,\ldots,m$, requires a subsequent application of convolution integrals. Since it becomes impracticable in problems where the number of sequences exceeds three an approximate procedure has to be employed to efficiently estimate the failure of structures subjected to time-varying load sequences. The procedure given in the paper takes its origin in the first order approximation method.

Let us assume a point $(\underset{\sim}{y}^o, \underset{\sim}{s}^o)$ to be an extension point of the linear approximation, $l^o(\underset{\sim}{y}, \underset{\sim}{s})=0$, of the limit condition, $h(\underset{\sim}{y}, \underset{\sim}{s})=0$. Random pulse amplitudes, S_j, of all sequences, $S_j(t)$, $j=1,2,\ldots,m$, are independent standard normal random variables. The probability distribution value of maximum of a number ρ_m of the pulse amplitudes S_m, i.e.

$$S_m^{o+} = \max_{\rho_m} S_m^{os} = \max_{\rho_m} (h_m^o S_m) \qquad (21)$$

in the point $s_m^{o+}=n_m^o s_m^o$ is simply defined as

$$F_{S_m^{o+}}(s_m^{o+}) = P[S_m^{o+} \le s_m^{o+}] = P^{\rho_m}[S_m^{os} \le s_m^{o+}] = P^{\rho_m}[h_m^o S_m \le s_m^{o+}] = \qquad (22)$$

$$= \Phi^{\rho_m}\left(\frac{s_m^{o+}}{h_m^o}\right)$$

Following the sequence of equations (14) the next random variable is

$$s_{m-1}^{o+} = \max_{\rho_{m-1}} s_{m-1}^{os} = \max_{\rho_{m-1}} [s_m^{o+} + h_{m-1}^o s_{m-1}] \tag{23}$$

All ρ_{m-1} random variables $s_m^o s_1$ are independent of each other. Thus, the probability distribution value of their maximum in the point $s_{m-1}^o = s_m^{o+} + h_{m-1}^o s_{m-1}^o = h_m^o s_m^o + h_{m-1}^o s_{m-1}^o$ is defined as

$$F_{S_{m-1}^{o+}}(s_{m-1}^{o+}) = P[S_{m-1}^{o+} \leq s_{m-1}^{o+}] = P^{\rho_{m-1}}[S_{m-1}^{os} \leq s_{m-1}^{o+}] =$$

$$= P^{\rho_{m-1}}[S_m^{o+} + h_{m-1}^o s_{m-1} \leq s_{m-1}^{o+}] \tag{24}$$

s_{m-1} is the standard normal random variable. Thus, the random variable s_m^{o+} is only transformed in the current iteration point s_m^{o+}

$$\Phi(\xi_m^o) = F_{S_m^{o+}}(s_m^{o+}) = \Phi^{\rho_m}\left(\frac{s_m^{o+}}{h_m^o}\right) \tag{25}$$

It leads to the following approximation of the probability distribution value (24)

$$F_{S_{m-1}^{o+}}(s_{m-1}^{o+}) \approx \Phi^{\rho_{m-1}}(l_{m-1}^o \cdot \delta_{m-1}^o) \tag{26}$$

where δ_{m-1}^o denotes the distance between the point (ξ_m^o, s_{m-1}^o) and the origin of the coordinate system (ξ_m, s_{m-1}), i.e.

$$(\delta_{m-1}^o)^2 = (\xi_m^o)^2 + (s_{m-1}^o)^2 \tag{27}$$

It should be noted that the value ξ_m^o is determined from the transformation (25) for $s_m = s_m^o$, i.e.

$$\xi_m^o = \Phi^{-1}\left[\Phi^{\rho_m}\left(\frac{s_m^{o+}}{h_m^o}\right)\right] \tag{28}$$

The sign parameter, I_{m-1}^o, is determined as usually from

$$I_{m-1}^o = -\mathrm{sign}\left\{h_m^o \cdot \Phi^{-1}\left[\left(\tfrac{1}{2}\right)^{1/\rho_m}\right] - s_{m-1}^{o+}\right\} \tag{29}$$

