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ON AN APPROXIMATE CRITERION FOR CHAOTIC MOTION IN A MODEL OF A BUCKLED BEAM

Summary: Periodic and chaotic motions in a methematical model of a bucked beam is studied by the aid of approximate theory of nonlinear vibration and computer simulation. It is shown that the first approximate harmonic solution of Small Orbit motion may provide an approximate criterion for chaotic motion to appear. While computer simulation shows a variety of subharmonic motions /period doubling bifurcation/ critical system parameters calculated on the assumption of harmonic solution prove to be close to the true boundaries of chaotic zone.

1. Introduction

Phenomena of chaotic motion in a system having three positions of equilibrium governed by an equation

$$\ddot{x} + h\dot{x} - \tfrac{1}{2}(1-x^2)x = P\cos\vartheta t,$$

has been attracting a great deal of attention in recent literature. The system studied first by Holmes [1,2] has become now a classical mathematical model of a buckled beam and of systems which exhibit chaotic motion. A main point of interst is a "strange behaviour" of the system: when ratio P/h exceeds a certain critical value periodic oscillations around one of two stable equilibrium positions turn into an irregular motion consisting essentially in random-like jumps from oscillations around one to oscillations around the other rest point. The phenomenon was observed experimentally by Tseng and Dugundji first as early as in 1971, [3] however did not draw much of an attention and was known and "snap through oscillations". Since then chaotic dynamics in deterministic systems has established itself as a new phenomenon in non-linear vibrations. In a series of papers [4-7] a major interest was focused on finding and recording essential characteristics of chaotic motion either with experimental or computer simulation methods, on theoretical analysis of strange attractors by means of topological methods and on routes to chaos. Some attempts to develop an approximate criterion for chaotic motion i.e. to determine critical parameters for which one might expect chaotic behaviour was made by Holmes and Moon [2,6]. However theoretical and experimental results of boundaries between regular and chaotic motion in [6] do not show satisfactory coincidance.

Hitherto results show clearly that chaotic motion may coexist and border upon periodic motion. Therefore if one recalls that approximate analytical methods make an effective tool to examine periodic solutions in nonlinear vibrating systems whether the systems are weakly or strongly nonlinear (see e.g. [8]) one might be interested in seeing zones of chaotic motion against a background of approximate periodic solutions (see [9, 10]).

It is an attempt of the present paper to study the first approximate harmonic solution of the system followed and verified by computer simulation analysis and to propose an approximate criterion for boundary of periodic motion and its transition to chaos on the forcing parameter - frequency plane. Although the computer simulation show a variety of subharmonic motions the harmonic solution with period of excitation appears to play an essential role in the task of determining critical values of system parameters for the chaotic motion to occur.

2. General equation and its fundamental properties

To derive the second order differential equation whose solutions are to be studied we consider partial differential equation for transverse large deflections of a beam accompanied with linear autonomous boundary conditions written in general form as

$$m\frac{\partial^2 w}{\partial \tau^2} + H\frac{\partial w}{\partial \tau} + EJ\frac{\partial^4 w}{\partial y^4} + F_0\frac{\partial^2 w}{\partial y^2} + L(w) = p(y)\cos\bar{\nu}\tau;$$

$$B_{i1}(w)\Big|_{y=0} = 0; \quad B_{i2}(w)\Big|_{y=l} = 0; \quad i = 1,2 \tag{1}$$

where F_0 represents axial compressive load and $L(w)$ - a nonlinear part of elastic forces due to large deflections /geometric nonlinearity/ and assume a single-mode solution

$$w(y,\tau) = \psi_1(y)\check{x}(\tau), \quad \psi_1(y_0) = 1; \tag{2}$$

where $\Psi_1(y)$ is a fundamental eigenfunction /normal mode shape/ of the linear system governed by equations

$$m\frac{\partial^2 w}{\partial \tau^2} + EJ\frac{\partial^4 w}{\partial y^4} = 0; \qquad (3)$$

$$B_{i_1}(w)_{w=0} = 0, \quad B_{i_2}(w)_{w=l} = 0.$$

On inserting eqs. (2) into eqs. (1) and applying the Galerkin procedure one arrives at a single second order differential equation for the normal coordinate $\check{x}(\tau)$. In the post-buckling conditions $F_0 > F_{0_{cr}}$ the equation transformed into a nondimentional form is written as [2,6,11] :

$$\ddot{x} + h\dot{x} - \tfrac{1}{2}(1-x^2)x = P\cos\nu t; \quad x = \frac{\check{x}}{l}; \, ,$$

$$0 < h \ll \tfrac{1}{2} \qquad (4)$$

where t is a nondimentional time, and the nonlinear term $\frac{1}{2}x^3$ is, in general case, the first term in Taylor expansion of the function representing nonlinear part of elastic force.

From the theory of nonlinear vibrations it is known that the system (4) at $P=0$ has three equilibrium positions:

$x_{(1)} = 0$ - unstable position /saddle point in x-$\dot{x}$ plane/;

$\left.\begin{array}{l} x_{(2)} = +1 \\ x_{(3)} = -1 \end{array}\right\}$ stable positions /stable foci in x-$\dot{x}$ plane/;

and that the complete system may exhibit two types of periodic steady state oscillations : around the stable equilibrium positions, called later "Small Orbit" and oscillations around all three equilibrium positions, called "Large Orbit" /see Fig. 1/. It depends on initial conditions which type of motion is really generated in the system.

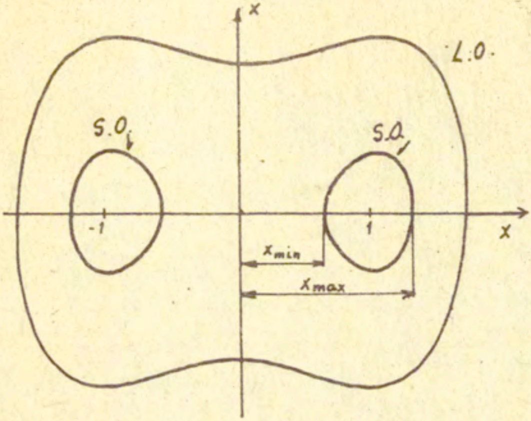
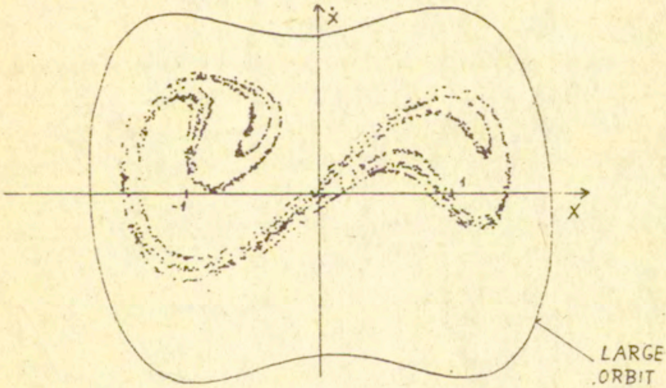


Fig. 1 . Phase portrait of Small and Large Orbit motion.



