4.41 — optymalizacja

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43/1985

WARSZAWA 1985

ISSN 0208-5658

Praca wpłynęła do Redakcji dnia 25 czerwca 1985 r.

N a   p r a w a c h   r ę k o p i s u

Instytut Podstawowych Problemów Techniki PAN

Nakład 140 egz. Ark.wyd. 2,1 Ark.druk. 3,25
Oddano do drukarni w listopadzie 1985 r.
Nr zamówienia 689/85

Warszawska Drukarnia Naukowa, Warszawa,
ul. Śniadeckich 8

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SOME PROBLEMS OF MULTICRITERIA OPTIMIZATION OF CABLE STRUCTURES

Summary

The paper deals with multicriteria structural optimization of cable structures. First, a general formulation of multicriteria optimization problem is presented and discussed. Next, some applications concerning the single and double layer-cable systems are considered. Minimum weight and maximum of the lowest natural frequency of free vibration are taken as optimization criteria. The permissible stresses and displacements are taken as behavioral constraints. The optimization problem is solved using nonlinear programming and metric function methods.

Introduction

The results of single criterion /scalar/ optimization of single and double layer-cable systems as well as cable nets under static loading have been presented in papers [25, 26, 27] . The present paper is concern with multicriteria structural optimization of single and double layer-cable systems under static and dynamic loading. The paper consists of three Chapters and one Appendix.

Chapter 1 is devoted to multicriteria optimization approach in optimum structural design. A few criteria of structural optimization have been listed and discussed.

Chapter 2 deals with two criteria optimization of single layer-cable systems. Minimum weight and maximum of natural frequency of free vibration are used as optimization criteria. The permissible stresses and displacements are taken as the behavioral constraints.

Chapter 3 cont aims two criteria optimization of double--layer-cable systems. The basic relations of dynamic response of double layer-cable have been presented and discussed. The minimum weight and maximum of natural frequency are choosed as optimization criteria. The permissible stresses and displacements are also taken as optimization constraints.

Appendix contains a few methods for selecting a preferable solution from the set of compromise solutions.

1. Multicriteria optimization in structural design

1.1. Characteristic of multicriteria optimization approach

Optimum structural design usually involve a number of requirements that should be met at the same time to obtain the fully useful design. In the case of single criterion optimization, one of the requirements is selected as the criterion, while the remaining ones are met by including them into the constraints set. But with such an approach, it is necessary to determine a priori the bounds which these requirements should fulfill.

Multiobjective /or multicriteria/ optimization enables us to take into account numerous criteria that are often mutually conflicting. It is then possible to find compromise and pre-preferable solutions which - although none of the criteria involved attains its extremum - guarantees meeting all the requirements in the best way possible.

The paper deals with the problem of formulating the multiobjective function and finding the set of compromise solutions and also with selecting a prefarable solution. In the following some criteria of structural optimization are listed and discussed, namely minimum volume or weight of a structure, minimum potential energy or maximum structural stiffness, minimum displacement at selected points or regions of the structure, maximum critical force, maximum natural frequency of free vibration, maximum moment of inertia and maximum safety or reliability. Some of these criteria will be considered in the following to solve optimization problems of cable structures.

The multicriteria optimization approach has been already discussed in many papers devoted to optimum structural design /see e.g. [15] and [18]. In papers [13] and [14] Eschenauer discussed the optimization problem of space structures that support radio-telescopes assuming the following criteria: minimum weight and minimum displacements of the radio teleskope surface from its initial configuration under different loading states. In [44] Sattler presented a survey of multicriteria optimization methods and their use for the optimization of a structure consisting of beam and truss elements. As optimization criteria he assumed the minimum weight of the lattice structure and the minimum displacements of the beam surface under different loading states. Koski [34, 35, 36] formulated the multicriteria optimization problem of bar structures assuming the minimum weight and minimum displacement of selected structural nodes as the objective functions. Stadler [47 - 50] applied two optimization criteria, namely minimum mass and minimum strain energy and calls the shapes thus determined the natural shapes.
In the general statement of the multicriteria optimization problems of structures given by Baier in [3, 4], the structural weight and energy stored under various loading states were assumed to be the optimization criteria. Carmichael [9] solved a multicriteria optimization problem by employing the method of constrained objective functions. The optimization problems of mechanical structures with numerous objective functions are also treated by Osyczka [37] and Rao and Hati [19].
The state-of-the art of multicriteria optimization approach in optimum structural design has been presented in [28,30,31].
In the following a brief formulation of the multicriteria

optimization problem will be presented. Also, the partial and global criteria of optimization will be discussed. They can be used to formulate the specific optimization problems. The multicriteria optimization approach will be demonstrated for cable structures.

1.2. Formulation of multicriteria optimization problem

The problem of multicriteria structural optimization is the generalization of a single-criteria optimization and it enables to get close to the real conditions that are decisive for the selection of a design solution. The problem of multicriteria optimization can be formulated as follows:

$$\min_{\underline{x} \in \Omega} \underline{f}(\underline{x})$$

where $\underline{f}: \Omega \rightarrow R^k$ is a vector objective function given by

$$\underline{f}^T(\underline{x}) = \{ f_1(\underline{x}), f_2(\underline{x}), \ldots, f_k(\underline{x}) \}$$

and $\Omega \subset R^n$ is a feasible domain defined by the equality and inequality constraints, i.e.

$$\Omega = \{ \underline{x} \in R^n \mid \underline{h}(\underline{x}) = 0, \ \underline{g}(\underline{x}) \leq 0 \}$$

The components $f_i : \Omega \rightarrow R$, $i=1,2,\ldots,k$ are called the criteria of optimization and $\underline{x}$ is the vector of design variables.

Since the particular components f_i of the objective vector are mutually conflicting, it is in general impossible to find the so called ideal feasible solution $f_i^{id} = \min_{\underline{x} \in \Omega} f_i$, $i=1,2,\ldots,k$. The problem of multicriteria optimization can be solved in two stages. The first stage consists in determining the Pareto solution. In the second stage a preferable solution will be found. A vector $\underline{\overset{*}{x}} \in \Omega$ is called Pareto optimal if and only if there exists no $\underline{x} \in \Omega$ such that $f_i(\underline{x}) \leqslant f_i(\underline{\overset{*}{x}})$ for $i \in K$, $K = \{1,2,\ldots,k\}$ with $f_i(\underline{x}) < f_i(\underline{\overset{*}{x}})$

for at least one i, $i \in K$. In other words, the above definition states that $\overset{*}{\underline{x}}$ is a Pareto optimal solution if there exist no feasible vector $\underline{x}$ which would decrease some criterion without causing a simultaneous increase in at least one criterion. There exist a number of methods which allow to generate the compromise set and they are discussed in numerous publications e.g. [7, 11, 19, 28, 29] They are broadly divided into two categories of non-preference techniques /including Pareto optimization/ and preference techniques. In the second stage, a preferable solution is determined on the basis of the compromise set. A few method for selection of a preferable solution are discussed in [16, 17, 22, 30, 34, 44] . A metric function method, method of utility functions, method of constrained objective functions and lexicographic method are often used to select the preferable solution.

1.3. Criteria used in structural optimization

1.3.1. Partial criteria of optimization

The partial criteria appearing most frequently in the optimization theory of structures are given in the following.

i) Minimum volume or weight of the structure. In accordance with the criterion of minimum material volume the optimal structure is one with the minimum material volume or the minimum weight that has been selected from all the possible structures satisfying the given conditions and are subjected to the same loads. If the structure has been build of a number of different materials, it may be required to minimize each one of them. Such requirement can concern e.g. the minimum volume of concrete and of the prestressing cables or the minimum volume of steel of different grades in the truss structure.

ii) The minimum potential energy or the maximum structural stiffness

In the case of this criterion, between all the possible structures satisfying the given conditions and subjected to the same loads, the optimal structure is one in which the work done by the loads due to strains has the minimum value. But the work done by the loads along displacements in the equilibrium state is equal to the energy /potential/ of elastic deformation and, thus, this energy has also its minimum value.

iii) Minimum displacements at selected points or regions of the structure. In some cases, it is required to minimize changes in the shape of the structure or its part under action of loads. Such a requirement concerns e.g. the support structure of radio-telescopes or some cable suspended structures.

iv) Maximum critical force. It is obvious, that buckling is important in compressed elements having considerable slenderness. Therefore, they should be shaped in such a way to obtain the maximum critical force. In some kinds of structures, the local buckling is most important and then it is necessary to ensure the maximum stiffness of particular structural elements.

v) Maximum natural frequency of free vibration. Dynamically loaded structures should be shaped in such a way to obtain the maximum natural frequency of free vibration. In same cases it is sufficient to take only into account the first frequency of free vibration /unimodal optimization/ but, in others, computations must be performed for the frequencies of higher vibration modes /multimodal optimization/. This criterion can be modified in such a way to require the maximum denuting of the vibration systems which consists in achieving the maximum separation between the frequencies of natural and forced vibration.

vi) Maximum moment of inertia. It is required to maximize the moment of inertia of bending and compressing members. The same requirement concerns vibrating systems. Thus, the criterion of the maximum moments of inertia is in line with many criteria mentioned above. For twisted members, it is necessary to maximize the polar moment of inertia.

vii) Maximum safety or reliability. These two criteria are always required but they are especially important in aerospace engineering and civil engineering structures build in seismic zones.

