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INTRODUCTION

The distribution of the near field produced by a piston has been examined by several authors [1][4]. The structure of the field, which is very complex as a result of interference was described in terms of acoustic pressure. Such a description, however, does not characterize sufficiently the properties of the field, for it supplies no information regarding the distribution of particle velocity, intensity or acoustic impedance.

ANALYSIS OF FIELD DISTRIBUTION

Rayleigh´s expression [3] determines the potential of particle velocity in a fluid medium, which arises in consequence of vibrations of the plane surface of a piston.

$$\varphi = -\frac{1}{2\pi}\int_F \frac{\partial\varphi}{\partial n}\frac{e^{-ikr}}{r}\,dF \qquad /1/$$

where F - surface of the piston, r - distance from the surface element dF to the point of the field in consideration, $k = 2\pi/\lambda$, λ- wave length. We assume a constant normal velocity w over the entire surface of the piston and a steady in time, thus $\partial\varphi/\partial n = w.e^{i\omega t}$ The acoustic pressure p is equal to $p = -\varrho\partial\varphi/\partial t$, where ϱ - density of the medium, $\omega = 2\pi f$ /f - frequency/.

So we obtain in a calyndric system of coordinates the following known expression for the acoustic pressure on the piston axis [4] :

$$p = \frac{i\omega\varrho w\lambda}{\pi} e^{i\omega t} \sin\left[\frac{k}{2}\left(\sqrt{a^2+l^2}-l\right) e^{-i\frac{k}{2}\left(\sqrt{a^2+l^2}+l\right)}\right] \qquad /2/$$

where a - radius of the piston, l - distance on the piston axis.

The distribution of the particle velocity on the axis of the piston can be calculated form the ralation $v_z = \partial\varphi/\partial z$, where z is the cylinder coordinate along the piston axis. So we obtain

$$v_z = -\frac{\lambda}{\pi} w e^{i\omega t} e^{-i\frac{k}{2}\left(\sqrt{a^2+l^2}+l\right)} \left\{\frac{k}{2}\left(\frac{l}{\sqrt{a^2+l^2}}-1\right)\cos\left[\frac{k}{2}\left(\sqrt{a^2+l^2}-l\right)\right] - \frac{ik}{2}\left(\frac{l}{\sqrt{a^2+l^2}}+1\right)\sin\left[\frac{k}{2}\left(\sqrt{a^2+l^2}-l\right)\right]\right\} \qquad /3/$$

The moduli of the particle velocity v_z calculated from the formula /3/ and the acoustic pressure p calculated from the formula /2/ are exemplified in Fig.1. Hence it follows that the distribution of the two quantities is different near the piston, however, the maxima and the minima are at the same distances. The maxima occur when [4]

$$l = \frac{\frac{a^2}{\lambda} - \left(n+\frac{1}{2}\right)^2}{\frac{2n+1}{\lambda}}, \qquad n = 0,1,2,\ldots \qquad /4/$$

The minima - when

$$l = \frac{\frac{a^2}{\lambda} - n^2}{\frac{2n}{\lambda}} \qquad /5/$$

The acoustic impedance on the axis of the piston will be determined from the quotient of /2/ and /3/.

The modulus and argument of the impedance in the function of the distance from the piston is shown in Fig. 2 and 3. For the distances which correspond to the maxima, when condition /4/ is satisfied, the particle velocity is

$$v_z = \frac{\lambda}{\pi} w e^{i\omega t} \left\{\frac{ik}{2}\left[\frac{l}{\sqrt{a^2+l^2}}+1\right]\right\} e^{-i\frac{k}{2}\left(\sqrt{a^2+l^2}+l\right)} \qquad /6/$$

From /2/ and /6/ we arrive at the acoustic impedance at the maxima of the field which is equal to

$$Z = \frac{2\varrho c}{1 + \frac{1}{\sqrt{1 + a^2/l^2}}} = \begin{cases} \varrho c & \text{when } l \gg a \\ 2\varrho c & \text{" } l \to 0 \end{cases} \qquad /7/$$

The acoustic impedance at minima equals to 0, as can be easy verified.

It is interesting to calculate the distribution of intensity on the axis of the piston. On account of the phase shift between the acoustic pressure and particle velocity we will introduce a formal notion of active intensity I_c which corresponds to active power, and the notion of reactive intensity I_b which corresponds analogically to reactive power. Both these quantities, that is,

$$I_c = \frac{1}{2} p v_z \cos(\psi_p - \psi_v) \qquad\qquad I_b = \frac{1}{2} p v_z \sin(\psi_p - \psi_v) \qquad /8/ /9/$$

are presented in Fig. 3 and 4, respectively. $(\psi_p - \psi_v)$ -denotes the phase difference between acoustic pressure and particle velocity p and v_z amplitudes of the two quantities.