We replace the subscripts (m-1) with j and m with (j+1), respectively, to obtain the whole sequence of approximations for all random variables from the sequence (14). The approximation of the probability distribution function of S_1^{o+} in the point

$$s_1^{o+} = \sum_{j=1}^{m} h_j^o s_j^o \tag{30}$$

eventually takes the following form

$$F_{S_1^{o+}}(s_1^{o+}) \approx \Phi^{\rho_1}(I_1^o \delta_1^o) \tag{31}$$

where

$$\begin{aligned}
(\delta_1^o)^2 &= \left\{\Phi^{-1}\left[\Phi^{\rho_2}(I_2^o \delta_2^o)\right]\right\}^2 + (s_1^o)^2 \\
&\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\
(\delta_j^o)^2 &= \left\{\Phi^{-1}\left[\Phi^{\rho_{j+1}}(I_{j+1}^o \delta_{j+1}^o)\right]\right\}^2 + (s_j^o)^2
\end{aligned} \tag{32}$$

for all j=1,2,..,(m-1)

and

$$I_j^o = -\mathrm{sign}\left\{\Phi^{-1}\left[\left(\tfrac{1}{2}\right)^{1/\rho_{j+1}}\right] - s_j^{o+}\right\} \quad \text{for } j=1,2,..,(m-1) \tag{33}$$

while

$$I_m^o \delta_m^o = s_m^o \tag{34}$$

is put into the last term in the sequence (32) for j=m-1.

Applying the approximation (31) to the transformation (16) we are able to estimate the failure probability, P_F, for any expansion point $(\underline{x}^{(k)}, \underline{s}^{(k)})$. Therefore, the optimization problem (19) which leads to the best first order ap-

proximation of the failure probability can be solved using any algorithm of reliability calculation in which the Rosenblatt transformation and the search of the minimum distance δ are applied.

It is easily seen that the approach can be staightforwardly generalized for groups of sequences of the same pulse duration. If there are m_j sequences in the j-th group then the values of s_j^{o2} in the respective expressions containing $(s_j^o)^2$ and s_j^{o+}, e.g. Eqs.(32) and (33), should be replaced with the sums

$$(s_j^o)^2 = \sum_{k=1}^{m_j} (s_{jk}^o)^2 \tag{35}$$

$$s_j^{o+} = \sum_{k=1}^{m_j} h_{jk}^o s_{jk}^o \tag{36}$$

where the couple subscript jk denotes the k-th sequence from the j-th group of sequences of the same pulse duration τ_j .

V. IMPORTANCE SAMPLING VERIFICATION

The importance sampling techniques are still calling an increasing attention as an interesting and very efficient method that can be applied to verify approximate solutions of time-independent reliability problems, e.g. [6÷9]. Just these techniques where the β-point is assumed as a centre of importance sampling probability distribution allowed to combine the advantages of the first order reliability method and the Monte Carlo simulation. This approach drastically reduces the variance of the failure probability estimator and gives an acceptable confidence interval of it even from a few dozen samples produced appropriately by the Monte Carlo simulation. The similar concept where a maximum of an integrated function is assumed as the centre of importance sampling probability distribution is investigated in estimation of mean outcrossing rate of stochastic processes and

fields, e.g. [10]. The first results confirm the usefulness and efficiency of this approach.

In this chapter we show how the importance sampling method can be used to verify the approximate results from the First Order Reliability Method when the actions are modelled by the FBC random load sequences, $R_j(t)$, $j=1,2,..,m$. We assume different pulse durations, τ_j, for every sequence and some ordering of them, $\tau_j > \tau_{j+1}$. It will be easily seen how to generalize the approach for groups of sequences of the same pulse durations.

We firstly assume that the original problem

$$P_F = 1 - P_S = 1 - P\Big[\max_{t\in[0,T]} g(\underset{\sim}{X},\underset{\sim}{R};t) \le 0\Big] \tag{37}$$

where $\underset{\sim}{X}$ and $\underset{\sim}{R}(t)$ are the vector of time-independent random variables and the vector of load sequences, respectively, has been transformed according to the Rosenblatt transformation (2) into the space of independent standard normal random variables, Y_i and $S_{j,1},\ldots,S_{j,r_j}$, which are the counterparts of the components X_i, $i=1,2,\ldots,n$, and of all pulse amplitudes, $R_{j,1},\ldots,R_{j,r_j}$ from every sequence, $j=1,2,\ldots,m$, within a time interval $[0,T]$, respectively. The random variables Y_i, $i=1,2,\ldots,n$, and the pulse amplitudes $S_{j,1},\ldots,S_{j,r_j}$, $j=1,2,\ldots,m$, are in a natural way grouped together in $(n+m)$-dimensional vectors, $(\underset{\sim}{Y},\underset{\sim}{S}^q)$, which for every $q=1,2,\ldots,r_m$ represent a combination of the random variables appropriate to the q-th time interval $[(q-1)\cdot\tau_m,\ q\cdot\tau_m]$ within $[0,T]$. In order to relate the serial number of the pulse amplitude from the j-th load sequence to the serial number of the load combination, q say, we introduce an integer function

$$\pi_j(\underset{\sim}{K}) = k_j + r_j \sum_{l=1}^{j-1} \frac{k_l - 1}{r_l} \tag{38}$$

with $k_l = 1,2,\ldots,p_l$ for every $l = 1,2,\ldots,j$.