It should be emphasized that to satisfy any of the criteria mentioned above. It is necessary to find the extremum of an objective function with taking into account the related constraints. In the theory of structures, the most important constraints are related with:

- permissible stresses or the safety factor of the structure under all possible loading states,
- permissible displacements in a given structure,
- minimum and maximum sizes that are permissible in view of services and constructional reasons, e.g. the minimum thickness of metal plates or the maximum height of a bridge structure following from the given conditions of the transportation system.

1.3.2. Global criteria of structural optimization

The solution obtained due to meeting a few partial criteria of optimization mentioned above has usually the form of the compromise set. The optimal structure should be selected from this set on the basis of a global criterion which can take various forms in different optimization problems. Its choice is particularly vital because it will be decisive for the result of the entire optimization problem. The formulation of a global criterion depends also on the partial criteria that are taken into account in the optimization problem. It is the minimum cost criterion which is most frequently assumed in structural optimization problems. The total costs can include

the costs of not only materials but also of construction works and exploitation costs of the complited object. In some cases, it is also possible to take into account the cost of demolition of the object after the prescribed period of its exploitation has elapsed.
The assumption of a minimum cost criterion enables to directly solve the problem without determining the compromise set. However, the compromise set provides much more information on the optimal structure than the solution of the scalar optimization problem. It should be noted that the compromise set is the source of information on the relation between the degree to which the particular partial criteria are satisfied and the value of global criterion.
If the structure has been build of a number of different materials and if it can be assumed that its cost is proportional to the amount of materials used, then the minimum cost criterion can be presented as the sum of the products of the volumes of particular materials and the coefficients which are the measures of their unit costs.
Problems formulated in this way are solved by employing the metod of utility function [26, 28, 32].
Another global criterion used in structural optimization problems is the minimum of metric function in which the preferable solution is the point belonging to the compromise set the distance of which is minimum from the ideal point usually not feasible. Problems formulated in such a way were solved in [22, 28, 30, 31].
A multicriteria optimization problem can be solved in such a way that one of the partial criteria is selected to be the global one, while the remaining criteria are taken as the constraints the levels of fulfilment of which to be properly determined. Such an approach can be used especially for the following criteria: minimum displacement, maximum critical force and maximum natural frequency of free vibration. To such formulation there corresponds the method of constrained objective functions [22, 28, 30].

The particular manner of formulating multicriteria optimization problems consists in determining the hierarchy of importance of partial criteria. A solution is obtained by fulfilling first the most important criterion and then, if possible, the successive criteria. A solution depends on the criteria of lower importance if the solution obtained due to the optimization on the basis of the more important criteria is not unique. The problems formulated in this way were solved by employing the lexicographic method in [22, 28, 30]. The following examples of optimum structural design of cable structures illustrates the application of multicriteria optimization.

2. Two criteria optimization of single layer-cable systems

2.1. Characteristic of cable-suspended structures

This Chapter deals with an optimization problem of single-layer cable systems which are often used as carrying elements in mechanical structures e.g. building machines as well as the load-carrying elements in electric power lines or hanging rope-ways. Single layer-cable systems are also used in large-span roofing structures as shown on Figure 1.
Cable suspended structures are substantially different from other kinds structures because they are capable of assuming a variety of shapes under action of different loadings.
That is why static and dynamic analysis of cable-suspended structures are different from those commonly known. The first difference consists in that account the actual shape of the structure. The principle of structural rigidity cannot be used here. The second difference consists in the fact that the principle of superposition is inapplicable to cable-suspended structures. This follows from geometric nonlinearity of cables caused by large changes in cables shapes due to varying loadings. Also the elongations of the large span cables can result in major displacements and deformations of the structural shape. Then, to write the conditions of equi-

librium and deformability it is necessary to take into account, each time, all the loads acting on the structure which has no a priori determined shape.

It is assumed that the cable can not resist bending and compression and constitutes a kinematically varying system. The dead weight of cable can be neglected in comparison to the live loads acting on the cable. The physical nonlinearity of cable depends on their material behaviour and construction of cables. However, for the sake of simplicity the stress--strain relationships can be assumed as linear because within the range of working stresses the behaviour of cables obeys the Hooke's law.

The purpose of optimization in the design of cable systems is to find the best shape of cable structure according to minimum weight criterion and/or maximum of natural frequency of free vibrations. The first criterion comes from economical consideration. The second one is derived from experience that the most dangerous for dynamically loaded structure is usually the lowest natural frequency /e.g. in the case of wind loading/.

The dynamic analysis of cable systems is closely connected with its static solution. The cable shape and static internal force coming from static loading have a large influence on the natural frequency of free vibrations and their amplitudes as well as on the dynamic internal force. In the classical theory of elastic vibrations of structures such phenomenon does not occur.

2.2. Basic relationships of static and dynamic response

2.2.1. Large sag inextensible cables

Saxon and Cahn [45] have considered the in-plane free vibrations of an inextensible cable fixed at the rigid supports which are situated on the same level /Fig. 2./. They have shown in form of a diagram /Fig. 3/ the relationship between the natural frequencies of free vibrations which are

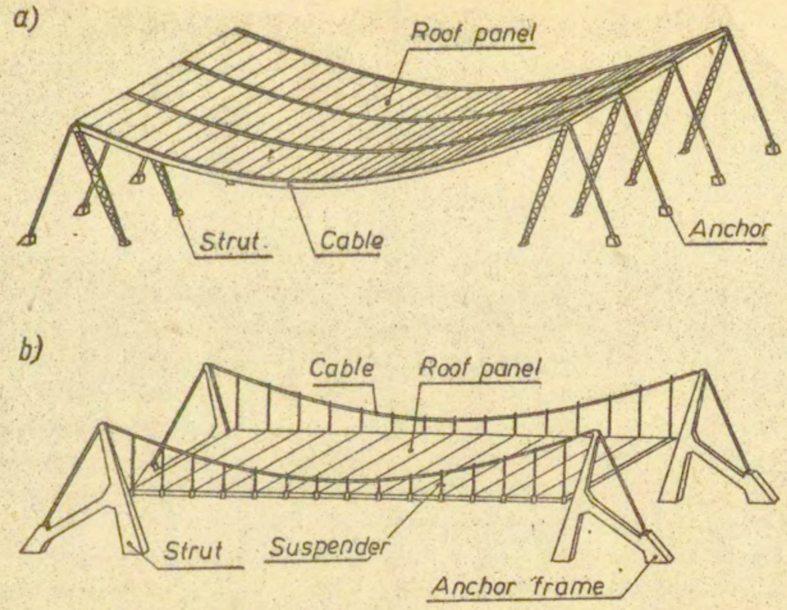


Fig. 1. Schematic diagrams of single layer-cable systems

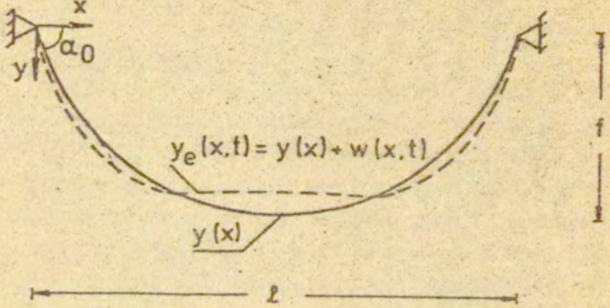


Fig. 2. A symmetric in-plane free vibration

determined by means of parameter A_n/π and the sag of the cable which can be calculated on the basis of an angle a_0 between tangent line to cable shape at the supports and horizontal line. This relationship is a monotonically decreasing function and maximal natural frequencies occurs for very small cable sags.

