PATTERN RECOGNITION AND STATISTICAL LEARNING IN STOCHASTIC MECHANICS

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1. Introduction

Modern engineering materials often display substantial heterogeneity in their properties at scales that are similar to those at which material damage initiates. Some examples are the variation of crystallographic orientation in polycrystalline metals and the elastic property mismatch between reinforcing particles/fibers and the matrix of composite materials. The efficient and safe use of these materials in demanding engineering applications such as aviation, space, and defense structures requires detailed understanding of the effect of these material heterogeneities on stress and strain fields developed under loading and the resulting effect on material service lifetimes.

Engineering analysis of materials with strong spatial heterogeneity of material properties poses many significant challenges, among which are the randomness of the geometry, the very strong gradients present in the material property fields, and the large difference in scales between the smallest material constituents and the material volumes used in engineering applications. All of these challenges make the use of standard finite element analysis difficult and computationally expensive, often requiring very fine meshes, and, in the case of materials with random microstructures, expensive Monte Carlo simulation to provide statistics of the expected response.

This presentation describes an alternative, approximate, method for the analysis of materials with spatially heterogeneous material properties that makes use of well-developed tools of pattern recognition and statistical learning. The objective is to develop a method that can predict, without solution of the governing equations of elasticity, the location of large elastic stress or strain concentration in a heterogeneous material subject to deterministic boundary conditions.

2. Problem Statement

Let $D \subset R^n$ be a domain occupied by a material with spatially varying elastic properties $C(x)$, subject to Dirichlet or Neumann boundary conditions, or a combination of the two types. The boundary conditions generate a response $r(x)$ in the material which can consists of the stress and strain fields $\sigma(x)$ and $\varepsilon(x)$. The critical regions of the material are those in which a condition of the type

$$r(x) > r_{threshold}$$

is satisfied. That is, locations at which the material response exceeds a threshold value. Examples include conditions on the allowable maximum principal stress or strain, the maximum shearing stress, or any other combination of stress or strain values. Typically, such a condition would be associated with a criterion for the onset of damage in the material. The goal is to identify the critical regions that are defined by

$x \in D_{critical}$ if $r(x) > r_{threshold}$.

3. Methods

The problem stated above is solved in an approximate fashion by detecting patterns in the material property fields that are associated with the criticality condition being met. The first step in the analysis is to identify such patterns through the analysis of a set of training data. These training data typically comprise a set of randomly generated microstructures for which the response field has been calculated using finite element analysis so that for each element of the training set the criticality can be determined.

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The second step in the analysis consists of identifying patterns in the set of training samples in which the response is critical. This step can be accomplished by a variety of data mining techniques and the two used here are Principal Components Analysis and analysis by the Sobol' Decomposition. At the end of this stage of analysis a set of basis vectors are established that can be used to represent the spatially varying material properties of the material.

Finally, using the training samples and the new basis vectors, classifiers are developed that predict, based on a projection of a random microstructure onto the new basis vectors, whether the particular microstructural configuration is likely to lead to critical material response that may in turn lead to damage initiation. The classifiers, either support vector machines or decision trees [1], can be implemented in a moving window algorithm to extract $D_{critical}$ from D. The success of the approach can be evaluated by assessing the number of true positive, true negative, false positive, and false negative results. In an engineering context, false negative results, which falsely indicate safety, are non-conservative, and the classifiers can be trained to avoid such errors.

4. Example Applications

This presentation describes the application of the above methods to two example application problems. In both cases the criticality criterion is based on maximum principal stress/strain and the material is assumed to remain elastic when subject to uniaxial extension. The first example considers a two dimension fiber-reinforced composite material [2,3] and the second example considers a two dimension polycrystalline material in which the grains have varying crystallographic orientation [4]. Results in both cases are good, with high true positive and low false negative rates. Figure 1 shows an example result for the polycrystalline case, in which the classifier broadly predicts the locations at which the stress in the material is highly elevated. The black pixels indicate critical locations, and panel (b) shows the prediction while panel (c) shows the 'exact' result obtained by finite element analysis.

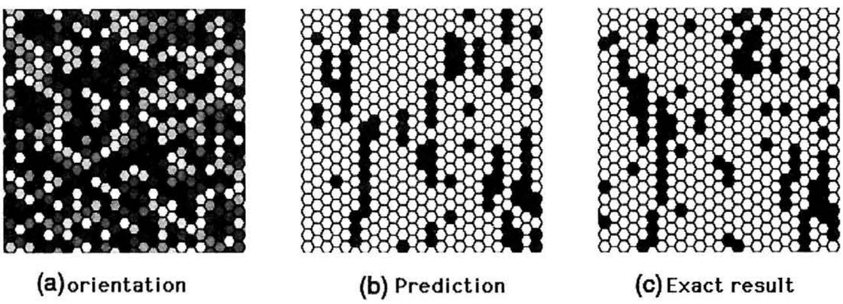

Figure 1: Example classification of polycrystalline microstructure. Orientation variation produces elastic property variation (a), classifier predicts location of stress concentration (b), finite element analysis provides validation result for classifier prediction (c).

5. References

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