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ON PHYSICAL FOUNDATIONS
OF VISCOPLASTICITY

28/1968

WARSZAWA



N a p r a w a c h r ę k o p i s u
D o u ż y t k u w e w n ę t r z n e g o

Zakład Mechaniki Ośrodków Ciągłych IPPT PAN
Nakład 150 egz. Ark. wyd.1,9. Ark. druk.2,5 .
Oddano do drukarni w październiku 1968 r.
Wydrukowano w grudniu 1968 r. Nr zam. 501/056

Warszawska Drukarnia Naukowa , Warszawa ,
ul.Śniadeckich 8

ON PHYSICAL FOUNDATIONS OF VISCOPLASTICITY[‡]

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1. Introduction

The basic object of the present paper is the discussion of the physical foundations of the theory of thermo-viscoplasticity. On the basis of thermodynamics of a rate sensitive plastic material some special cases of the constitutive equations are considered. It has been shown^{§§§} that thermodynamic theory of viscoplasticity for finite deformations can be established within the framework of thermodynamics with internal state variables.

The analysis of the basic assumptions of the theory of viscoplasticity in the light of microdynamics of plastic flow and the physical theory of thermally activated deformations of metals are given. The constitutive equations for viscoplasticity and inviscid plasticity are compared with those predicted by thermally activated process^{§§§§} and are discussed on the basis of dislocation theory of crystalline metals^{§§§§§}. Theoretical predictions are compared with experimental results for dynamic finite deformations of single crystals^{§§§§§§} and polycrystals^{§§§§§§§}.

‡ This paper has been prepared for the 12th International Congress of Applied Mechanics, Stanford, August 26 - 31, 1968.

§§ Cf. Perzyna and Wojno [1968, 28].

§§§ Vid. Seeger [1954, 30, 31], [1955, 32].

§§§§ Vid. Gilman [1965, 9], [1966, 10], [1967, 11].

§§§§§ Vid. Lindholm and Yeakley [1965, 18] for aluminium.

§§§§§§ Vid. Hauser, Simmons and Dorn [1961, 12] for aluminium; Marsh and Campbell [1963, 21] for mild steel; Chiddister and Malvern [1963, 4] for aluminium; Lindholm [1964, 17] for aluminium, lead and copper; Lindholm [1967, 19] for aluminium; Campbell and Cooper [1967, 3] for low-carbon steel.

The theoretical predictions are also compared with those experimental results which based on the measurement of the average velocity of dislocations^{*}.

These comparisons have shown that a hypothesis that the rate dependence in metals is due primarily to a thermally activated process is valid for certain metals and for certain ranges of strain rate only. This analysis has indicated simultaneously that introduction of the nonlinear function $\Phi(F)$ in the constitutive assumption and its choice on the basis of experimental results may be treated as a very well founded hypothesis.

The description of the strain-rate and temperature history effects on the behaviour of a material are discussed.

2. Thermodynamic theory of a rate sensitive plastic material

The conception of the description of a rate sensitive plastic material as a material with internal state variables has been presented in the previous paper^{**}. In this theory no connection between the deformation tensor and the inelastic deformation tensor has been postulated. The deformation tensor ζ and temperature θ have been assumed as thermodynamic state variables and the inelastic deformation tensor ζ^i as internal state variable. The deformation tensor is implied by kinematics of the given motion for a body \mathcal{B} and the inelastic deformation tensor is determined by the solution of the initial value problem for ordinary first order differential equation.

The full system of the constitutive equations for a rate sensitive plastic material has the following

* Vid. Johnston and Gilman [1959, 13] for lithium fluoride and Stein and Low [1960, 33] for silicon-iron.

** Vid. Perzyna and Wojno [1968, 28].

form*

$$/2.1/ \quad \psi = \hat{\psi}(\underline{c}, \vartheta, {}^i c),$$

$$/2.2/ \quad \underline{\tilde{T}} = 2 \varrho_R \partial_{\underline{c}} \hat{\psi}(\underline{c}, \vartheta, {}^i c),$$

* In order to define a thermodynamic process in \mathcal{B} , which is compatible with the condition for the balance of linear momentum /Cauchy's first law of motion/

$$\text{Div}(\underline{F}\underline{\tilde{T}}) + \varrho_R \underline{b} = \varrho_R \underline{\ddot{x}}$$

and with the balance of energy /the first law of thermodynamics/

$$\frac{1}{2} \text{tr}(\underline{\tilde{T}}\underline{\dot{c}}) - \text{Div} \underline{q}_R - \varrho_R (\dot{\psi} + \vartheta \dot{\eta} + \dot{\vartheta} \eta) + \varrho_R \tau = 0,$$