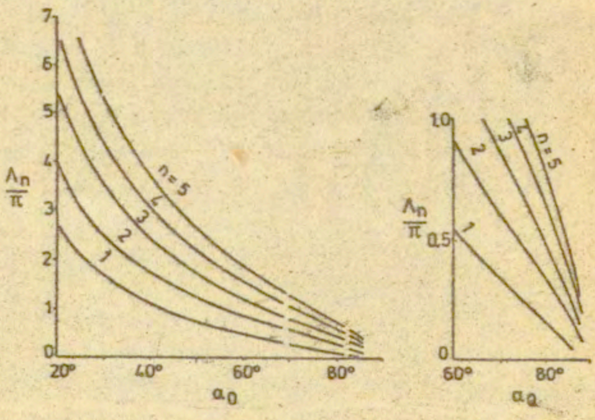


Fig. 3. Diagram of relationship between the parameter A_m/π describing the natural frequency and the angle a_0 [45]

On the other hand, it has been proved that the minimum weight of single layer-cable systems corresponds to the large cable sags [24]. This can be seen on Figure 4 showing the diagram of cable weight ρ with respect to cable sag $\eta = f/l$ with the optimal value of $\eta = 0.258$; /i.e. rather a large cable sag/.

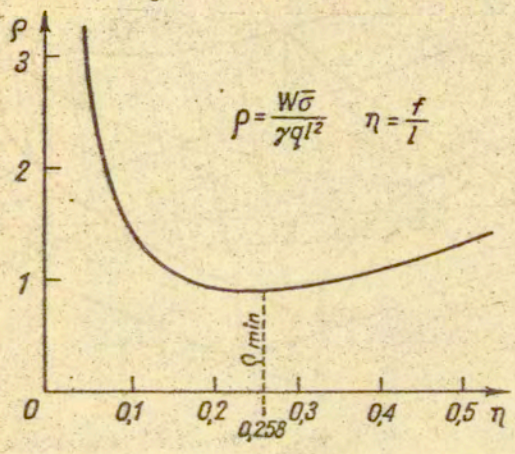


Fig. 4. Diagram of cable weight ρ versus cable sag η

From the comparison of results discussed above it can be seen that the objective functions, minimum weight and maximum of natural frequencies of free vibrations, are in conflict. It means that a compromise solution should be found. In order to get such a solution it is necessary to formulate and solve the multiobjective optimization problem for single layer-cable systems.

2.2.2. Flat sag extensible cables

The in-plane free vibrations of the extensible flat sag cables fixed at supports situated at the same level have been considered by Ananiev [2] and later by Rzanicyn [42]. Hajduk and Osiecki have developed the dynamic analysis of single layer-cable systems. They have presented a nomogram

/Fig. 5/ for determining the values of the lowest /first/ natural frequencies for symmetric in-plane free vibrations. It can be seen from Figure 5 that maximal values of the lowest natural frequencies occur for small cable sags i.e. η = 0.03 - 0.05 corresponds to the interval /20 - 200/ of the parameter ξ = ql/A, respectively.
On the other hand, the same extensible flat sag cables were optimized according to minimum weight criterion /see e.g. [8]/. It has been shown that optimal values of cable sags determined according to minimum weight criterion occur always on the boundary of the feasible domain /Fig. 6/ which was determined by the permissible sag /$\bar{f}$/, stress /$\bar{\sigma}$/ and displacement /$\bar{w}$/ constraints.
A similar conclusion as for large sag cables can be drawn also for flat sag cables by comparing the results discussed above: it means that a set of compromise solutions should be found by use of the multiobjective optimization approach.

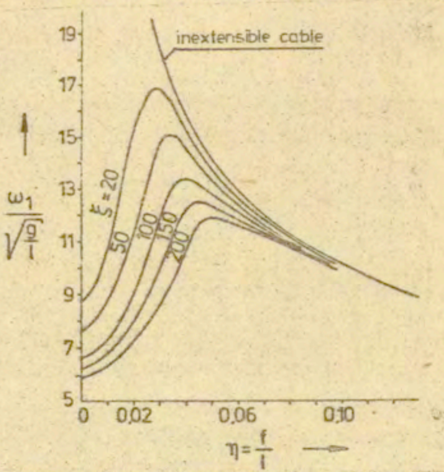


Fig. 5. Nomogram for determination of the lowest frequency for symmetric in-plane free vibrations of the cable /after [20] /

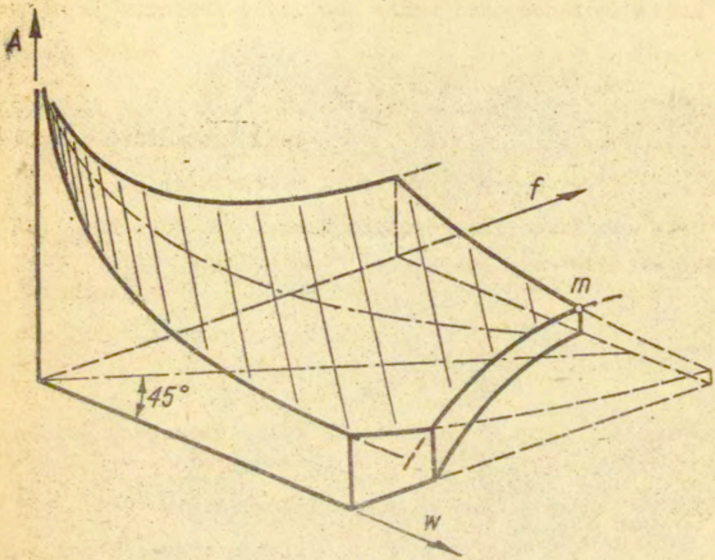


Fig. 6. Diagram of cross-section area A versus sag f and displacement w

2.3. Formulation of two criteria optimization problem

2.3.1. A general problem formulation

In general, the multiobjective optimization of single layer-cable systems can be formulated in the following way. Find the following design variables: cable sag $\eta = f/l$, cross-sectional area A and eventually also the material properties /e.g. modulus of elasticity E/ for given cable span l, static loading q(x), dynamic loading p(x,t) and permissible stresses $\tilde{\sigma}$ which minimize the weight of the single layer-cable system, i.e.

$$\min_{\eta \in \Omega} W = \int_0^s A(\eta, E)\, ds, \qquad /1/$$

and maximize the lowest natural frequency of the in-plane free vibrations

$$\max_{\eta \in \Omega} \omega_1 = \omega_1(\eta, A, E). \qquad /2/$$

Ω is the feasible domain of solutions and it is described by the following constraints:

- the static equilibrium equations

$$\frac{d}{dx}\left[T(x)\sin a_0(x)\right] + q_y(x) = 0$$
$$\frac{d}{dx}\left[T(x)\cos a_0(x)\right] + q_x(x) = 0 \qquad /3/$$

- the dynamic equilibrium equations

$$\frac{m}{\cos a_0(x)} \frac{\partial^2 w\, x,t}{\partial t^2} = \frac{\partial}{\partial x}\left[T(x)\frac{\frac{\partial w(x,t)}{\partial x} - \varepsilon(x,t)y'(x)}{1+\varepsilon(x,t)}\cos a_0(x)\right]$$
$$+ \frac{\partial}{\partial x}\left[\frac{N(x,t)\left[\frac{\partial w(x,t)}{\partial x} + y'(x)\right]}{1+\varepsilon(x,t)}\cos a_0(x)\right] + p_y(x,t)$$

/4/

$$\frac{m}{\cos a_0(x)} \frac{\partial^2 u(x,t)}{\partial t^2} = \frac{\partial}{\partial x}\left[T(x)\frac{\frac{\partial u(x,t)}{\partial x} - \varepsilon(x,t)}{1+\varepsilon(x,t)}\cos a_0(x)\right] +$$
$$+ \frac{\partial}{\partial x}\left[N(x,t)\frac{1 + \frac{\partial u(x,t)}{\partial x}}{1+\varepsilon(x,t)}\cos a_0(x)\right] + p_x(x,t)$$

- the geometric nonlinear equation

$$\varepsilon(x,t) = \left[\frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial u}{\partial x}\right)^2\right]\cos^2 a_0(x) +$$
$$-\left[\frac{\partial w}{\partial x}\left(\frac{dy(x)}{dx}\right) + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2\right]\cos^2 a_0(x) \qquad /5/$$

one of the following physical equations depending on the structural material behaviour taken into considerations

a/ Hooke's law

$$N(x,t) = E\,A\,\varepsilon(x,t) \qquad /6a/$$

b/ linear reological laws

$$\Pi(t)\,N(x,t) = A\,\Gamma(t)\,\varepsilon(x,t).$$

$\Pi(t)$ and $\Gamma(t)$ are linear differential operators with respect to time depending on the reological model of the material:

- For Voigt-Kelvin material:

$$\Pi(t) = 1, \quad \Gamma(t) = E + \bar{\eta}\,\frac{\partial}{\partial t}$$

where $\bar{\eta}$ - coefficient of internal damping: it has been obtained

$$N(x,t) = E\,A\,e(x,t) + \bar{\eta}\,A\,\frac{\partial e}{\partial t}; \qquad /6b/$$

- For the standard model:

$$\Pi(t) = 1 + \frac{\bar{\eta}}{E}\,\frac{\partial}{\partial t}, \quad \Gamma(t) = E + \left(1 + \frac{E}{E'}\right)\bar{\eta}\,\frac{\partial}{\partial t}$$

it has been found

$$N(x,t) + \frac{\bar{\eta}}{E'}\,\frac{\partial N}{\partial t} = E\,A\,e(x,t) + \left(1 + \frac{E}{E'}\right)\bar{\eta}\,A\,\frac{\partial e}{\partial t} \qquad /6c/$$

c/ plastic deformability

$$N(x,t) = \begin{cases} E\,A\,e(x,t) & \text{for } \varepsilon \le \varepsilon_e \\ A\left[E\,e(x,t) + \phi\,(\varepsilon - \varepsilon_e)\right] & \text{for } \varepsilon > \varepsilon_e \end{cases} \qquad /6d/$$

where ε_e is the elastic limit deformation and ϕ is the post-elastic behaviour function for the cable /e.g. Ramberg - Osgood law for postelastic material behaviour/

d/ rigid cable

$$\varepsilon(x,t) = 0 \qquad /6e/$$

- and the mechanical constraints concerning allowable stresses and displacements, e.g.

$$\sigma_{max} \leqslant \bar{\sigma} \qquad /7/$$

$$w_1(x,t)_{max} \leqslant \bar{w}_1 \quad \text{or} \quad w_2 = \int_0^l w(x,t)\,dx \leqslant \bar{w}_2 \qquad /8/$$

- and side constraints: $0.01 \leqslant \eta \leqslant 0.1$

The above system of nonlinear partial differential equations was derived on the basis of continuous model of mass distribution in dynamic analysis of single layer-cable systems. A solution of such a system of nonlinear equations cannot be found easily. But the system of equations can be linearized for flat sag cables and elastic material behaviour. In what follows, the system of linearized equations will be used to solve the multiobjective optimization problem for single layer-cable systems.

2.3.2. Problem formulation for the extensible flat sag cables

The multiobjective optimization problem of single layer--cable systems with the assumptions of flat sags and Hooke's law for material behaviour can be formulated as follows. Find the cable sag $\eta = f/l$ and cross-sectional area A for the given cable span l, static loading $q(x)$, modulus of elasticity E, dynamic loading $p(x,t)$ and permissible stress $\bar{\sigma}$ such that the weight of a single layer-cable system

$$W = \gamma A \cdot s = \gamma A\, l\left(1 + \frac{8}{3}\eta^2\right) \qquad /9/$$

is minimized, γ is the bulk density of the cable material. It was assumed here that the catenary can be replaced by a parabolic curve of second order

$$y = 4f\left(\frac{x}{l} - \frac{x^2}{l^2}\right); \qquad /10/$$

because of the flatness. In this case the approximated length of the cable is

$$s = l\left(1 + \frac{8}{3} n^2\right). \qquad /11/$$

In addition the first natural frequency of the in-plane free vibrations

$$\omega_1 = 2\, a_1 \sqrt{\frac{H_{st}}{ql}} \sqrt{\frac{g}{l}} \qquad /12/$$

has to be maximized.

This corresponds to the symmetric in-plane eigenmode

$$X_1(x) = C_1\left(1 - \cos a_1 \frac{2x}{l} - \tan a_1 \sin a_1 \frac{2x}{l}\right) \qquad /13/$$

where C_1 is a constant and a_1 can be determined from the following transcendental equation [20]

$$\tan a_1 = a_1 + a_1^3 \frac{H_{st}}{16\eta^2 EA} = 0. \qquad /14/$$

The optimal solution should satisfy the following system of inequality and equality constraints

- side constraints

$$0.01 \leq \eta \leq 0.1, \quad A > 0 \qquad /15/$$

- static governing equation

$$\eta = \frac{1}{8}\left[\left(\frac{ql}{H_{st}}\right)^2 - 24 \frac{H_{st}}{EA}\right]^{1/2} \qquad /16/$$

- stress constraint

$$\sigma_{max} = \frac{H_{st}+H_d}{A \cos a_{max}} = \frac{H_{st}+H_d}{A}\left(1+16\eta^2\right) \leq \bar{\sigma} \qquad /17/$$

where $H_d(t) = \frac{EA}{l} \int_0^l \frac{\partial w}{\partial x}\, y'(x)dx$

with $w(x,t) = \phi(t) \cdot X_1(x)$

- dynamic displacement constraint

$$w_{max} = \phi(t) \cdot X_1(x) \leq \bar{w} \,, \qquad /18/$$

where $\phi(t)$ is determined by the dynamic magnification factor regarding only the steady state response. The last two constraints arise from the dynamic response and can be calculated on the basis of the dynamic loading represented e.g. by

$$p(x,t) = p_0 \sin \Omega t \,. \qquad /19/$$

Substituting equation /16/ into /9/ gives the following cable weight function

$$W = \gamma A l \left[1 + \frac{1}{24}\left(\frac{ql}{H_{st}}\right)^2 - \frac{H_{st}}{EA} \right]. \qquad /20/$$

2.4. Solution of the optimization problem

2.4.1. Determination of the sets of the feasible and compromise solutions

In order to solve the multiobjective optimization problem it is necessary to determine the set of feasible solutions in the design space /A, η/ and the set of compromise solutions in the objective space /ω, W/. The sets of feasible and compromise solutions should satisfy the constraints (14)÷(18) as given above.

To find the sets of feasible and compromise solutions two problem formulations have been checked. In the first formulation we maximize the first natural frequency of free vibrations for a given cable weight W = const. and take the constraints /14/ - /18/ into account. The second one deals with the minimization of cable weight for a given frequency ω considering the same group of constraints /14/ - /18/. Both formulations give the same sets of feasible and compromise solutions as shown on Figures 7 and 8.

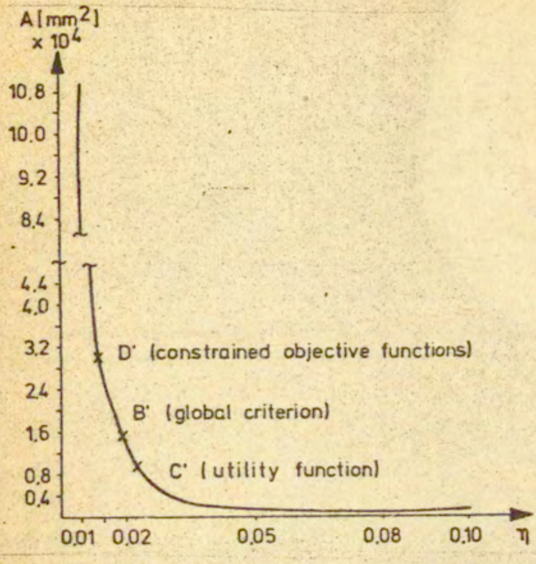


Fig.7 Representation of the design variable space

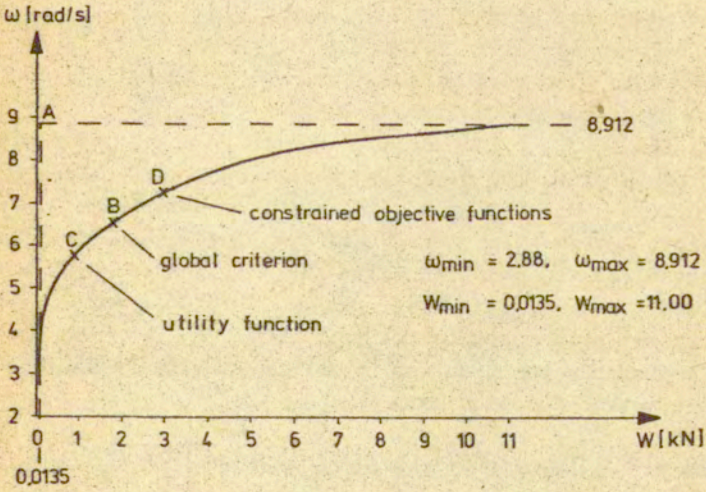


Fig.8 Representation of the objective function space

The optimization problems were solved with two different methods. The first one uses the method of Lagrangian multipliers [46] and require the gradients of the objective functions and the constraints. The gradients were evaluated numerically.
The second algorithm is based on the evolution strategy [41], which works with random numbers and can learn by the improvements of the objective function. This algorithm does not require any evaluation of gradients.
The two algorithms were used to compare their capability in solving the optimization problem described above and to increase the probability of attaining a global optimum.