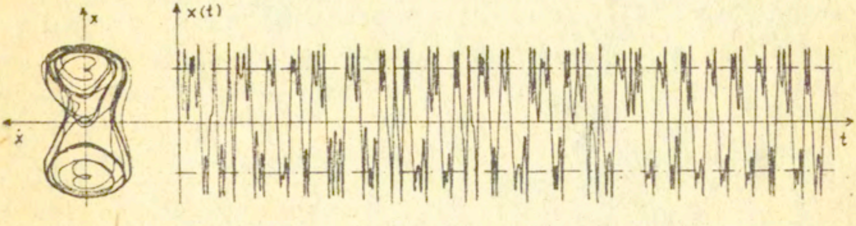


Fig. 2 . Time history, phase portrait and Poincare map (strange attractor) of chaotic motion: $P=0.14$; $h=0.10$; $\nu=0.8$;

For better insight into the Small Orbit motion one may shift the origin of cooridinates on introducing new variable

$$Z = X + X_{(2),(3)} , \tag{5}$$

and transform eqs. (4) into the form

$$\ddot{z} + h\dot{z} + z \mp \frac{3}{2} z^2 + \frac{1}{2} z^3 = P\cos\nu t . \tag{6}$$

Thus it is seen that the natural linear frequency of the Small Orbit is equal to one : $\omega_0 = 1$.

Indeed the system governed by eqs. (4) behaves that way i.e. exhibits periodic oscillations either of Small Orbit or Large Orbit type provided the amplitude of forcing function P at given h does not exceed a certain critical value. On may expect that on increasing the parameter P Small Orbit motion would jump into Large Orbit oscillations - the jump phenomena belong to the most characteristic features of nonlinear vibrating systems. However jump phenomena of the type does not occur in the case. Instead, when P exceeds a critical value Small Orbit turns into an irregular motion consisting essentially on random-like jumps from oscillations around $X_{(2)} = +1$ to oscillations around $X_{(3)} = -1$ and back. The motion does not decay and in a long time interval shows properties of a "steady-state" behaviour. This is shown in Fig. 2 where chaotic motion is illustrated by means of three descriptors: time history, phase portrait and Poincare map. The Poincare map, which is a stroboscopic picture of the motion on the phase plane synchronous with the forcing term $P\cos\nu t$ shows a set of points called "strange attractor". The nature of strange attractor although being a point of great interest from both experimental and theoretical point of view is not studied in this paper.

Experimental results presented by Moon [6] allow to make an observation that critical values of the parameter for the change of Small Orbit into the chaotic motion take minimum values in the region of frequency close to the principal resonance i.e. at $\nu \approx 1$. In this paper an attention is focused on this zone of the excitation frequency.

3. The first approximate solution and peculiar features of resonance curves

In the neighbouhood of the principal resonance of Small Orbit and at sufficiently low value of the parameter P/h it is legitimate to seek the first approximate periodic solution as /see e.g. [8]/ :

$$x(t) = A_0 + A_1 \cos(\nu t + \vartheta) = x(t+T) \; ; \; T = \frac{2\pi}{\nu} ; \qquad (7)$$

where $A_0 \neq 0$ for Small Orbit solution and

$A_0 = 0$ for Large Orbit motion.

The unknown coefficients A_0, A_1, ϑ are determined by the harmonic balance method i.e. they satisfy equations

$$\begin{aligned} &(A_0^2 + \tfrac{3}{2} A_1^2 - 1) A_0 = 0, \\ &-\nu^2 A_1 + \tfrac{3}{8} A_1^3 + \tfrac{3}{2} A_0^2 A_1 - \tfrac{1}{2} A_1 = P \cos\vartheta, \\ &-h \nu A_1 = P \sin\vartheta \end{aligned} \qquad (8)$$

Equations (8) are derived on inserting eqs. (7) into eqs. (4) and equating a constant term and coefficients of $\cos(\nu t + \vartheta)$ $\sin(\nu t + \vartheta)$ and $\cos(\nu t + \vartheta)$ separately to zero. /see e.q. [8] /. Solving of eqs. (8) gives the sought amplitudes A_0, A_1 as functions of frequency at $A_0 \neq 0$

$$A_1 = \frac{P}{\sqrt{(1-\frac{15}{8}A_1^2-\nu^2)^2+h^2\nu^2}} \quad ; \quad A_0 = \mp\sqrt{1-\tfrac{3}{2}A_1^2} \tag{9}$$

It comes out that the backbone curve of Small Orbit:

$$\omega(A_1) = 1 - \frac{15}{8}A_1^2 , \tag{10}$$

is of the soft type and hence the resonance curves $A_1(\nu)$, $A_0(\nu)$ are bowed to the left. The resonance curve $A_1(\nu)$ shows a peculiar feature: peak amplitude $A_{1\,max}$ takes a finite value for sufficiently low value of the parameter P/h only i.e. at

$$P/h < \sqrt{\frac{2}{15}} \tag{11}$$

Character of the resonance curves at $P/h < \sqrt{\frac{2}{15}}$ and $P/h > \sqrt{\frac{2}{15}}$ is sketched in Fig. 3a and 3b. At the particular value of P/h the peak amplitudes are

$$(P/h)_1 = \sqrt{\frac{2}{15}} \quad ; \quad A_1 = A_1(\nu_B) = \sqrt{\frac{4}{15}} \quad ; \quad \nu_B = \sqrt{\frac{1}{2}} \tag{12}$$

While the resonance curve in Fig. 3a takes the classical shape of that in Duffing equation with soft nonlinearity, the one in Fig. 3b looks like a resonance curve in undamped system. Since the system considered is damped $(h>0)$ the only reasonable explanation of the peculiar behaviour of $A_1(\nu)$ is that the first approximate harmonic solution (7) is not adequate one for so high value of the parameter P/h.

In spite of the observation we study the harmonic solution at $P/h > (P/h)_1$ because surprising and promising results are obtained where instead of the amplitudes A_1 and A_0