/where operator Div is computed with respect to the material coordinates \underline{X} , ϱ_R is the mass density in the reference configuration \mathcal{R} , the dot denotes the material differentiation with respect to time t , \underline{b} is the body force per unit mass and τ the heat supply per unit mass and unit time, \underline{F} denotes the deformation gradient and \underline{c} is the right Cauchy-Green deformation tensor, $\underline{c} = \underline{F}^T \underline{F}$ /, it suffices to prescribe the seven functions $\{\underline{x}, \underline{\tilde{T}}, \psi, \underline{q}_R, \eta, \vartheta, {}^i c\}$. These functions have following interpretations. The function of motion $\underline{x}(\underline{X}, t)$ determines the spatial position occupied by the material point \underline{X} at time t , which in the reference configuration \mathcal{R} occupied the position \underline{X} , i.e., $\underline{x} = \underline{x}(\underline{X}, t)$; $\underline{\tilde{T}}(\underline{X}, t)$ is the second Piola-Kirchhoff stress tensor, $\psi(\underline{X}, t)$ denotes the specific free energy per unit mass, $\underline{q}_R(\underline{X}, t)$ is the heat flux vector per unit surface in the reference configuration \mathcal{R} , $\eta(\underline{X}, t)$ is the specific entropy, $\vartheta(\underline{X}, t)$ the local absolute temperature and ${}^i c(\underline{X}, t)$ is the inelastic deformation tensor.

$$/2.3/ \quad \eta = -\partial_{\dot{\gamma}} \hat{\psi}(\underline{C}, \gamma, \dot{\underline{C}}),$$

$$/2.4/ \quad \underline{q}_R = \hat{q}_R(\underline{C}, \gamma, \text{Grad} \gamma, \dot{\underline{C}})$$

$$/2.5/ \quad \dot{\underline{C}} = \gamma(\dot{\gamma}) \langle \Phi(\mathcal{F}) \rangle \underline{M}(\bar{\underline{I}}, \gamma, \dot{\underline{C}}),$$

where \mathcal{F} denotes the statical yield condition and is defined as follows

$$/2.6/ \quad \mathcal{F} = \frac{f(\bar{\underline{I}}, \gamma, \dot{\underline{C}})}{\alpha} - 1,$$

if the isotropic work-hardening parameter α is determined by

$$/2.7/ \quad \alpha = \text{tr} \{ \underline{N}(\bar{\underline{I}}, \gamma, \dot{\underline{C}}) \dot{\underline{C}} \}.$$

The functions f , \underline{M} and \underline{N} are tensor functions. The function $\Phi(\mathcal{F})$ is introduced to describe the influence of the rate of deformation and the temperature on the yield limit of a material. The symbol $\langle \Phi(\mathcal{F}) \rangle$ is defined as follows

$$/2.8/ \quad \langle \Phi(\mathcal{F}) \rangle = \begin{cases} 0 & \text{for } \mathcal{F} \leq 0, \\ \Phi(\mathcal{F}) & \text{for } \mathcal{F} > 0, \end{cases}$$

and $\gamma(\dot{\gamma})$ denotes a temperature dependent viscosity coefficient.

The equation /2.5/ postulates that the rate of the inelastic deformation tensor $\dot{\underline{C}}$ is the function of the excess of stress over the static yield condition. This equation yields to the following dynamical yield criterion

$$/2.9/ \quad f(\bar{\underline{I}}, \gamma, \dot{\underline{C}}) = \alpha \left\{ 1 + \Phi^{-1} \left[\frac{(\text{tr} \dot{\underline{C}}^2)^{1/2}}{\gamma(\dot{\gamma})} (\text{tr} \underline{M}^2)^{-1/2} \right] \right\}.$$

This relation describes the actual change of the yield surface during the thermodynamic process. This change is caused by isotropic and anisotropic work-hardening effects and by influence of the rate of inelastic deformation tensor and temperature on the yield limit of a material.

Notice that the equation /2.5/ defines the inelastic deformation tensor \dot{c} .