The vector $\underset{\sim}{K}=\underset{\sim}{K}^q=[K_1^q, K_2^q, \ldots, K_m^q]$ uniquely determines the serial number, $q=\pi_m(\underset{\sim}{K}^q)$, of the load pulse amplitude combination within the shortest pulse duration, $[(q-1)\cdot\tau_m, q\cdot\tau_m]$.

For every consecutive combination, $q=1,2,\ldots,r_m$, the survival domain for the structure can be defined in $(n+r_1+\ldots+r_m)$-dimensional space of the vectors $\underset{\sim}{Y}$ and all pulse amplitudes, $S_{j,1},\ldots,S_{j,r_j}$, of all load component histories, $j=1,2,\ldots,m$, as follows

$$D_S^q = \{(\underset{\sim}{y}, s_{1,1},\ldots,s_{1,r_1},\ldots,s_{m,1},\ldots,s_{m,r_m}): h(\underset{\sim}{y},\underset{\sim}{s}^q) \geq 0\} \quad (39)$$

where

$$\underset{\sim}{s}^q = \left[s_{1,\pi_1(\underset{\sim}{K}^q)}, s_{2,\pi_2(\underset{\sim}{K}^q)}, \ldots, s_{m,\pi_m(\underset{\sim}{K}^q)}\right] \quad (40)$$

while $\underset{\sim}{K}^q$ and q are related to each other by the function (38) for $j=m$, i.e. $q=\pi_m(\underset{\sim}{K}^q)$.

The complete survival domain, D_S, in the $(n+r_1+r_2+\ldots+r_m)$-dimensional space is given by an intersection over all domains (39), i.e.

$$D_S = \bigcap_{q=1}^{r_m} D_S^q \quad (41)$$

The survival probability from (37) can be expressed in the integral form in this space as follows

$$P_S = \int\cdots\int_{D_S} \prod_{i=1}^{n} \varphi(y_i) \sum_{K_1=1}^{\rho_1}\cdots\sum_{K_m=1}^{\rho_m} \varphi(s_{1,\pi_1(\underset{\sim}{K})})\cdots \quad (42)$$

$$\cdots \varphi(s_{m,\pi_m(\underset{\sim}{K})})\, dy_1\cdots dy_n\, ds_{1,1}\cdots ds_{1,r_1}\cdots ds_{m,1}\cdots ds_{m,r_m}$$

In the crude Monte Carlo method the integral (42) or the similar one appropriately expressed in the space of the original basic variables, $(\underset{\sim}{x}, R_{1,1},\ldots,R_{1,r_1},\ldots,R_{m,1},\ldots,R_{m,r_m})$, is the basis of numerical calculations. The samples $(\underset{\sim}{y}, s_{1,1},\ldots,s_{1,r_1},\ldots,s_{m,1},\ldots,s_{m,r_m})$ or $(\underset{\sim}{x}, r_{1,1},\ldots,r_{1,r_1},\ldots,r_{m,1},\ldots,r_{m,r_m})$ are generated from the respective probability distributions

and put as arguments into the limit state functions, $h(\underset{\sim}{y},\underset{\sim}{s}^q)$ or $g(\underset{\sim}{x},\underset{\sim}{r}^q)$, to check the survival condition for every $q=1, 2,\ldots,r_m$. If the survival condition is satisfied for every q then the sequence is the safe one and it is considered as a "success". The number of "successes" divided by the total number of samples gives a statistical estimate of the survival probability. It is well-known that convergence of the crude Monte Carlo method is very slow in practice where the interesting failure probabilities are small. In order to improve the efficiency of the estimation there have been recently proposed some methods in which the variance of the estimator is drastically reduced making the method applicable whenever the verification of approximate results is needed. The estimator variance reduction methods have been developed to estimate the failure probabilities in time invariant problems. In the following we propose an approach which generalizes the importance sampling technique used in the time invariant problems to the problems where the random load effect sequences of the Ferry-Borges-Castanheta type are considered to be basic variables.

In order to make the approach more effective we take into account the information about β-point. This point determines a neighbourhood which contains the main part of the failure probability measure. Therefore it is reasonable there to concentrate the samples which are generated and eventually used in probability estimation.