2.4.2. Choosing a preferable solution

The set of compromise solutions shown in Figure 8 contains a number of solutions. It has to be decided which one should be taken as the preferable solutions. There exist a few methods of choosing a preferable solution from the set of compromise solutions.
Using a metric function /A.1. in Appendix/ to find a preferable solution we have $k = 2$ and we choose $p = 2$. The following data have been taken in numerical solutions:
$q = 1$ kN/m, $E = 200$ kN/mm^2, $\sigma = 1{,}2$ kN/mm^2, $\rho = 0.001$ kN/m, $\bar{\omega} = 1$ rad/s, $\delta = 0.06$, where δ is the logarithmic decrement of damping. The objective functions are $f_1(\underline{x}) = W(\eta, A)$ and $f_2(\underline{x}) = \omega_1(\eta, A)$. The ideal solution satisfying the constraints /14/ - /18/ was found numerically and shown in Figure 8 as the point A with the coordinates: $W_{min} = 0.0135$ kN, $\omega_{max} = 8.91$ rad/s. It does not belong to the set of compromise solutions.
The preferable solution was found numerically by minimization of the metric function with $p = 2$, i.e.

$$\min_{\eta \in \Omega} F^{(2)} = \left[(W - W^{id})^2 + \mu^2 (\omega_1 - \omega^{id})^2 \right]^{1/2}$$

where $\mu = 1$ kN s/rad

The preferable solution obtained by metric function is shown on Figure 8 as the point B / W^{pr} = 1.60 kN , ω^{pr} = = 6.46 rad/s / corresponding to the point B' on Fig. 7 /η = 0.0185, A = 15990 mm^2/.

Next we determine the preferable solution using the utility function method in the form of /A-3/ in Appendix with weighing factors w_1 = 0.5 and w_2 = 0,5. To find the preferable solution it is necessary to minimize the utility function

$$U(\underline{f}) = \sum_{i=1}^{2} w_i f_i(\underline{x}) = w_1 W(\eta, A) - w_2 \omega_1(\eta, A)$$

subject to the constraints /14/ - /18/. The preferable solution found numerically is shown on Fig. 8 as the point C /W^{pr} = 0.97 kN , ω^{pr} = 5.92 rad/s / corresponding to the point C' on Fig. 7 /η = 0.023, A = 9700 mm^2/.

The preferable solution can also be found by employing the method of constrained objective functions /see Appednix - A3/ A first naturaly frequency of free vibrations has been chosen as objective function which should be maximized, i.e.

$$\max_{\eta \in \Omega} \omega_1 = 2\, a_1 \sqrt{\frac{H_{st}}{ql}} \sqrt{\frac{g}{l}} \;,$$

subject to /14/ - /18/ and additional constraints concerning cable weight

$$\underline{W} \leq W(\eta, A) \leq \overline{W} \;.$$

The lower and upper limits of cable weight can be established on the basis of the permissible interval of cable sags which was taken as follows:

$$\underline{\eta} \leq \eta \leq \overline{\eta}$$

i.e. $W = W(\bar{\eta})$ and $\bar{W} = W(\underline{\eta})$. We have taken $\underline{\eta} = 0.015$ and $\bar{\eta} = 0.1$. The preferable solution is shown in Fig. 8 as the point D /$W^{pr} = 3.00$ kN , $\omega^{pr} = 7.176$ rad/s / corresponding to the point D' on Fig. 7 /$\eta = 0.015$, $A = 30000$ mm^2/.

3. Two-criteria optimization of double layer-cable systems

3.1. Basic relationships of static and dynamic analysis

3.1.1. Characteristic of double layer-cable systems

The double-layer cable systems are build of two layers - - upper and lower - and of struts in between. The strutes can work on tension or compression depending how the carrying and prestressing cables are mutually situated /see Fig. 10/. The double-layer cable systems can be used as elements for covering large areas as shown on Figure 9 . The value of initial prestressing should be determined on the basis that the forces in carrying and prestressing cables must be always positive.

Let us consider in details the double layer-cable system shown on Figure 10d. It is assumed that the struts are always vertical and inextensible i.e. the displacement of the lower and upper cables are the same. It is also assumed that the struts are continuously distributed and the cable sags are small.

3.1.2. The governing equation for double layer-cable systems

It is assumed that the carrying and prestressing cables are situated horizontally in initial configuration /colinearly with x-axis/. The deformed /actual/ configuration under static and dynamic loading is shown in Figure 10d.

The dynamic equations of motion using d'Alambert principle can be written in the following form [23, 38]:

- for the upper cable

$$\frac{\partial}{\partial x}\left(N_{22}\cos\theta_2\right) - m_2\frac{\partial^2 u_{22}}{\partial t^2} = 0 \qquad /21/$$

$$\frac{\partial}{\partial x}\left(N_{22}\sin\theta_2\right) - m_2\frac{\partial w_{22}}{\partial t^2} - 2\frac{\partial w_{22}}{\partial t} + q_e - q - p(x,t) = 0$$

- for the lower cable

$$\frac{\partial}{\partial x}\left(N_{12}\cos\theta_1\right) - m_1\frac{\partial^2 u_{12}}{t^2} = 0$$

$$/22/$$

$$\frac{\partial}{\partial x}\left(N_{12}\sin\theta_1\right) - q_e - m_1\frac{\partial^2 w_{12}}{\partial t^2} - \eta_1\frac{\partial w_{12}}{\partial t} = 0$$