To ensure the fulfilment of the thermodynamic postulate^{**}

$$/2.10/ \quad -\dot{\psi} - \dot{\eta} + \frac{1}{2\zeta_r} \text{tr}(\bar{\mathbb{T}} \dot{c}) - \frac{1}{\zeta_r \dot{\gamma}} \hat{q}_{r\kappa} \cdot \text{Grad} \dot{\gamma} \geq 0,$$

the constitutive equations /2.1/ - /2.5/ should satisfy the general dissipation inequality

$$/2.11/ \quad \text{tr}[\partial_{i_c} \hat{\psi}(\underline{c}, \dot{\gamma}, \dot{c}) \dot{c}] + \frac{1}{\zeta_r \dot{\gamma}} \hat{q}_{r\kappa}(\underline{c}, \dot{\gamma}, \text{Grad} \dot{\gamma}, \dot{c}) \cdot \text{Grad} \dot{\gamma} \leq 0.$$

In the case $\dot{\gamma}(\dot{\gamma}) \rightarrow \infty$ the dynamical yield criterion /2.9/ yields

$$/2.12/ \quad f(\bar{\mathbb{T}}, \dot{\gamma}, \dot{c}) = \infty,$$

the material loses its viscosity properties and behaves as an elastic-plastic material. The plastic deformation tensor \dot{c} is defined as follows^{**}

$$/2.13/ \quad \dot{c} = \lambda \langle \text{tr}(\partial_{\bar{\mathbb{T}}} f \dot{\mathbb{T}}) + \partial_{\dot{\gamma}} f \dot{\gamma} \rangle M(\bar{\mathbb{T}}, \dot{\gamma}, \dot{c}),$$

where

* This inequality is implied by the Clausius-Duhem inequality.

** Vid. Perzyna and Wojno [1968, 28].

$$/2.14/ \quad \lambda = \{ \text{tr} [(\underline{N}(\underline{\bar{T}}, \underline{v}, \underline{p}_c) - \partial_{\underline{p}_c} f) \underline{M}(\underline{\bar{T}}, \underline{v}, \underline{p}_c)] \}^{-1} > 0,$$

and

$$/2.15/ \quad \langle [\text{tr}(\partial_{\underline{\bar{T}}} f \underline{\dot{\bar{T}}}) + \partial_{\underline{v}} f \underline{\dot{v}}] \rangle = \begin{cases} [] \text{ if } f = \alpha \text{ and } [] > 0, \\ 0 \text{ if } f = \alpha \text{ and } [] \leq 0 \\ \text{or if } f < \alpha. \end{cases}$$

Thus, the full system of constitutive equations for inviscid theory of plasticity can be written as follows^{*}

$$/2.16/ \quad \underline{\psi} = \hat{\psi}(\underline{c}, \underline{v}, \underline{p}_c),$$

$$/2.17/ \quad \underline{\bar{T}} = 2 \zeta_{\underline{c}} \partial_{\underline{c}} \hat{\psi}(\underline{c}, \underline{v}, \underline{p}_c),$$

$$/2.18/ \quad \eta = - \partial_{\underline{v}} \hat{\psi}(\underline{c}, \underline{v}, \underline{p}_c),$$

$$/2.19/ \quad \underline{q}_R = \hat{q}_R(\underline{c}, \underline{v}, \text{Grad } \underline{v}, \underline{p}_c),$$

$$/2.20/ \quad \underline{p}_c = \lambda \langle \text{tr}(\partial_{\underline{\bar{T}}} f \underline{\dot{\bar{T}}}) + \partial_{\underline{v}} f \underline{\dot{v}} \rangle \underline{M}(\underline{\bar{T}}, \underline{v}, \underline{p}_c).$$

Notice that the system /2.16/ - /2.20/ should satisfy the thermodynamic postulate /2.10/.

For a rate sensitive plastic material by /2.2/ we have

* Vid. Perzyna and Wojno [1968, 28].

$$/2.21/ \quad \bar{\underline{T}} = \hat{\underline{T}}(\underline{C}, \underline{\nu}, \dot{\underline{C}}).$$

After material differentiation of the equation /2.21/ we get

$$/2.22/ \quad \dot{\underline{C}} = \underline{H}(\underline{C}, \underline{\nu}, \dot{\underline{C}})[\dot{\underline{T}}] + \underline{K}(\underline{C}, \underline{\nu}, \dot{\underline{C}})[\dot{\underline{C}}] + \underline{L}(\underline{C}, \underline{\nu}, \dot{\underline{C}})\dot{\underline{\nu}},$$

where

$$\underline{H}(\underline{C}, \underline{\nu}, \dot{\underline{C}}) = [\partial_{\underline{C}} \hat{\underline{T}}(\underline{C}, \underline{\nu}, \dot{\underline{C}})]^{-1},$$

$$/2.23/ \quad \underline{K}(\underline{C}, \underline{\nu}, \dot{\underline{C}}) = [\partial_{\underline{C}} \hat{\underline{T}}(\underline{C}, \underline{\nu}, \dot{\underline{C}})]^{-1} [\partial_{\dot{\underline{C}}} \hat{\underline{T}}(\underline{C}, \underline{\nu}, \dot{\underline{C}})],$$

$$\underline{L}(\underline{C}, \underline{\nu}, \dot{\underline{C}}) = [\partial_{\dot{\underline{C}}} \hat{\underline{T}}(\underline{C}, \underline{\nu}, \dot{\underline{C}})]^{-1} [\partial_{\underline{\nu}} \hat{\underline{T}}(\underline{C}, \underline{\nu}, \dot{\underline{C}})].$$