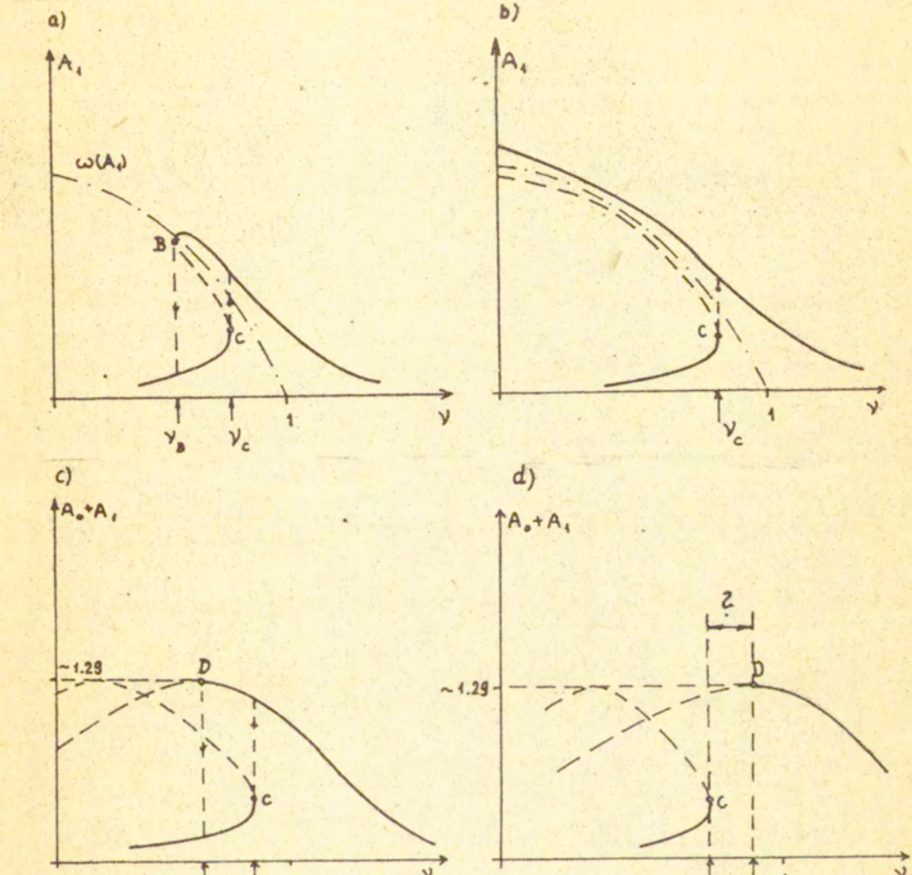


Fig. 3 . Resonance curves of the approximate harmonic solution

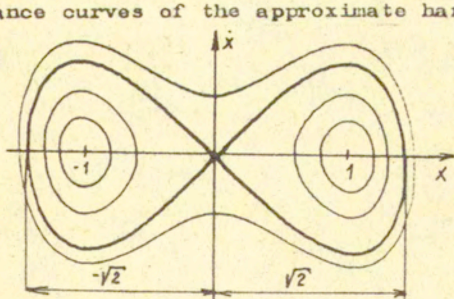


Fig. 4 . Phase portrait of hamiltonian system

maximum and minimum of displacement is considered:

$$A_0 + A_1 \cong A_{max} \equiv A_{max}(\nu) \, ; \; A_0 - A_1 \equiv A_{min} \cong A_{min}(\nu) \, .$$

Indeed resonance curves $A_{max}(\nu)$ and $A_{min}(\nu)$ appear to show a behaviour close to x_{max} and x_{min} obtained by computer simulation even for very large values of the parameter P/h and become a key to the sought approximate criterion for chaotic motion to appear.

At $P/h > \sqrt{\frac{2}{15}}$ a characteristic feature of the resonance curve $A_0 + A_1 \cong A_{max}(\nu)$ is an appearance of a point with horizontal tangent. Coordinates of the point denoted by D in Fig. 3c are derived as follows: since $\frac{dA_1}{d\nu} \neq 0$ and takes finite value on the resonant branch of $A_1(\nu)$, the condition

$$\frac{d}{d\nu}(A_0 + A_1) = \frac{d}{dA_1}(A_0 + A_1) \cdot \frac{dA_1}{d\nu} = 0 \qquad (13a)$$

is equivalent to the condition

$$\frac{d(A_0 + A_1)}{dA_1} \qquad (13b)$$

The latter is easily derived by virtue of eqs. (8) with the results

$$A_1(\nu_D) = \sqrt{\frac{4}{15}} \; ; \quad A_0(\nu_D) = \sqrt{\frac{3}{15}} \; ;$$

$$A_{max}(\nu_D) = A_1(\nu_D) + A_0(\nu) = \sqrt{\frac{4}{15}} + \sqrt{\frac{3}{15}} \cong 1.29 \qquad (13c)$$

$$\nu_D^2 = \frac{1}{2}\left[1 - h^2 + \sqrt{h^4 - 2h^2 + 15P^2}\right];$$

$$h << \frac{1}{2}$$

It is worth noticing that the value $A_{max}(\nu_p)$ thus calculated is constant, independent of the parameters P and h. The observation brings immediately an idea to compare the result with maximum displacement of Small Orbit in hamiltonian system governed by equation

$$\ddot{x} - \frac{1}{2}(1 - x^2)x = 0 ; \qquad (14a)$$

One can easily find out that at the separatrix which is a boundary line between Small and Large Orbit motion the maximum deflection is /see Fig. 4/:

$$x^{(h)}_{max} = \sqrt{2} = 1.41 > A_{max}(\nu_p) \qquad (14b)$$

This is what one expects intuitively: the forced Small Orbit of the complete system (4) should remain inside the separatrix loop of the hamiltonian system /see also [6] /.

The other characteristic point in Fig. 3 c,d - the point of the vertical tangent C is the classical stability limit of the harmonic solution (7) . The point lies on non-resonant branch of the resonance curve and hence a dependence of frequency ν_c on the damping coefficient at the low values of h considered is negligible. At $h = 0$ the frequency is given by

$$\nu_c^2 = 1 - \frac{3}{2}\sqrt[3]{\frac{15}{4}P^2} ; \qquad (15)$$

Although one can not prove that the point with horizontal tangent, /point D / is also a stability limit, a physical intuition says that the resonant branch of the maximum displacement $A_{max}(\nu)$ where $\frac{dA_{max}}{d\nu} > 0$ should not be realized in real systems. On using this argument the branch of $A_{max}(\nu)$ on the left from the point D has been denoted as "unstable" branch in Fig. 3c,d. Therefore as long as

$$\nu_p < \nu_c$$

as it is sketched in Fig. 3c the stable harmonic solution of Small Orbit exists in the whole range of frequency considered and classical jump phenomena from nonresonant to resonant branch of the resonance curve at $\nu = \nu_c$ on increasing ν and from resonant to nonresonant solution at point $\nu = \nu_p$ on decreasing frequency can be expected.

However on further increase of the forcing parameter the frequency ν_p grows and then exceeds the frequency ν_c /see Fig. 3d/. In this case a stable harmonic solution of Small Orbit does not exist in the region $\nu_c \div \nu_p$ and an appearance of "strange phenomena" can be expected on reaching frequency ν_c and ν_p . This observation makes an essential point in the attempt of proposing an approximate criterion for chaotic motion based on the first approximate solution (7) .

Frequencies of the two characteristic points ν_c and ν_p evaluated by eqs. /13c/ and (15) are displayed on $P-\nu$ plane at two values of the damping coefficient /see Fig. 5/. Indeed the two curves $P = P(\nu_c)$ and $P = P(\nu_p)$ cross each other at a point whose coordinates have been denoted by P_{cr_t} and ν_{cr_t} . By virtue of eqs. /13c/ and /15/ the critical point satisfies relations

$$\nu_c^2 = \nu_p^2$$

$$1 - \frac{3}{2}\sqrt[3]{\frac{15}{4}P^2_{cr_t}} = \frac{1}{2}\left(1 - h^2 + \sqrt{h^4 - 2h^2 + 15P^2_{cr_t}}\right) \qquad (16a)$$