Let us rotate the n-dimensional coordinate subsystem given by the respective coordinates, $y_1, y_2, \ldots, y_n$, so that the n-th new coordinate, v_n, of the new coordinate subsystem given by $(v_1, v_2, \ldots, v_n)$ is parallel to the vector given by the following directional cosines in the (n+m)-dimensional space, $(\underset{\sim}{y},\underset{\sim}{s}^q)$, which can be defined for every pulse combination

$$\underset{\sim}{\alpha}_v = \begin{bmatrix} \underset{\sim}{\alpha}_Y^* \\ \underset{\sim}{0} \end{bmatrix} \qquad (43)$$

where the vector $\underset{\sim}{\alpha}^*_Y$ denotes the normalized projection of the gradient of $h(\underset{\sim}{\chi},\underset{\sim}{s})=0$ in the β-point, $(\underset{\sim}{\chi}^*,\underset{\sim}{s}^*)$, on the n-dimensional subspace defined by the coordinates $y_1, y_2, \ldots, y_n$, i.e.

$$\alpha^*_{Y,i} = \left[\frac{\partial h(\underset{\sim}{\chi},\underset{\sim}{s})}{\partial y_i}\right] \cdot \left\{ \sum_{i=1}^{n} \left[\frac{\partial h(\underset{\sim}{\chi},\underset{\sim}{s})}{\partial y_i}\right]^2 \right\}^{-1} \Bigg|_{(\underset{\sim}{\chi},\underset{\sim}{s})=(\underset{\sim}{\chi}^*,\underset{\sim}{s}^*)} \tag{44}$$

Moreover, the rotation should assure the mixed second order derivatives with respect to coordinates v_i and v_j, $j \neq i$, of the transformed limit condition $v_n = f(\underset{\sim}{v},\underset{\sim}{s})$ to vanish in the β-point. The similar transformation is used in time invariance reliability problems in SORM and importance sampling [8]. In the present case it can be written as

$$\begin{bmatrix} \underset{\sim}{v} \\ v_n \\ \underset{\sim}{s} \end{bmatrix} = \underset{\sim}{C} \cdot \begin{bmatrix} \underset{\sim}{\chi} \\ \underset{\sim}{s} \end{bmatrix} \tag{45}$$

where $\underset{\sim}{C}$ denotes the orthogonal transformation matrix which assures the conditions given above to be satisfied.

In the new coordinate system the survival domain for the q-th load combination can be expressed in the following form

$$D_S^q = \{(\underset{\sim}{v}, v_n, s_{1,1}, \ldots, s_{1,r_1}, \ldots, s_{m,1}, \ldots, s_{m,r_m}) : v_n \leq f(\underset{\sim}{v},\underset{\sim}{s}^q)\} \tag{46}$$

Since the rotation does not change the probability distribution in the whole $(n+r_1+r_2+\cdots+r_m)$-dimensional space and the random variables, $\underset{\sim}{V}, V_n, S_{1,1}, \ldots, S_{1,r_1}, \ldots, S_{m,1}, \ldots, S_{m,r_m}$, remain independent, standard and normal the survival probability can be written in the new coordinate system as follows

$$P_S = \int_{\bigwedge_{q=1}^{r_m} v_n \leq f(\underset{\sim}{v},\underset{\sim}{s}^q)} \cdots\cdots \varphi(v_n) \cdot \prod_{i=1}^{n-1} \varphi(v_i) \cdot \sum_{k_1=1}^{\rho_1} \cdots \sum_{k_m=1}^{\rho_m} \varphi(s_{1,\pi_1(\underset{\sim}{K})}) \cdots \tag{47a}$$

$$\cdots \varphi(s_{m,\pi_m(\underset{\sim}{K})})\,dv_1\cdots dv_n\,ds_{1,1}\cdots ds_{1,r_1}\cdots ds_{m,1}\cdots ds_{m,r_m} =$$

$$= \int\limits_{R_m^{(n-1+r_1+\cdots+r_m)}} \cdots\cdots\cdots\cdots \Phi[\min_{q=1}^{r_m} f(\underset{\sim}{y},\underset{\sim}{s}_n^q)] \cdot \prod_{i=1}^{n-1} \varphi(v_i) \cdot \sum_{K_1=1}^{\rho_1}\cdots\sum_{K_m=1}^{\rho_m} \varphi(s_{1,\pi_1(\underset{\sim}{K})}) \tag{47b}$$

$$\cdots \varphi(s_{m,\pi_m(\underset{\sim}{K})})\,dv_1\cdots dv_{n-1}\,ds_{1,1}\cdots ds_{1,r_1}\cdots ds_{m,1}\cdots ds_{m,r_m}$$

Eq.(47b) has similar form to the integrals which are usually defined in the importance sampling approach to estimate the failure or survival probability by effective simulation. It defines the survival probability as the mean

$$P_S = E\left[\Phi\left[\min_{q=1}^{r_m} f(\underset{\sim}{y},\underset{\sim}{s}^q)\right]\right] \tag{48}$$

with respect to the independent, standard, normal random variables $v_1,\ldots,v_{n-1},s_{1,1},\ldots,s_{1,r_1},\ldots,s_{m,1},\ldots,s_{m,r_m}$.