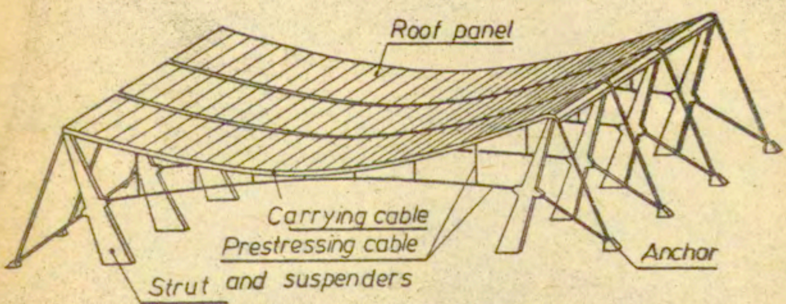


Fig. 9a. Double layer prestressed hanging roof

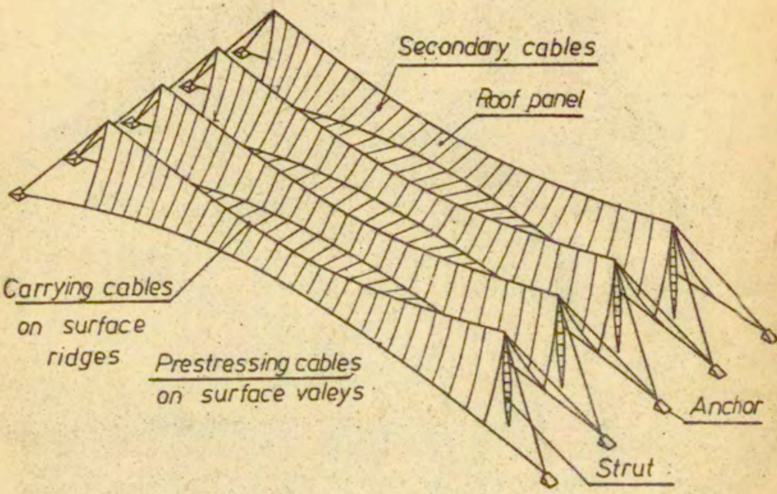


Fig. 9b. Double layer prestressed hanging roof

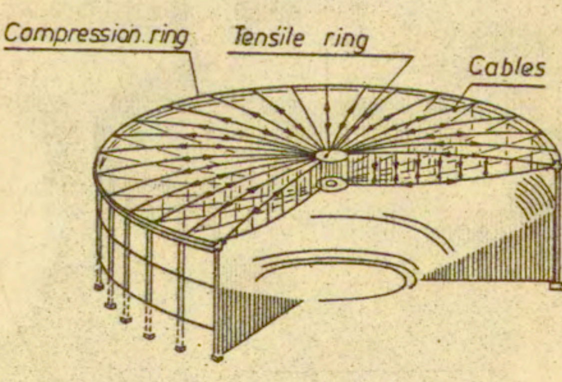
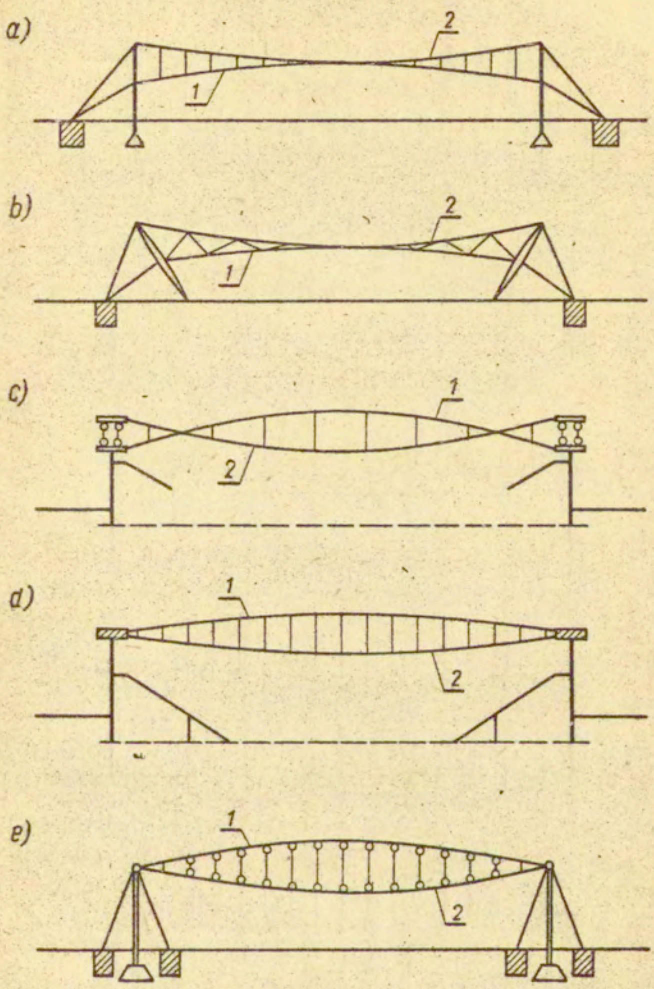


Fig. 9c. Double layer prestressed hanging roof



2-load-carrying cable 1-prestressing cable

Fig. 10 Double-layer cable systems

In the above equations the following notation is used:

m_1, m_2 - uniformly distributed masses of lower and upper cables, respectively,

q - static loading acting on the upper cable,

q_e - equivalent interaction between lower and upper cable,

$p/x,t/$ - forced /dynamic/ loading acting on the upper cable,

η - coefficient of viscous damping

w_{1j}, w_{2j} - total vertical displacement of lower and upper cables caused by initial, static and dynamic loading[x],

u_{1j}, u_{2j} - total horizontal displacement of lower and upper cables caused by initial, static and dynamic loading,

N_{1j}, N_{2j} - forces in lower and upper cables in initial, static and dynamic configurations,

θ_{ij} - an angle between the tangent to lower or upper cable and horizontal line, respectively.

Taking the previously mentioned assumptions the following formulaes are valid:

$$\sin \theta_{ij} = \frac{\dfrac{\partial w_{ij}}{\partial x}}{\left[\left(1 + \dfrac{\partial u_{ij}}{\partial x}\right)^2 + \left(\dfrac{\partial w_{ij}}{\partial x}\right)^2\right]^{1/2}}$$

---

[x] Indices ij at N, H, u and w denote the type of cable and the configuration, respectively: i=1,2 denote the load-carrying and prestressing cables, j=0,1,2 denote the initial, static and dynamic configurations, respectively.

$$\cos\theta_{ij} = \frac{1 + \frac{\partial u_{ij}}{\partial x}}{\left[\left(1 + \frac{\partial u_{ij}}{\partial x}\right)^2 + \left(\frac{\partial w_{ij}}{\partial x}\right)^2\right]^{1/2}} \qquad /23/$$

$$N_{ij} = E_i A_i \left\{ \left[\left(1 + \frac{\partial u_{ij}}{\partial x}\right)^2 + \left(\frac{\partial w_{ij}}{\partial x}\right)^2\right]^{1/2} - 1 \right\}, \qquad /24/$$

$$H_{ij} = \text{const. i.e. } \frac{\partial}{\partial x}\left(H_{ij}\right) = 0 \text{ where } H_{ij} = N_{ij}\cos\theta_{ij} \qquad /25/$$

H_{ij} are the horizontal components of cable forces /thrusts/ in initial and actual /static, dynamic/ configurations. E_i and A_i denote Young's modulus and cross-section areas for lower and upper cables, respectively.
Taking into account /23/ - /25/ and using power series expension of the expressions with roots and neglecting small of higher order the following equation is obtained from /21/ and /22/:

$$H_{12}\frac{\partial^2 w_{12}}{\partial x^2} + H_{22}\frac{\partial^2 w_{22}}{\partial x^2} - q - \eta\frac{\partial \xi}{\partial t} - m\frac{\partial^2 \xi}{\partial t^2} = -p(x,t) \qquad /26/$$

where $m=m_1+m_2$, $\eta = \eta_1+\eta_2$ and $\xi(x,t)$ denote the dynamic displacement of truss system /the same for lower and upper cables/ and

$$H_{12} = H_{10} + \frac{E_1 A_1}{2l}\int_0^l \left(\frac{\partial w_{12}}{\partial x}\right)^2 dx$$

$$H_{22} = H_{20} + \frac{E_2 A_2}{2l}\int_0^l \left(\frac{\partial w_{22}}{\partial x}\right)^2 dx \qquad /27/$$

H_{10} and H_{20} are the values of initial prestressing for the lower and upper cables, respectively. The total displacement w_{i2} can also be written as follows

$$w_{12} = w_{11}(x) + \xi(x,t)$$
$$w_{22} = w_{21}(x) + \xi(x,t) \qquad /28/$$

Static displacement w_{i1} can be presented in the trygonometric series form i.e.

$$w_{11}(x) = \sum_{i=1}^{n} a_{1i} \sin \frac{i\pi x}{l}$$
$$w_{21}\, x = \sum_{i=1}^{n} a_{2i} \sin \frac{i\pi x}{l} \qquad /29/$$

where $i=1,2,\dots,n$ and $n \ll N$; N denotes number of struts.

Using generalized coordinate approach the dynamic displacement $\xi(x,t)$ of truss system is taken in the following form:

$$\xi(x,t) = \sum_{i=1}^{n} \phi_i(t) \sin \frac{i\pi x}{l} \qquad /30/$$

Taking into account /29/ and /30/ the cable forces can be written as follows:

$$H_{12} = H_{1o} + \frac{E_1 A_1}{4\, l^2} \sum_{j=1}^{n} j^2 \left(a_{1j} + \phi_j\right)^2$$
$$/31/$$
$$H_{22} = H_{20} + \frac{E_2 A_2}{4\, l^2} \sum_{j=1}^{n} j^2 \left(a_{2j} + \phi_j\right)^2$$

Using the relationship /29/ ÷ /31/ and assuming $p(x,t) = B(x)\, b(t)$ the following system of differential equations based on /26/ has been obtained:

$$\ddot{\phi}_i + \nu \dot{\phi}_i + \alpha_i \phi_i + \delta_{i1}\left(a_{1i} + \phi_i\right) \sum_{j=1}^{n} j^2 \left(a_{1j} + \phi_j\right)^2 +$$

$$+ \delta_{i2}\left(a_{2i} + \phi_i\right) \sum_{j=1}^{n} j^2 \left(a_{2j} + \phi_j\right)^2 - \delta_{i1} a_{1i} \sum_{j=1}^{n} j^2 a_{1j}^2 - \qquad /32/$$

$$- \delta_{i2} a_{2i} \sum_{j=1}^{n} j^2 a_{2j} = S_i\, b(t)$$

where $b(t)$ is a certain time function. The following notation is used above

$$\nu = \frac{\eta}{m}\,, \quad \alpha_i = \frac{i^2 \pi^2 \left(H_{1o} + H_{2o}\right)}{ml^2}$$

$$\delta_{i1} = \frac{i^2 \pi^4 E_1 A_1}{4ml^4}\,, \quad \delta_{i2} = \frac{i^2 \pi^4 E_2 A_2}{4ml^2}$$

$$S_i = \frac{2}{ml} \int_0^l B(x) \sin \frac{i\pi x}{l}\, dx\,.$$

3.1.3. Free and forced vibration of double layer-cable system

a/ For free vibration

$$\nu = 0 \quad \text{and} \quad S_i = 0 \qquad /33/$$

should be put into /32/. The trygonometric approximation method is taken to solve a problem i.e.