CONCLUSIONS

The distribution of particle velocity is different than of acoustic pressure, what is more distinct near the piston /Fig. 1/. It follows from /2/ that the maxima of acoustic pressure reach the value

$$p_{max} = 2\varrho c w \qquad /10/$$

and thus are twice as great as in the case of a plane homogeneous wave with the particle velocity w - which is well known. However, the maxima of particle velocity - according to /3/ are

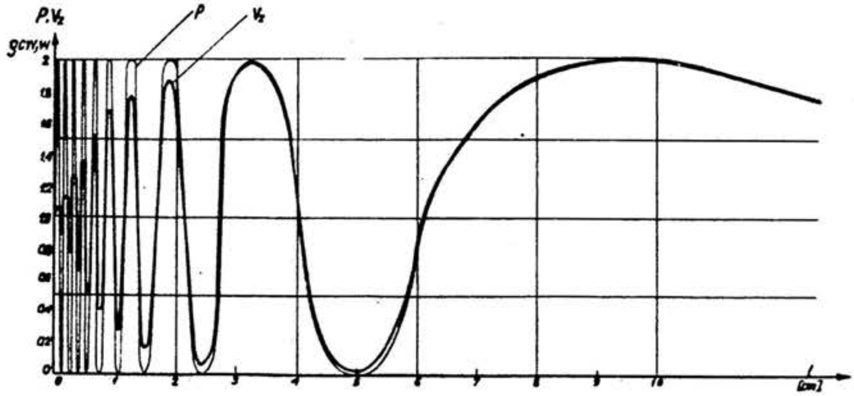
$$v_{z\,max} = \left(\frac{l}{\sqrt{a^2+l^2}} + 1\right) w \quad \begin{cases} 2w & \text{when } l \gg a \\ w & \text{" } l \to 0 \end{cases} \qquad /11/$$

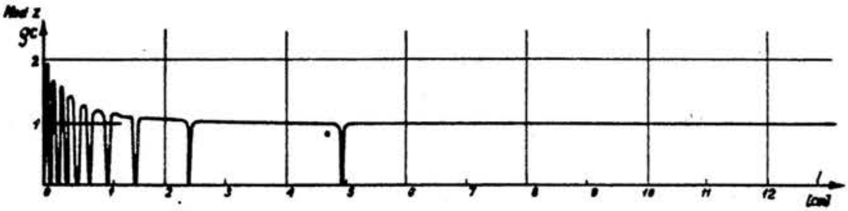
Thus the greatest maximal amplitude $v_{z\ max}$ far from the piston is twice as great as on the piston itself, on which by our assumptions the velocity must be w /see page 1/. The phase shift, which occurs between the acoustic pressure and particle velocity, is the greatest at minima and it amounts then to $\pm 90^{\circ}$.

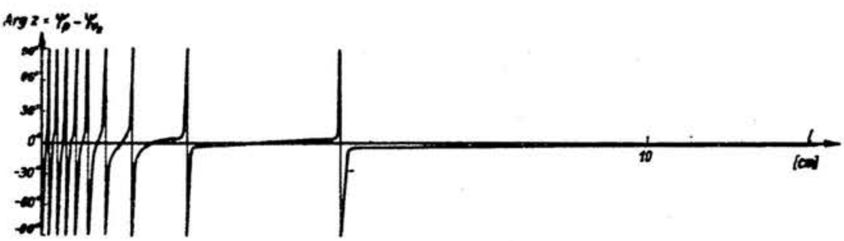
Of interest is the distribution of acoustic impedance. It is equal to ϱc only at the maxima /Fig.2/ and only when $l \gg a$. Near the piston this quantity grows and assumes a value twice as great /see formula 7/. At minima, the impedance reaches zero and its phase changes by 180°.

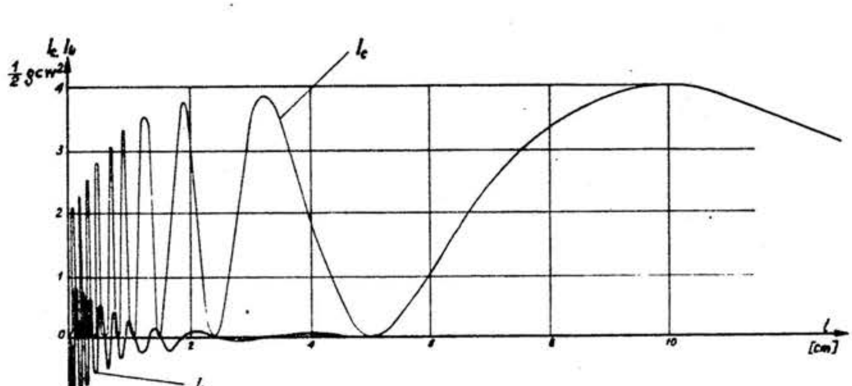
The intensity /active component/ shows also a number of maxima and minima, and the maxima distindly decrease nearer to the piston. At the farthest maximum, that intensity is four times greater than the intensity of a plane homogeneous wave with the particle velocity w. The reactive component of intensity which increase near the piston assumes alternatively a plus and minus sign /Fig.4/.

[5]









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