The estimator, $\tilde{P}_S$, of the survival probability can be written as

$$\tilde{P}_S = \frac{1}{N}\sum_{K=1}^{N} \Phi\left[\min_{q=1}^{r_m} f(\underset{\sim}{y}^K,\underset{\sim}{s}^{q,K})\right] \tag{49}$$

where $\underset{\sim}{y}^K$ and $\underset{\sim}{s}^{q,K}$ are the samples of the random vector $\underset{\sim}{y}$ and the random load effects which enter the q-th load combination, respectively. The efficiency of the estimator can be significantly improved by taking into account the curvature of the limit condition in the β-point, [9], [10]. In the case of the limit conditions $v_n=f(\underset{\sim}{y},\underset{\sim}{s}^q)$, $q=1,2,\ldots,r_m$, the curvature correction of the importance sampling probability distributions can be introduced for the random variables V_i, $i=1,2,\ldots,(n-1)$. Additionally, in order to concentrate the samples of $\underset{\sim}{y}$ about the β-point the projection of that point, $\underset{\sim}{w}$, on the hyperplane $v_n=0$ is taken as the centre of the importance sampling distribution for V_i, $i=1,2,\ldots,(n-1)$. All of this leads to the more efficient estimator in the following form

$$\tilde{P}_S = \frac{1}{N} \sum_{k=1}^{N} \Phi\left[\min_{q=1}^{r_m} f(\underset{\sim}{w}^k, \underset{\sim}{s}^{q,k})\right] \cdot \frac{\varphi(\underset{\sim}{w}^k)}{\prod_{i=1}^{n-1} \frac{1}{\sigma_i} \varphi\left[\frac{w_i^k - \bar{w}_i}{\sigma_i}\right]} \tag{50}$$

where the samples w_i^k are generated from the normal probability distribution with the mean $\bar{w}_i$ and variance

$$\sigma_i^2 = \left[1 + \frac{\kappa_i \Psi(-\beta_Y)}{1 + \kappa_i \omega}\right]^{-1} \tag{51}$$

which depends on the main curvature, κ_i, of the limit condition in the β-point and on the projection length, β_Y, of the β-distance in the n-dimensional subspace given by y_i's, i.e.

$$\beta_Y^2 = \sum_{i=1}^{n} (y_i^*)^2 \tag{52}$$

ω is the switching coefficient depending on the curvature κ_i and β_Y, i.e.

$$\omega = \begin{cases} 0 & \text{for } \kappa_i \geq 0 \\ \beta_Y - \dfrac{\Psi(-\beta_Y)}{\beta_Y}(1 + \beta_Y) & \text{for } -\dfrac{1}{\beta_Y} \leq \kappa_i \leq 0 \end{cases} \tag{53}$$

while

$$\Psi(-\beta) = \frac{\varphi(\beta)}{\Phi(-\beta)} \tag{54}$$

The value of the estimator (50) for N independent samples of $\underset{\sim}{w}$ and of the load history components $s_{j,1}, \ldots, s_{j,r_j}$, $j=1,2,\ldots,m$, gives an estimate of the survival probability (48). The quality of the estimate can be easily controlled by variation coefficient and confidence interval of the estimator. It allows to terminate the simulation for such a sample size which assures the required quality.

VI. SIMPLIFIED IMPORTANCE SAMPLING APPROACH

The importance sampling as it is defined by the estimator (50) requires the calculation of r_m function values $f(\underset{\sim}{w}^K, \underset{\sim}{s}^{q,K})$, $q=1,2,\ldots,r_m$, for every sample of the load combination history $(\underset{\sim}{w}^K, s^K_{j,1},\ldots,s^K_{j,r_j}; j=1,2,\ldots,m)$, $K = 1, 2,\ldots,N$. Unfortunately, the function is usually not defined explicitly. The Rosenblatt transformation (2), the rotations of the coordinate system (45) and, especially, a zero search algorithm to find the function values sequentially lead from the original limit condition $g(\underset{\sim}{x}, \underset{\sim}{r}^q)=0$ through the transformed one $h(\underset{\sim}{y}, \underset{\sim}{s}^q)=0$ and, eventually, to the functions $f(\underset{\sim}{w}, \underset{\sim}{s}^q)$. All this can make the calculation of the $\min_{q=1}^{r_m} f(\underset{\sim}{w}, \underset{\sim}{s}^q)$ so time consuming that the method becomes inefficient in solution of practical problems. Therefore, it seems to be important to propose an approach which saves the computational time without losing too much of valuable features of the exact importance sampling.