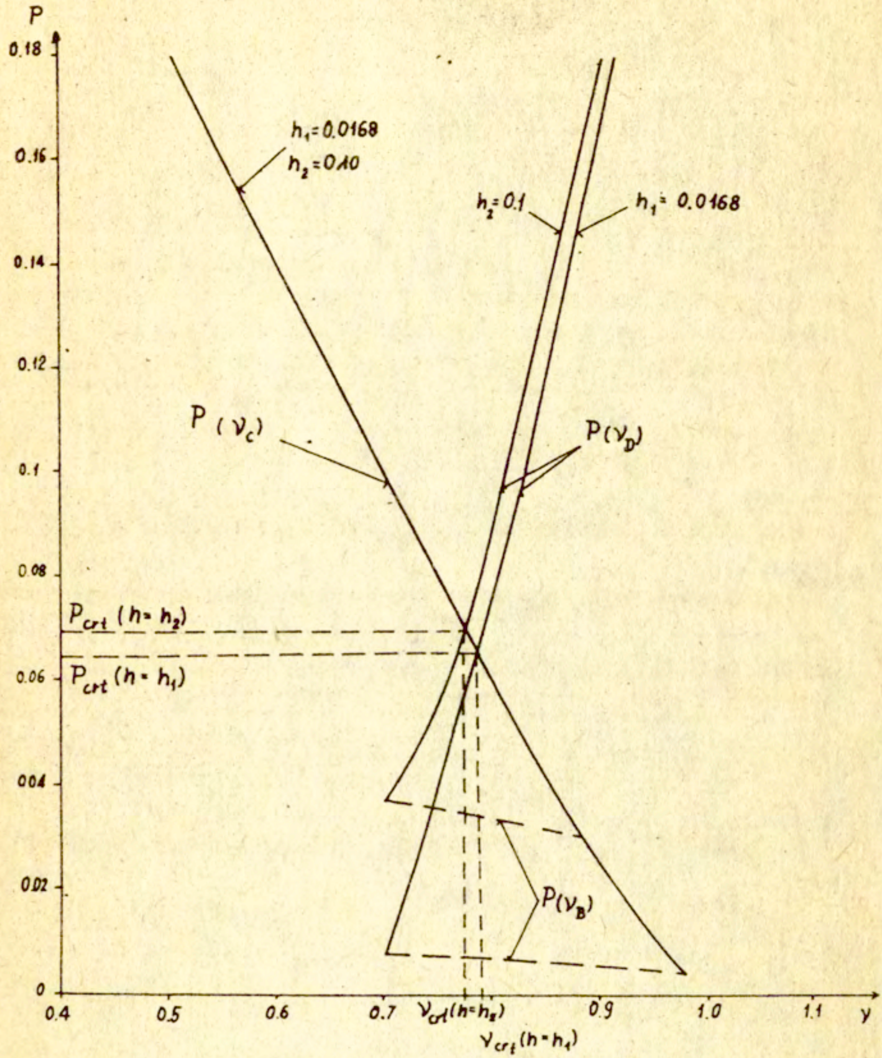


Fig. 5 . Loci of characteristic points B, C, D (denoted in Fig.4.3) on $P-\nu$ plane at $h_1 = 0.0168$ and $h_2 = 0.10$;

or in a reduced form as

$$\nu_{cr_t}^2 = 1 - 2.33\, P_{cr_t}^{\frac{2}{3}}$$

$$h^2 = \frac{15 P_{cr_t}^2 - (1 - 4.66\, P_{cr_t}^{\frac{3}{2}})^2}{4(1 - 2.33\, P_{cr_t}^{\frac{2}{3}})} \qquad (16b)$$

It is clearly seen that the region where "strange phenomena" can be suspected is definied by equations

$$P > P_{cr_t}$$

$$\nu_C < \nu < \nu_D \qquad (16c)$$

Loci of the other characteristic point on the resonance curve which appears only at $P/h < \sqrt{\frac{2}{15}}$ - point B in Fig. 3a - are also drawn in Fig. 5.

4. Computer simulation analysis

To verify the theoretical predictions and to investigate true behaviour of the system, equation of motion (4) was simulated on an electronic computer /both analog and digital computers were used/ and response of the system was observed, transormed and recorded. The simulation results are displayed in Figs. 2,6-12. First in Fig. 6 periodic motion is characterized by resonance curves of maximal and minimal displacements of Small Orbit - $x_{max}(\nu)$, $x_{min}(\nu)$ /see Fig. 1/ drawn together with the theoretical values $A_{max}(\nu)$ and $A_{min}(\nu)$ respectively. Maximal displacement of Large Orbit motion, generated in the system on applying appropriate, sufficiently large initial conditions, is also shown together with amplitude of harmonic solution $A_1(\nu)$ at $A_0 \equiv 0$.

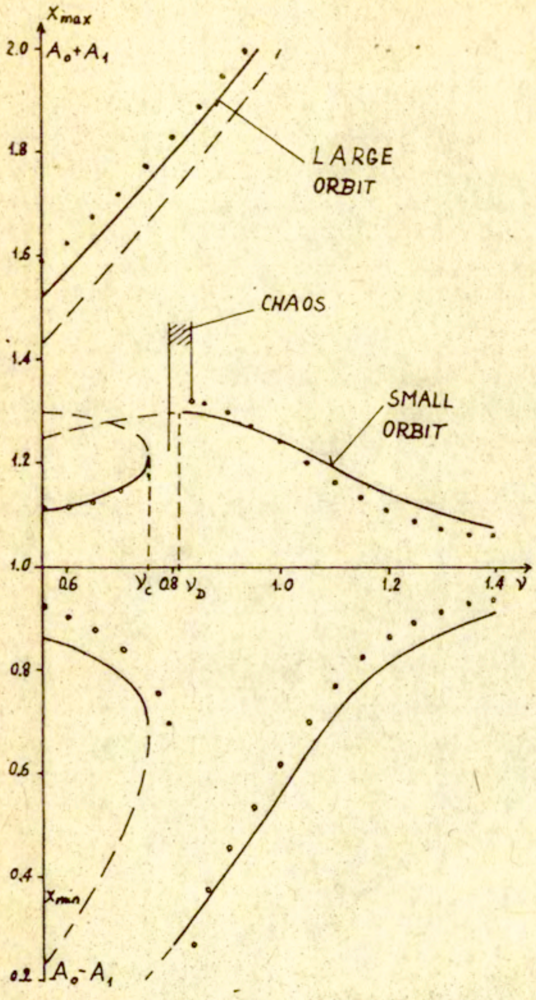


Fig. 6 . Resonance curves of maximal and minimal displacement in Small and Large Orbit motion at $P=0.08$; $h=0.0168$; ——— - - - theoretical, o o • • simulation results.

It is readily seen that the theoretical and simulation results show, in general, good coincidance, in spite of rather high value of the forcing parameter satisfying the condition $\nu_D > \nu_C$. Computer simlation confirm the theoretical prediction that "strange phenomena" may appear in this case. Indeed chaotic behaviour was observed in the region of frequency denoted as CHAOS in Fig. 6. The region is very close to the theoretical zone where stable Small Orbit harmonic solution does not exists i.e. the zone definied by eqs. (16c) .