$$\phi_i = C_i \cos \omega_i t \qquad /34/$$

Substituting /34/ into /32/ and neglecting the higher order harmonics and also combination of modes which are weakly nonlinear the following relation for frequency of free vibration has been found

$$\omega_i = \sqrt{\alpha_i}\left[1 + \frac{3\delta_{i1}i^2a_{1i}^2}{2\alpha_i} + \frac{3\delta_{i1}i^2C_i^2}{8\alpha_i} + \right.$$

$$+ \frac{\delta_{i1}}{2\alpha_i}\sum_{\substack{j=1\\ /j\neq i/}}^{n} j^2\left(\frac{C_j^2}{2} + a_{1j}^2\right) + \frac{3\delta_{i2}i^2a_{2i}^2}{2\alpha_i} + \frac{3\delta_{i2}i^3C_i^2}{8\alpha_i} +$$

$$\left. + \frac{\delta_{i2}}{2\alpha_i}\sum_{\substack{j=1\\ /j\neq i/}}^{n} j^2\left(\frac{C_j^2}{2} + a_{2j}^2\right)\right]. \qquad /35/$$

b/ The steady-state harmonic vibration. Assuming $b(t) = \sin\bar{\omega}t$ and taking solution in the form $\emptyset_i = d_i + C_i\sin(\bar{\omega}t + \varphi_i)$ on the basis of equation /29/ the following relations have been found

$$\alpha_i d_i + \delta_{i1}\sum_{k=1}^{n} k^2\left[(a_{1k}+d_k)^2(a_{1i}+d_i) + (a_{1i}+d_i)\frac{C_k^2}{2} + \right.$$

$$/36/$$

$$+ (a_{1k}+d_k)C_kC_i\cos(\varphi_k-\varphi_i)\Big] + \delta_{i2}\sum_{k=1}^{n} k^2\left[(a_{2k}+d_k)^2(a_{2i}+d_i) + \right.$$

$$\left. + (a_{2i}+d_i)\frac{C_k^2}{2} + (a_{2k}+d_k)C_kC_i\cos(\varphi_k-\varphi_i)\right] - \delta_{i1}a_{1i}\sum_{k=1}^{n} k^2a_{1k} -$$

$$- \delta_{i2}a_{2i}\sum_{k=1}^{n} k^2a_{2k} = 0,$$

$$C_i = \frac{S_i}{\sqrt{D_i^2 + E_i^2}} \quad \text{and} \quad \operatorname{tg}\varphi_i = -\frac{E_i}{D_i};$$

where

$$D_i = -\bar{\omega}^2 + \alpha_i + \delta_{i1}\sum_{k=1}^{n} k^2 \Big[(a_{1k} + d_k)^2 + \frac{C_k^2}{2} + \frac{C_k^2}{4}$$

$$\cos 2(\varphi_k - \varphi_i) + 2(a_{1k} + d_k)(a_{1i} + d_i)\frac{C_k}{C_i}\cos(\varphi_k - \varphi_i)\Big] +$$

$$+ \delta_{i2}\sum_{k=1}^{n} k^2 \Big[(a_{2k} + d_k)^2 + \frac{C_k^2}{2} + \frac{C_k^2}{4}\cos 2(\varphi_k - \varphi_i) +$$

$$+ 2(a_{2k} + d_k)(a_{2i} + d_i)\frac{C_k}{C_i}\cos(\varphi_k - \varphi_i)\Big] . \qquad /37/$$

$$E_i = \nu\bar{\omega} + \delta_{i1}\sum_{k=1}^{n} k^2 \Big[\frac{C_k^2}{4}\sin 2(\varphi_k - \varphi_i) + 2(a_{1k} + d_k)(a_{1i} + d_i)$$

$$\frac{C_k}{C_i}\sin(\varphi_k - \varphi_i)\Big] + \delta_{i2}\sum_{k=1}^{n} k^2 \Big[\frac{C_k^2}{4}\sin 2(\varphi_k - \varphi_i) +$$

$$+ 2(a_{2k} + d_k)(a_{2i} + d_i)\frac{C_k}{C_i}\sin(\varphi_k - \varphi_i)\Big] ,$$

which allows to determine the invariable component d_i, amplitude C_i and phase angle φ_i in term of frequency $\bar{\omega}$. The relationships $d_i(\bar{\omega})$, $C_i(\bar{\omega})$ and $\varphi_i(\bar{\omega})$ can be found using sequential iteration method. For example, as the initial /zero/ approximation can be taken C_{io}, φ_{io} and $\bar{\omega}_o$ which corresponds to keeping only the terms $k=i$ under the sign $\sum_{k=1}^{n}$. In this way for a given d_i the following quantities $C_{io}/d_i/$, $\varphi_{io}/d_i/$ and $\bar{\omega}_o(d_i)$ can be determined. More exact values of C_i, φ_i and d_i can be found by substituting their initial /zero/ approximation into /36/ and /37/. Finally, on the basis of above calculations, the diagrams $C_i(\bar{\omega})$ and $d(\bar{\omega})$ can be found.

3.2. Formulation of optimization problem

In general, the multiobjective optimization of double layer-cable systems can be formulated in the following way. Find the cable sags w_{11}, w_{21} and cross-section areas A_1, A_2 as well as the values of initial prestressing H_{10}, H_{20} for a given truss system span l, static loading q, dynamic loading $p(x,t)$ and permissible stress $\bar{\sigma}$ which minimize the weight of structure i.e.

$$\min W(w_{i1}, A_i, H_{io}) = \gamma \int_0^l \left\{ A_1 \left[1 + \frac{1}{2} \left(\frac{dw_{11}}{dx} \right)^2 \right] + A_2 \left[1 + \frac{1}{2} \left(\frac{dw_{21}}{dx} \right)^2 \right] \right\} dx \qquad /38/$$

and maximize the lowest natural frequency of the in-plane free vibration given in explicite form by equation /35/.
The optimal solution should satisfy the following system of inequality and equality constraints:

- side constraints

$$0.01 \leqslant \frac{w_{i1}}{l} \leqslant 0.1, \qquad A_i > 0, \qquad i=1,2 \qquad /39/$$

- static governing equation

$$q = H_{11}\frac{d^2 w_{11}}{dx^2} + H_{21}\frac{d^2 w_{21}}{dx^2} \qquad /40/$$

where $$H_{11} = H_{10} + \frac{E_1 A_1}{2l} \int_0^l \left(\frac{dw_{11}}{dx} \right)^2 dx$$

$$H_{21} = H_{20} + \frac{E_2 A_2}{2l} \int_0^l \left(\frac{dw_{21}}{dx} \right)^2 dx \qquad /41/$$

- stress constraints

$$\sigma_{k\max} = \frac{H_{ij}}{\cos\Theta_{ij\max}} \frac{1}{A_i} \leq \bar{\sigma} \quad i=1,2,\ j=0,1,2 \qquad /42/$$

- dynamic displacement constraints

$$\xi_{\max}(x,t) = \sum_{i=1}^{n} \phi_i(t) \sin\frac{i\pi x}{l} \leq \bar{\xi} \qquad /43/$$

where $\phi_i(t)$ is determined by steady-state response.

The optimization problem as formulated above can be solved in a similar way as in Chapter 2 using sequental quadratic programming methods [46] .

4. Conclusions

The following conclusions might be drawn on the basis of the results presented above:

i) The optimal cable sags obtained separately according to minimum weight and maximum first frequency of free vibrations occur in different intervals of the feasible domain.

ii) In order to satisfy two conflicting criteria mentioned above the multicriteria optimization problem has been formulated and the sets of feasible and compromise solutions has been found.

iii) An advantage of multicriteria optimization approach is in getting much more information about the optimal solution then from the single criterion optimization.

iv) The best compromise solution can be determined on the basis of user preferences or according to utility function.

v) If there are not preferences and it is difficult to build the utility function then the metric function can be used to found the preferable solution.

vi) The compromise set is an objective notion but the preferable solution depends on global criterion. In the case of metric function the preferable solution depends also on the form of dimensionless metric function.