It is seen that for every sample $(\underset{\sim}{w}^K, s^K_{j,1},\ldots,s^K_{j,r_j}; j=1,2,\ldots,m)$, $K=1,2,\ldots,N$, it is necessary to determine such a serial number q_K of the load combination $(\underset{\sim}{w}^K, \underset{\sim}{s}^{q_K,K})$ for which

$$f(\underset{\sim}{w}^K, \underset{\sim}{s}^{q_K,K}) = \min_{q=1}^{r_m} [f(\underset{\sim}{w}^K, \underset{\sim}{s}^{q,K})] \tag{55}$$

In order to avoid multiple computation of the numerically given functions $f(\underset{\sim}{w}^K, \underset{\sim}{s}^{q,K})$, $q=1,2,\ldots,r_m$, we substitute them with simple analytical approximations, $\tilde{f}(\underset{\sim}{w}^K, \underset{\sim}{s}^{q,K})$, and then find the relevant combination $(\underset{\sim}{w}^K, \underset{\sim}{s}^{q_K,K})$. Moreover, it is important to notice that the samples $\underset{\sim}{w}^K$ are mostly concentrated around the mean $\underset{\sim}{w}$ and the samples $s_{j,1},\ldots,s_{j,r_j}$, cover most likely the range $(-s^*_j, s^*_j)$, $j=1,2,\ldots,r_j$. Thus, the functions $f(\underset{\sim}{w}^K, \underset{\sim}{s}^{q,K})$ should be well approximated in the neighbourhood of the mean $\underset{\sim}{w}$ and in the ranges $(-s^*_j, s^*_j)$, $j=1, 2,\ldots,r_j$. It is proposed to use a parabolic approximation of

$f(\underset{\sim}{w},\underset{\sim}{s}^{q})$ in the following form

$$\tilde{f}(\underset{\sim}{w},\underset{\sim}{s}) = \delta_o + \sum_{i=1}^{n-1} (w_i - \bar{w}_i)\cdot[a_i\cdot(w_i - \bar{w}_i)+b_i] + \sum_{j=1}^{m} s_j\cdot(c_j\cdot s_j+d_j) \tag{56}$$

where $\delta_o = f(\underset{\sim}{\bar{w}},\underset{\sim}{0})$ and the parameters a_i, b_i, c_j and d_j are determined, respectively, from the following equations:

- a_i and b_i from

$$\delta_o + (\gamma_i^w\cdot\sigma_i - \bar{w}_i)\cdot[a_i\cdot(\gamma_i^w\cdot\sigma_i - \bar{w}_i)+b_i] = f(w_1,\ldots,(\gamma_i^w\cdot\sigma_i - \bar{w}_i),\ldots,w_{n-1},\underset{\sim}{0}) \tag{a}$$

(57)

- c_j and d_j from

$$\delta_o + \gamma_j^s\cdot s_j^*\cdot(c_j\cdot\gamma_j^s\cdot s_i^* + d_j) = f(\underset{\sim}{0},0,\ldots,\gamma_j^s\cdot s_j^*,\ldots,0) \tag{b}$$

where the coefficients γ_i^w and γ_j^s are rather arbitrarily chosen to assure the approximation (56) to be equal to the function $f(\underset{\sim}{w},\underset{\sim}{s})$ in some points from the likeliest range of simulation. In the examples the values $\gamma_i^w = 1.75$ and $\gamma_j^s = 0.75$ will be assumed.

The approximation (56) is used instead of the functions $f(\underset{\sim}{w}^K,\underset{\sim}{s}^{q,K})$ in e.g. (50) to determine the load combination $\underset{\sim}{s}^{q_K}$ which gives the minimum of the approximation. For the q_K-th load combination a value of the function $f(\underset{\sim}{w}^K,\underset{\sim}{s}^{q_K})$ is put into the estimator (50) as $\min_{q=1}^{r} f(\underset{\sim}{w}^K,\underset{\sim}{s}^{q,K})$. Thus, the estimate of the survival probability takes the following form

$$P_S \approx \frac{1}{N}\sum_{K=1}^{N} \Phi\left[f(\underset{\sim}{w}^K,\underset{\sim}{s}^{q_K})\right]\cdot\frac{\varphi(\underset{\sim}{w}^K)}{\prod_{i=1}^{n-1}\frac{1}{\sigma_i}\varphi\left[\frac{w_i^K - \bar{w}_i}{\sigma_i}\right]} \tag{58}$$

VII. EXAMPLES

Two simple examples are presented to demonstrate some possibilities of the proposed methods. Parameters of the random variables and pulse sequences are chosen so that the crude Monte Carlo simulation was also possible to be carried out to have some reference results.