To illustrate various types of response close to and inside the chaotic zone time histories, phase portraits and Poincare maps were recorded at several values of frequency and displayed in Fig. 7 a-e. First Small Orbit motion at frequency near the top of $X_{max}(\nu)$ curve i.e. at $\nu = 0.85$ and $\nu = 0.835$ slightly higher than ν_D is shown in Fig. 7a and 7b. This is the region of frequency where simulation value X_{max} is clearly higher than the theoretical A_{max} amplitude. The time history, phase portrait and harmonic analysis of $x(t)$ in Fig. 7a give reason for the discrepancy. The response $x(t)$ is still T-periodic here, but the second harmonic component appears to take relatively large value and one may conclude that a legitimate approximate solution at $\nu \approx 0.85$ is:

$$x(t) = A_0 + A_1 \cos(\nu t + \vartheta_1) + A_2 \cos(2\nu t + \vartheta_2) \tag{17}$$

The response at slightly lower frequency $\nu = 0.835$ in Fig. 7b is no longer T-periodic, but appears to be 2T-periodic /two points on Poincare map/. Then the 2T-periodic motion turns into chaotic motion at $\nu \approx 0.78$ /see Fig. 7c/. Due to low damping the strange attractor does not show a clear-cut structure, but time history and phase portrait are of the same character as those in Fig. 2 obtained at higher value

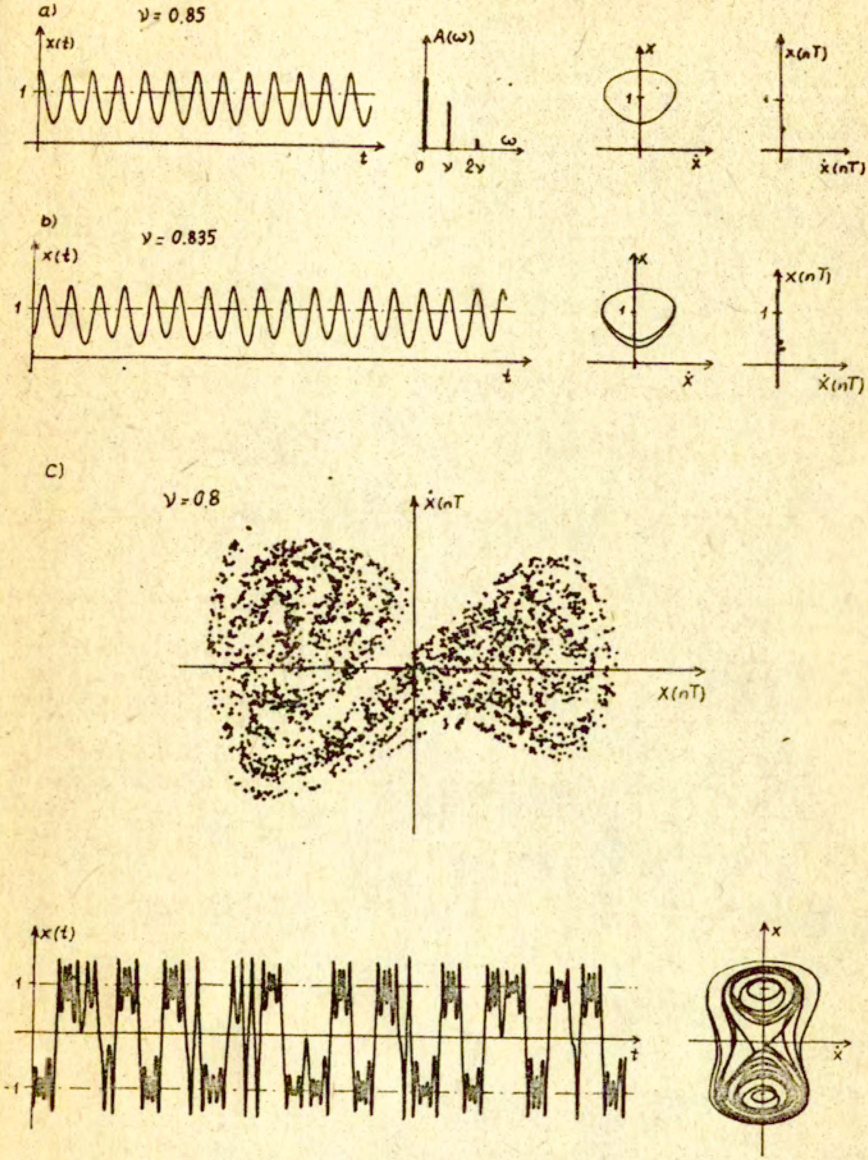
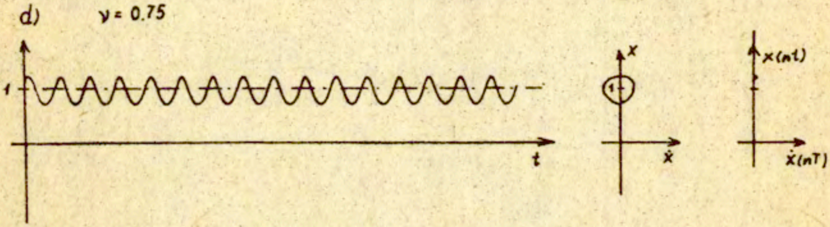


Fig. 7 . Time histories, phase portraits and Poincare maps in the neighbouhood and inside chaotic zone at $P = 0.08$; $h = 0.0168$;



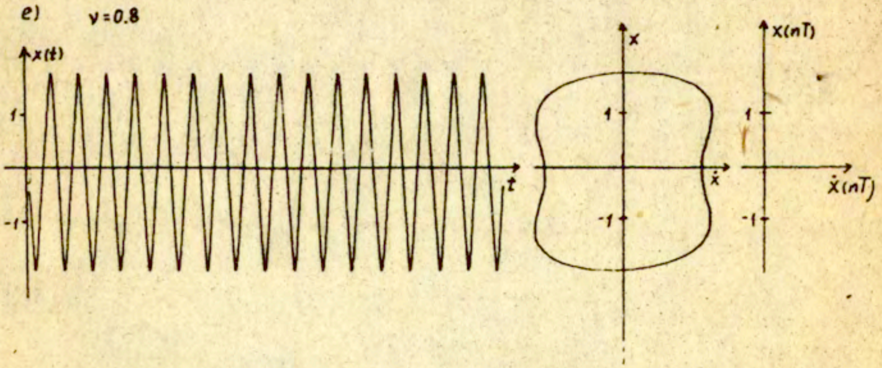


Fig. 7 . continued.

of the damping coefficient $h = 0.10$. On further decrease of frequency chaotic motion turns rapidly into the T-periodic Small Orbit motion /see Fig. 7d/. It is worth to point out that no hysteresis in the system behaviour was observed - responses were the same whether frequency was decreasing or increasing. Large orbit motion shown in Fig. 7e was generated in the system on applying appropriate, sufficiently large initial conditions only.

This type of the system behaviour - transition from resonant to nonresonant Small Orbit, or vice versa, through chaotic motion zone was observed at certain region of the forcing parameter i.e. at /see Fig. 11 and 12/:

$$P_{cr} < P < P_{cr}^{(h)}$$

At $P > P_{cr}^{(h)}$ a distincly new phenomenon was noticed - a hysteresis type behaviour accompanied with a jump from Small Orbit to Large Orbit motion. This is illustrated in Fig. 8-10. First we notice that resonance curves in Fig. 8 do not show any peculiarities compare to those on Fig. 6. However now the system behaviour on decreasing and increasing frequency is essentially different, as it is sketched in Fig. 9. When ν is growing the nonresonant Small Orbit jumps into Large Orbit motion /at $\nu \approx 0.74$ in Fig. 8/ and does not turn back to the resonant branch of the Small Orbit, so that no chaotic behaviour is observed. The chaotic behaviour occurs only on decreasing frequency: the resonant Small Orbit turns into chaotic motion and then jumps into the Large Orbit /see Fig. 9b and 10/. Therefore at the high values of the forcing parameter P/h the system response tends to turn Small Orbit into the Large Orbit and the chaotic motion occurs only as a transition zone between resonant Small Orbit and the Large Orbit motion at a decrease of ν.