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APPENDIX: Methods for selecting a preferable solution

A.1. Metric function method

The metric function method can be used to solve the following problem: find the minimum of the vector objective function, i.e.

$$\min_{\underline{x} \in \Omega} \underline{f}(\underline{x})$$

where $\Omega = \left\{ \underline{x} \in R^n \mid \underline{h}(\underline{x}) = 0,\ \underline{g}(\underline{x}) \leq 0 \right\}$.

The first step consists in finding the ideal solution, that is, the vector $f_j(\underline{x}^{id})$, j=1,2,...,k which satisfies the minimum condition of each objective function $f_j(\underline{x})$ considered independently of the remaining ones. Then, the metric function is formulated-the distance between the optimal and ideal point

$$F^{(p)} = \left(\sum_{j=1}^{k} \left| f_j(\underline{x}) - f_j(\underline{x}^{id}) \right|^p \right)^{1/p}, \quad 1 \leq p < \infty \qquad /A.1./$$

There exist the following cases with respect of p

$$p=1 \qquad F^{(1)} = \sum_{j=1}^{k} f_j(\underline{x}) - f_j(\underline{x}^{id})$$

$$p=2, \qquad F^{(2)} = \left\{ \sum_{j=1}^{k} \left[f_j(\underline{x}) - f_j(\underline{x}^{id}) \right]^2 \right\}^{1/2}$$

$$p \rightarrow \infty \quad F^{(\infty)} = \max_{j=1,2,\ldots,k} \left| f_j(\underline{x}) - f_j(\underline{x}^{id}) \right|.$$

The optimal vector is one that minimizes the metric function. The optimal solution depebds substantially on the parameter p.

For example, Boychuk and Ovchinnikov [7] propose to assume $p=1$, while Salukvadze [43] suggests to put $p=2$.

If the particular functions $f_j(\underline{x})$ involve different units, then they are multiplied by coefficients $\mu=1$ that include the corresponding units so that the expressions $\mu_j f_j(\underline{x})$ become dimensionless.

In numerous works e.g. [12,32,33,44] the metric function is assumed in the form

$$F^{(p)} = \left\{ \sum_{j=1}^{k} \left(\frac{f_j(\underline{x}) - f_j(\underline{x}^{id})}{f_j(x^{id})} \right)^p \right\}^{1/p} , \qquad /A.2/$$

However, it should be noted that in the case of /A.1/ and $p=2$, the distance between the points defined in the objective space by the vectors $f_j(x^{pr})$ and $f_j(x^{id})$ is minimum, that is $f_j(x^{pr})$ represents the point that belongs to the compromise set and is closest to the ideal solution. In the case of the criterion /A2/ and $p=2$, the solution $f_j(x^{pr})$ is not the point being nearest to the ideal solution. In the case of the criterion /A1/ the preferable solution is, however, dependent of the units of the function $f_j(\underline{x})$.

Numerical computation can be simplified by presenting all the objective functions in the dimensionless form. To this end, it is possible to introduce the following vector objective function

$$\underline{\tilde{f}}(\underline{x})^T = \left[\tilde{f}_1(\underline{x}), \tilde{f}_2(\underline{x}), \ldots, \tilde{f}_k(\underline{x})\right],$$

the components of which have the form

$$\tilde{f}_i(\underline{x}) = \frac{f_i(\underline{x}) - \min f_i(\underline{x})}{\max f_i(\underline{x}) - \min f_i(\underline{x})}$$

Thus, the values of each normalized objective function vary within the interval /0,1/. On the other hand, the minimum distance between the optimal and ideal points can be written in the following form

$$\min_{f \in f(\Omega)} F^{(p)}\left(\tilde{\underline{f}}(\underline{x}), 0\right).$$

However, the preferable solution depends on the manner in which the objective function $f_i(\underline{x})$ has been normalized.

The properties of the preferable solution are as follows

i) If the compromise set is a compact set, then there exists a solution for every $p \geq 1$.

ii) The overall deviation from the ideal solution represented by the number $F^{(p)}$ attains its minimum.

iii) For $1 \leq p < \infty$ the set $F^{(p)}$ is a subset of the compromise set.

iv) If the compromise set is convex, then the solution is unique.

v) If $F^{(p)}$ is interpreted as an estimate of the solution $\underline{x}$ from the viewpoint of the n-th criterion, then it could be shown [51] that with a small p all the criteria $f_j(\underline{x})$ are more rigorously taken into account, while with a large p the largest deviations from the ideal solution would be minimized.

A.2 Method of utility function

Employing this method, the problem of multicriteria optimization is formulated as follows: find the minimum of the function U i.e.

$$\min_{\underline{f} \in f(\Omega)} U = U(f_1, f_2, \ldots, f_k)$$

where $\Omega = \left\{ \underline{x} \in E^n \mid \underline{h}(\underline{x}) = 0, \quad \underline{g}(\underline{x}) \leq 0 \right\}$

The function $U(f_1,f_2,\dots,f_k)$ is called the multicriteria--utility function. In many cases, it can be difficult to define this function. This problem was treated by, among others, Farguhar [16] Fishburn [17], Huber [21] and Keeney and Raiffa [32,33].

The solution of the problem is the contact point between the compromise set and the countour lines of the function U /for details see e.g. Chankong and Haimes [10]/.

The utility function $U(f_1,f_2,\dots,f_k)$ can take various forms. It is most frequently additive and disjunctive with respect to the objective function, that is

$$U(f_1,f_2,\dots,f_k) = U_1(f_1) + U_2(f_2) + \dots + U_k(f_k).$$

In a particular case, the prioritization factors of individual objective functions can be given and then

$$U(f_1,f_2,\dots,f_k) = \sum_{j=1}^{k} w_j f_j(\underline{x}). \qquad /A.3/$$

Other form of the utility function can be e.g.

$$U(f_1,f_2,\dots,f_k) = \prod_{j=1}^{k} U_j(f_j).$$

The advantage of this method is its simplicity and the reduction of the problem of multicriteria optimization to the optimization with a single objective function. The principal difficulty lies in the determination of a utility function.

A.3 Method of constrained objective functions

This method is applicable provided that it is possible to determine the maximum values to be attained by the particular objective function. If this is possible, the problem of multicriteria optimization can be formulated as follows: find

$$\min_{\underline{x} \in \Omega} f_r(\underline{x})$$

where $\Omega = \{ \underline{x} \in R^n \mid \underline{h}(\underline{x}) = 0,\ \underline{g}(\underline{x}) \leq 0,\ l_j \leq f_j(\underline{x}) \leq u_j,\ j = 1,2,\dots,k,\ j \neq r \}$.

In using this method, the main difficulty consists in finding such constraints l_j and u_j that would ensure the attainment of particular objectives and the existence of a non-empty objective region. One can also very these values and perform trade-off analysis as in the surrogate worth trade-off method /see e.g. Haimes and Hall [19] /. Moreover, it is necessary to make a decision which of the objective functions should be selected as a criterion in solving the problem.

A.4 Lexicographic method

This method requires to order the components of the objective function vector in accordance with their importance. In this method the preferable solution is one that minimizes the objective function with beginning from that having the most important rank and running through those having the less and less important ranks.

If the indices from 1 to k represent importance rank in the hierarchy, then the optimization problem can be formulated as follows: find

$$\min_{\underline{x} \in \Omega} f_1(\underline{x})$$

$$\Omega = \{ \underline{x} \in R^n \mid \underline{h}(\underline{x}) = 0, \underline{g}(\underline{x}) \leq 0 \}$$

The solution of this problem are $\underline{x}^*$ and $f_1^* = f_1(\underline{x}_1^*)$. If this solution is unique, then $\underline{x}^*$ is considered to be the preferable solution of the primary problem. Otherwise, it is necsible solve the second problem: find

$$\min_{\underline{x} \in \Omega} f_2(\underline{x})$$

subject to the additional constraints

$$f_1(\underline{x}) = f_1^*$$

If this solution is unique, then $\underline{x}^*$ is the preferable solution of the primary problem. Otherwise, it is possible to apply the preceding procedure and to find

$$\min_{\underline{x} \in \Omega} f_j(\underline{x})$$

with satisfying the additional constraints

$$f_l \; \underline{x} = f_l, \qquad l = 1,2,\ldots,j-1.$$

The procedure is stopped when a unique solution is obtained. Objective functions of lower important ranks in the hierarchy are disregarded.

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