Example 1: i.e. cylinder.

Limit condition forms a parabola in the five-dimensional space of original basic variables

$$g(\underset{\sim}{x}) = x_1 + \frac{1}{4}\sum_{i=2}^{5} x_i^2 - 2.55 = 0 \qquad (59)$$

The survival probability can be written as

$$P_S = P[\max_{t\in[0,T]} g(\underset{\sim}{X};t) \leq 0]$$

It means that the structure survives if all load combinations within the time interval [0,T] assure the limit condition to be negative.

All random variables (time-invariant or pulse intensities) are assumed to have log-normal probability distributions with means equal to 1.0 and coefficient of variations, respectively, 0.1, 0.2, 0.1, 0.2, 0.1. The random variable X_1 remains time-invariant. Other random variables will be assumed alternatively as time-invariant or as pulse sequences.

In Table 1 the β-values from the first-order approximation, β_{FORM}, e.g. (20), crude Monte Carlo simulation, β_{MC}, and importance sampling, β_{IS}, e.g. (58), are given. For the both simulation methods the number of load sequences, N_{MC} and N_{IS}, which was necessary to reach the 5% value of variation coefficient of the estimator of failure probability and the number of function calls, I_{MC} and I_{IS}, which were needed to get the solutions express the calculation effort.

Table 1.

	Repetition numbers r_2, r_3, r_4, r_5				
	1,1,1,1	1,1,1,100	1,1,50,100	1,25,50,100	5,25 50 100
β_{FORM}	2.56	2 .01	1.05	1.02	0.88
β_{MC}	2.33	1.72	0.77	0.74	0.58
N_{MC}	$\sim 4 \cdot 10^5$	6000	1420	1340	1020
I_{MC}	----	$\sim 6 \cdot 10^5$	$\sim 1.4 \cdot 10^5$	$\sim 1.3 \cdot 10^5$	$\sim 1.0 \cdot 10^5$
β_{IS}	2.30	1.76	0.86	0.77	0.59
N_{IS}	19	66	350	380	480
I_{IS}	132	550	2490	2920	5620

Example 2:

In the second example a parallel structure is considered with two parabolic limit conditions

$$g_1(\underset{\sim}{x}) = x_1 + \frac{1}{4} \sum_{i=2}^{5} x_i^2 - 2.55 = 0$$

$$g_2(\underset{\sim}{x}) = -x_1 + \frac{1}{4} \sum_{i=2}^{5} x_i^2 - 0.50 = 0 \qquad (60)$$

functions g_1 and g_2

The structure fails if the both conditions are positive. If some of the variables represent pulse sequences the probability of structural failure is defined as

$$P_F = P\Big[\bigvee_{\tau_5^*} [g_1(\underset{\sim}{x}; t) > 0 \wedge g_2(\underset{\sim}{x}; t) > 0] \Big] \qquad (61)$$

τ_5^* is the time interval of the pulse with the greatest repetition number, r_m, within which a load combination assures the both limit condition to be positive.

The parameter values of the random variables and the art of presentation of the results are similar as in *Example*

1. The results are collected in Table 2.

Table 2.

	Repetition numbers r_2, r_3, r_4, r_5			
	1,1,1,1	1,1,1,100	1,1,50,100	1,25,50,100
β_{FORM}	2.65	2 .09	1.08	1.05
β_{MC}	2.65	2.04	1.12	1.10
N_{MC}	$\sim 1 \cdot 10^6$	~20000	2660	2550
I_{MC}	----	$\sim 2 \cdot 10^6$	$\sim 2.5 \cdot 10^5$	$\sim 2.4 \cdot 10^5$
β_{IS}	2.62	2.10	1.26	1.14
N_{IS}	600	150	550	980
I_{IS}	5500	1180	4930	11000

It is worth to notice that the parameters of the limit conditions are chosen so that the β-point lies on the crossing of them. Moreover, though the nonlinearity of the limit conditions the first order approximation, β_{FORM}, and the Monte Carlo solution, β_{MC}, which can be considered as the approximately exact one almost coincide. This fact makes very unfavourable situation for any importance sampling. The case from the first column of the Table 2 where all random variables are assumed to be time-invariant confirms this difficulty. Nevertheless, the proposed importance sampling approach at presence of pulse sequences significantly reduces the computational effort even for small β-values.