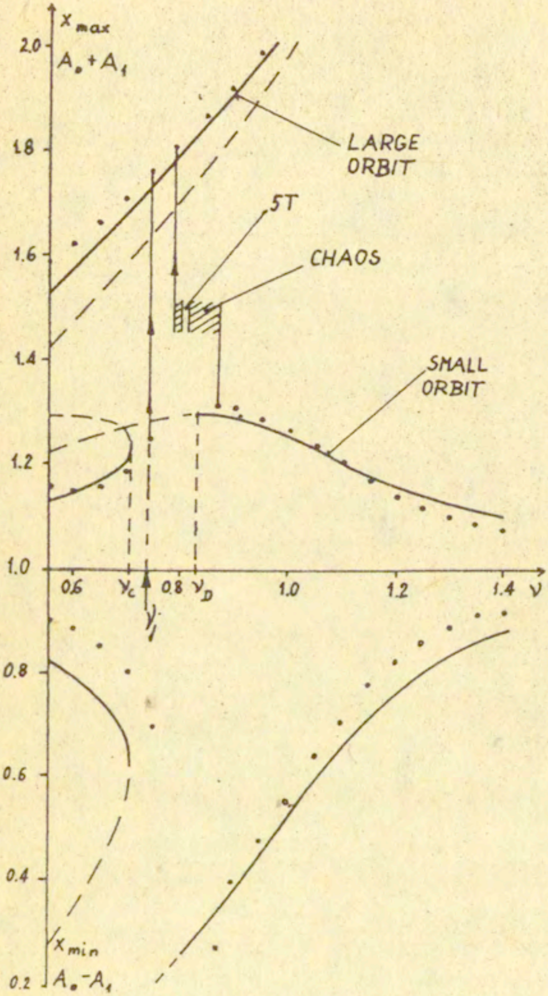


Fig. 8 . Resonance curves of maximal and minimal displacement in Small and Large Orbit motion at $P=0.10$; $h=0.0168$; ——— — — — theoretical, o o o • • simulation results ;

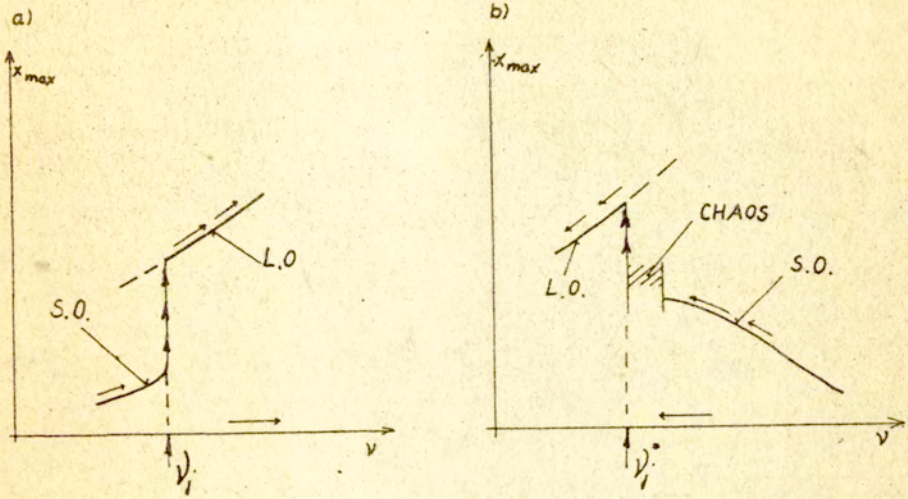


Fig. 9 . Hysteresis type behaviour at $P > P_{cr}^{(h)}$

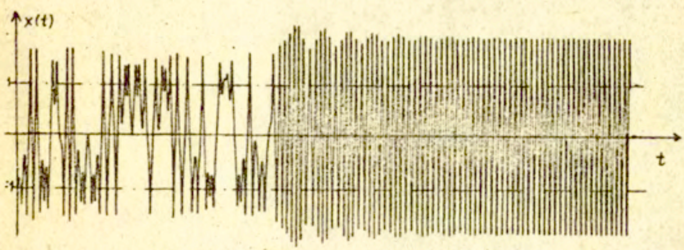


Fig. 10 . Time history of transient between chaotic and Large orbit motion at $\nu = \nu_1^*$

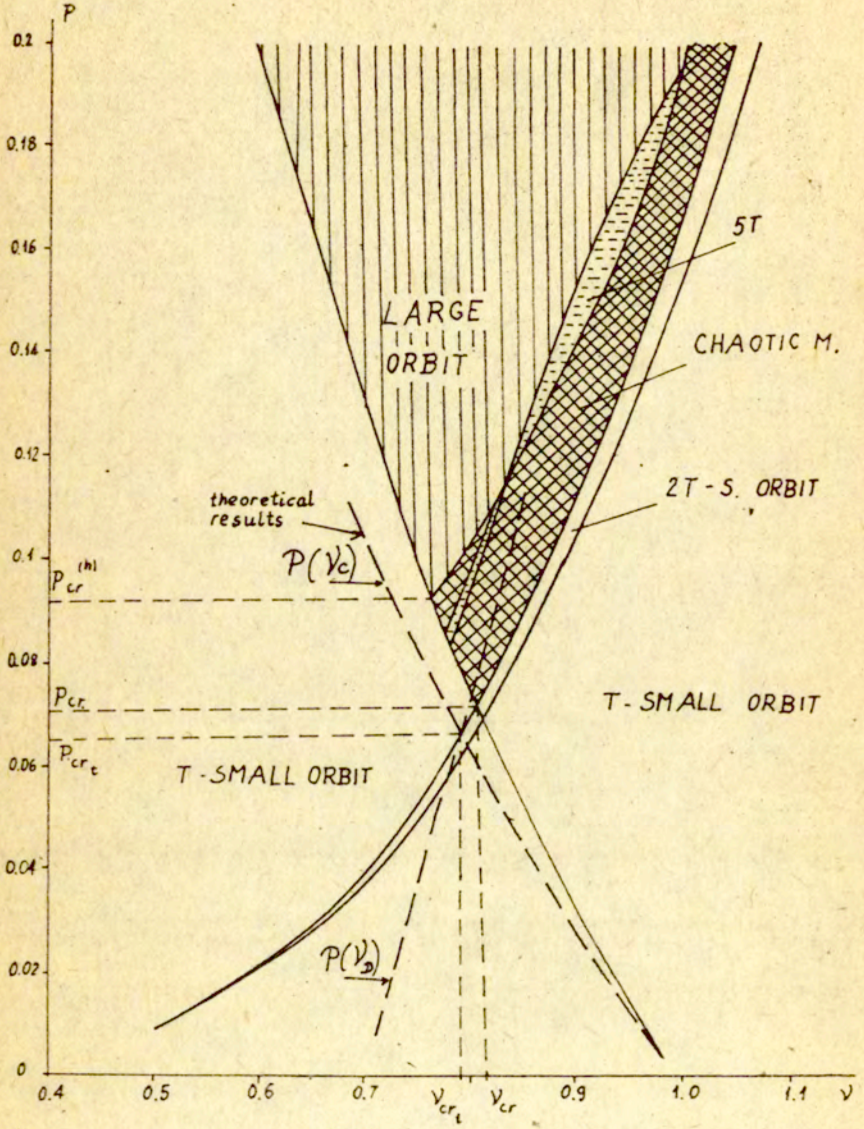


Fig. 11 . Regions of various periodic motion and chaotic behaviour in $P-\nu$ plane at $h=0.0168$ - simulation results.
— — — — theoretical curves $P=P(\nu_c)$ and $P=P(\nu_D)$;

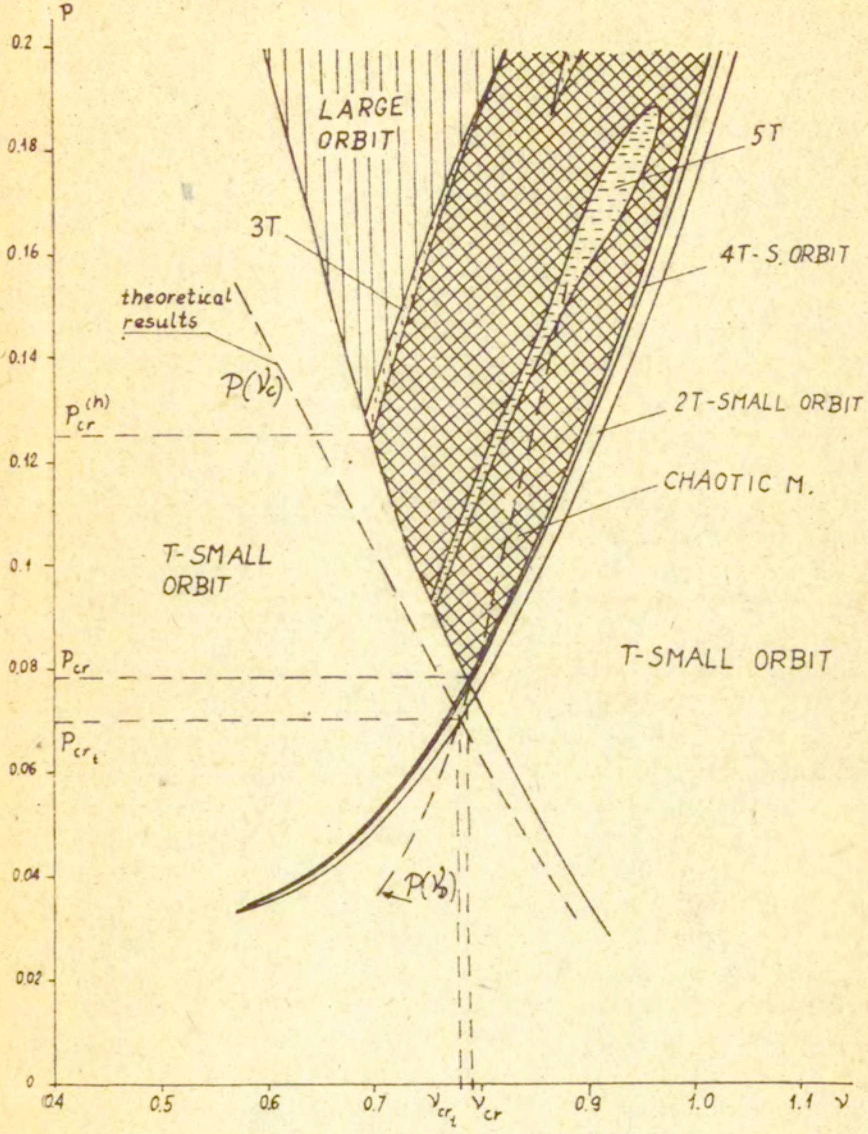


Fig. 12 . Regions of various periodic motion and chaotic behaviour at $h=0.10$ – computer simulation results ; —— — theoretical $P\equiv P(\nu_C)$ and $P\equiv P(\nu_D)$ curves.

It is interesting to notice that at the higher value of the damping coefficient /see Fig. 12/ chaotic zone is preceded by two period doubling bifurcations. The T-periodic Small Orbit first turns into 2T-periodic and then 4T-periodic Small Orbit motion in certain narrow frequency zones /see Fig. 13 a,b,c./. Characteristic windows of regular motion with period 3T and 5T, which can be interpreted as regular combinations of Small and Large Orbit motion, observed inside and on edges of chaotic zone are illustrated in Fig. 13 d,e.

5. The approximate criterion for chaotic motion

In the search for an approximate criterion for chaotic motion to appear based on the first approximate harmonic solution the theoretical curves $P \equiv P(\nu_c)$ and $P \equiv P(\nu_D)$ shown in Fig. 5 where drawn in Fig. 11 and 12 together with simulation results. It is clearly seen that the lowest critical value of the forcing parameter P_{cr_t} and the associated ν_{cr_t} are very close to those obtained by computer simulation. This is an essential observation: in spite of the fact that true response close to chaotic zone differs considerably from harmonic motion the theoretical lowest value of the forcing parameter P_{cr_t} when chaotic behaviour is predicted shows suprisingly good coincidance with the true value P_{cr} The theoretical value P_{cr_t} evaluated by eqs. (16b) is shown in Fig. 14 in two configurations - on $P-\nu$ and $P-h$ planes.

At higher values of the parameter $P > P_{cr}$ theoretical range of frequencies where Small Orbit motion was predicted do disappear is shifted to the left compared with computer simulation results. Still a concidance of the results is so good that the region on $P-\nu$ plane determined by eqs. (16c) is proposed as the approximate criterion for chaotic motion at the forcing parameter P slightly exceeding P_{cr_t}.