VIII. CONCLUDING REMARKS

The β-distance and the β-point which can be found according to the method presented in chapters III and IV coincide with the results of Rackwitz-Fissler algorithm in the case of one limit condition (single failure mode). The pre-

sent method, however, has an advantage which takes its origin in the consequent application of the Rosenblatt transformation. Any optimization algorithm can be used to find the β-point in the both original and transformed spaces of basic variables. Therefore, multimode failure modes can be successfully considered as well. It is presented in *Example 2*. The proposed method allows us to easily include the load combination problem into any existing numerical package which uses the Rosenblatt transformation for solving the reliability problems of structural systems.

The β-point in the transformed space of standard, independent, normal random variables (time-invariant and pulse intensities) is used to construct a relatively effective procedure of numerical simulation to calculate the failure probability of structural system. The crude Monte Carlo method fails totally in load combination problems because of the computational effort. It is clearly seen even in the very simple examples given in the previous chapter that the number of limit state function calls which is necessary for a result with an acceptable confidence level reaches an impractical value even for moderate failure probability and repetition numbers.

The present approach is based on the importance sampling technique which has been developed for time-invariant problems. Appropriate rotation of the coordinate system, dependence of the importance sampling probability distribution on the main curvatures of limit condition in the β-point follow the main features of that technique. In order to lessen the number of limit function calls in searching for the relevant load combination within a sample of pulse sequence a paraboloid surface is introduced. The estimator value, however, is calculated with regard to the real value of the appropriately transformed limit function for the relevant load combination which is found for the approximation. Therefore, it is important rather to care for the simplicity and general tendency agreement between the approximation and

the real limit function than to construct the approximation as close to the real limit function as possible. It seems that the paraboloid approximation with parameters as it was proposed in eqs.(56) and (57) satisfies well this necessary condition. Other approximations are possible as well.

The adaptation of the importance sampling approach to time-dependent structural reliability problems allows us to verify the results obtained according to any other approximate method. The calculation effort does not depend on the failure probability value. It mostly depends on the maximal repetition number, r_m, because r_m values of the approximate limit condition have to be checked to find the relevant load combination. A simple analytical form of this approximation in the transformed coordinate system, e.g. the hyperparaboloid, reduces significantly the searching time and makes the method sufficiently effective to be used whenever the more precise information about failure probability is needed.

Though the proposed simulation is even several order of magnitude faster than the crude Monte Carlo method the calculation effort still remains considerable. The acceleration effect of the importance sampling is slightly reduced by the rotation of the coordinate subsystem only given by the time-invariant random variables. Therefore, the more the time-invariant random variables are in the (n+m)-dimensional set of basic variables the more effective the method becomes. It is also seen in the given examples.

REFERENCES

1. FERRY BORGES J., CASTANHETA M., Structural Safety, 2nd edition, Laboratorio Nacional de Engenharia Civil, Lisbon, 1972;

2. RACKWITZ R., FISSLER B., Structural reliability under combined random load sequences, Computers&Structures, vol.9, pp. 489-494, 1978;

3. DITLEVSEN O., Principle of normal tail approximation, J.Eng.Mech. ASCE, vol.107, EM6, pp. 1191-1208, 1981;

4. DITLEVSEN O., OLESEN R., MOHR G., Solution of a class of load combination problems by directional simulation, Structural Safety, vol.4, pp. 95-109, 1987;

5. ROSENBLATT M., Remarks on a multivariate transformation, Ann.Math.Stat., vol.23, pp. 470-472, 1952;

6. HARBITZ A., An efficient sampling method for probability of failure calculation, Structural Safety, vol. 3, (2), 1986, pp. 109-115;

7. FUJITA M., RACKWITZ R., Updating first- and second-order reliability estimates by importance sampling, Structural Engineering/Earthquake Engineering, JCSE, vol. 5, (1), pp. 31-37, 1988;

8. HOHENBICHLER M., RACKWITZ R., Improvement of second-moment reliability estimates by importance sampling, J. Eng.Mech., ASCE, vol.114, No.12, 1988;

9. DOLINSKI K., Importance sampling techniques in reliability calculation, IFTR Reports, no. 37, 1988;

10. DOLINSKI K., Discussion to "Improvement of second-moment reliability estimates by importance sampling" Hohenbichler M., Rackwitz R., , ASCE, 1989;