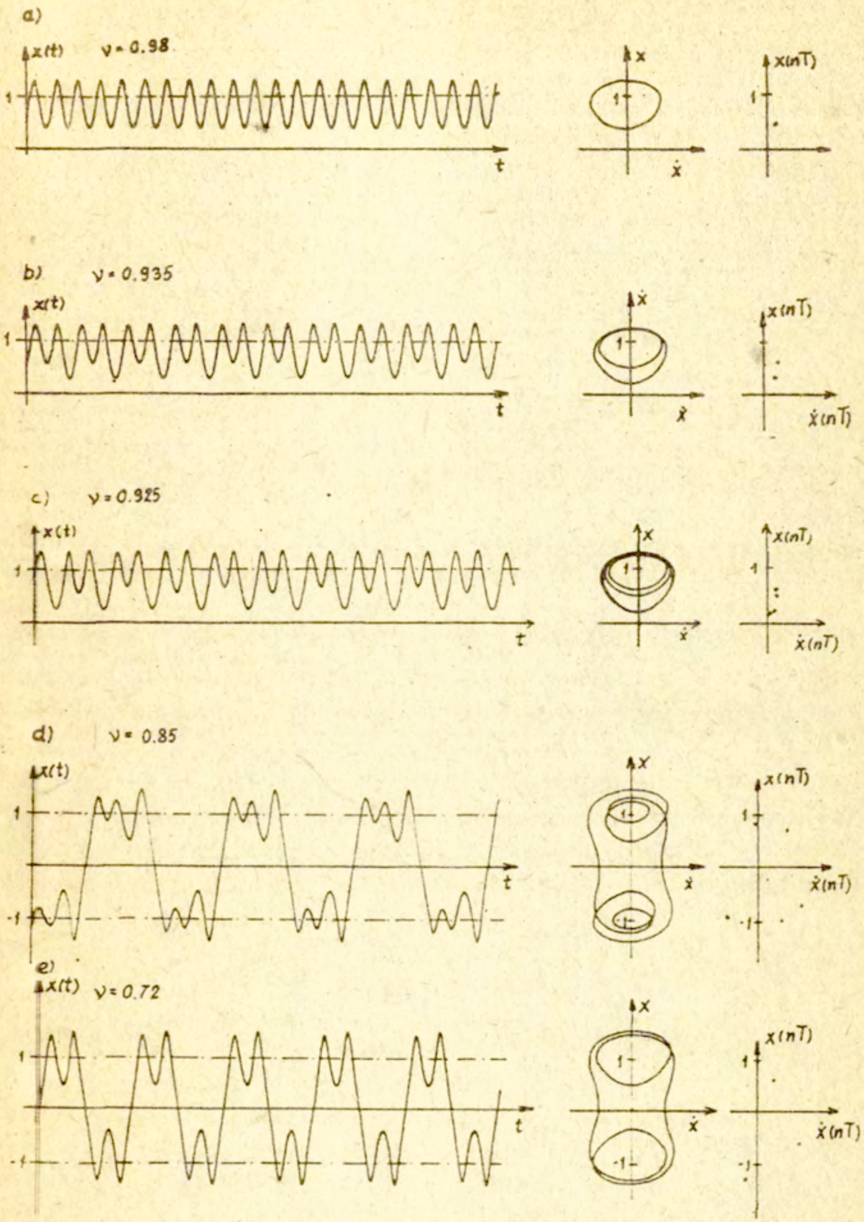


Fig. 13 . Various types of periodic motion in the neighbouhood of chaotic zone at $P = 0.14$; $h = 0.10$;

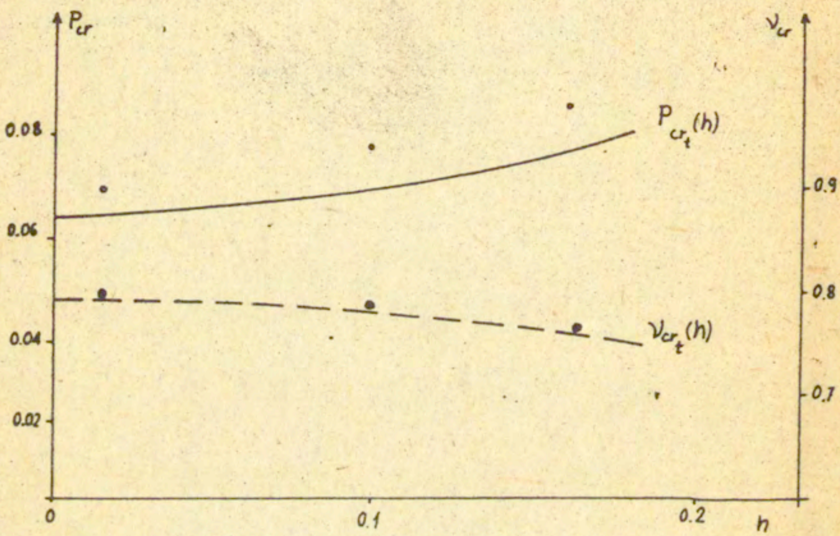


Fig. 14 . The lowest critical values P_{cr} and ν_{cr} as functions of damp-ing coefficient.

6. Conclusions

The detail study of mathematical model of a buckled beam by making use of the approximate theory of nonlinear vibrations and computer simulation technique gives a deeper insight into relations between periodic and chaotic behaviour near principal resonance of Small Orbit motion. Resonance curves of maximal and minimal displacement of Small Orbit calculated on an assumption of harmonic solution prove to be of the same character as true ones even when the forcing parameter reaches such high value that chaotic phenomena occur in certain frequency region.

It is stated that the region of system parameters where stable Small Orbit harmonic solution cease to exist i.e. where $\nu_D > \nu_C$ is that where strange phenomena may occur. Boundary of the region at forcing parameter slightly exceeding the lowest critical value satisfying the condition is proposed as an approximate criterion for chaotic motion to appear.

Although true response close to chaotic zone differs considerably from the T-periodic approximate harmonic solution, an appearance of the second harmonic and then components with periods 2T and 4T preceding the chaotic behaviour, theoretical critical values of system parameters are very close to those obtained by computer simulation.

Computer simulation shows two distincly different types of the system behaviour: first at

$$P_{cr} < P < P_{cr}^{(h)}$$

the chaotic motion appears as a transition zone between two branches - resonant and nonresonant one - of Small Orbit motion whether frequency is decreasing or increasing. Then at $P > P_{cr}^{(h)}$ Small Orbit tends to jump into Large Orbit in the neighbouhhod of the principal resonance considered. Chaotic motion occurs as a transition zone between resonant Small Orbit and Large Orbit on decreasing frequency only.

References

[1] P. Holmes, Strange phenomenon in dynamical systems and their physical implications, Appl. Math. Modelling, 1, 1977 362-366.

[2] P. Holmes A nonlinear oscillator with a strange attractor Phil. Trans. of the Royal Society of London, 292, 1979, 419-448.

[3] W.Y. Tseng and J. Dugundji, Nonlinear vibration of a buckled beam under harmonic excitation, J. Applied Mech. 38, 1971, 467-476.

[4] P. Holmes, Averaging and chaotic motion in forced Oscillations, SIAM, J. Appl. Math. 38, 1980, 65-80.

[5] J. Guckenheimer and P. Holmes, Nonlinear Oscillations, Dynamical Systems and Bifurcation of Vector Fields, New York, Springer-Verlag, 1983.

[6] F.C. Moon, Experiments on chaotic motion of a forced nonlinear oscillator: strange attractors, J. Appl.Mech. 47, 1980, 638-644.

[7] P. Holmes and F.C. Moon, Strange attractors and chaos in nonlinear mechanics, J. Appl. Mech. 50, 1983, 1021-1032.

[8] Ch. Hayashi, Nonlinear Oscillations in Physical Systems, New York, McGraw Hill Book Co., 1964.

[9] W. Szemplińska-Stupnicka and J. Bajkowski, The 1/2 subharmonic resonance and its transition to chaotic motion in a nonlinear oscillator, Int. J. Non-Linear Mech. /to appear/.

[10] W. Szemplińska-Stupnicka, Secondary resonances and an approximate model of transition to chaotic motion in nonlinear oscillators, J. Sound and Vibration, /to appear/.

[11] W.W. Bolotin, Dynamic Stability of Elastic Systems, San Francisko, Holden Day, 1964.

PRZYBLIŻONE KRYTERIUM WYSTĘPOWANIA RUCHU CHAOTYCZNEGO DLA MODELU WYBOCZONEJ BELKI

Streszczenie:

Drgania okresowe i chaotyczne w modelu wyboczonej belki zbadano przy użyciu przybliżonej teorii drgań nieliniowych oraz symulacji komputerowej. Wykazano, że rozwiązanie harmoniczne pierwszego przybliżenia dla ruchu po "małej" orbicie może być podstawą do wyprowadzenia przybliżonego kryterium występowania w układzie drgań chaotycznych. Chociaż przy użyciu symulacji komputerowej na granicy przejścia od ruchu okresowego do chaotycznego stwierdzono występowanie rozwiązań subharmonicznych /bifurkacje typu "period doubling"/ wyliczone na podstawie rozwiązania harmonicznego krytyczne parametry układu znajdują się w pobliżu rzeczywistej granicy ruchu chaotycznego.