Substitution of /2.5/ into /2.22/ gives*

$$/2.24/ \quad \dot{\underline{C}} = \underline{H}(\underline{C}, \underline{\nu}, \dot{\underline{C}})[\dot{\underline{T}}] + \nu(\underline{\nu}) \langle \Phi(\underline{\nu}) \rangle \underline{M}^*(\bar{\underline{T}}, \underline{\nu}, \dot{\underline{C}}) + \underline{L}(\underline{C}, \underline{\nu}, \dot{\underline{C}})\dot{\underline{\nu}},$$

where

$$/2.25/ \quad \underline{M}^* = \underline{K}[\underline{M}].$$

Similarly, for an elastic-plastic material we can find the result

$$/2.26/ \quad \dot{\underline{C}} = \underline{H}(\underline{C}, \underline{\nu}, \dot{\underline{C}})[\dot{\underline{T}}] + \lambda \langle \text{tr}(\partial_{\bar{\underline{T}}} \dot{\underline{T}}) + \partial_{\underline{\nu}} \dot{\underline{\nu}} \rangle \underline{M}^*(\bar{\underline{T}}, \underline{\nu}, \dot{\underline{C}}) + \underline{L}(\underline{C}, \underline{\nu}, \dot{\underline{C}})\dot{\underline{\nu}}$$

with

* This result may be compared with the constitutive assumption proposed in the previous paper of the present author [1967, 27].

$$\underline{H}(\underline{C}, \nu, \rho \underline{C}) = [\partial_{\underline{C}} \hat{T}(\underline{C}, \nu, \rho \underline{C})^{-1}],$$

$$/2.27/ \quad \underline{M}^*(\underline{C}, \nu, \rho \underline{C}) = \underline{K}(\underline{C}, \nu, \rho \underline{C}) [\underline{M}(\hat{T}, \nu, \rho \underline{C})],$$

$$\underline{L}(\underline{C}, \nu, \rho \underline{C}) = [\partial_{\underline{C}} \hat{T}(\underline{C}, \nu, \rho \underline{C})]^{-1} [\partial_{\nu} \hat{T}(\underline{C}, \nu, \rho \underline{C})].$$

The equations /2.24/ and /2.26/ show that the assumption^{*}
 $\dot{\underline{C}} = \dot{\underline{C}}^e + \dot{\underline{C}}^i$ is not in general justified for finite strain. The functions \underline{H} and \underline{L} depend on the internal tensor $\dot{\underline{C}}^i$.

3. Dynamic nature of plastic deformations

A modern theory of plastic flow must be based on microscopic investigations, because plastic deformation changes not only the external shape of a body but also its internal structure. Another essential feature of plasticity is that this theory must be intrinsically dynamical. Although plastic flow sometimes appears to be time independent, this is a result of particular combinations of circumstances and has no fundamental significance^{**}. Since plastic flow occurs by means of the motions of dislocation lines, the rate at which it takes place depends on how fast the dislocations move, how many dislocations are moving in a given volume of material and how much displacement is carried by each dislocation. Dislocation theory shows that the plastic strain rate is

$$/3.1/ \quad \dot{\underline{E}} = \alpha N b V,$$

* This assumption has been introduced in the paper [1967, 27] and may be treated as a simplification.

** Vid. Gilman [1966, 10].

where α is an orientation factor, N the mean density of mobile dislocations, b is the displacement per dislocation line which is called the Burgers vector and V denotes the mean dislocation velocity.

It is generally agreed that the finite stress needed to cause plastic flow is due to obstacles impeding the motion of dislocations through a crystal^{*}. It is convenient to divide obstacles into two groups according to the distance over which they interact with the glide dislocations:

1. Those that possess long-range stress fields /of the order of 10 atomic diameters or greater/.
2. Those that possess short-range stress fields /i.e. of less than about 10 atomic diameters/.

The energy required to overcome the former type of obstacle may be so large that the thermal fluctuations cannot assist the applied stress in the temperature range under consideration. Thermal activation thus plays no role in overcoming these long-range obstacles, and they are termed athermal obstacles.

Thermal fluctuations can assist the applied stress in overcoming short-range obstacles, and they are termed thermal obstacles. It is these thermal obstacles that are responsible for the dynamic aspects of plastic deformation^{**}.

The mechanism of overcoming the dislocation forest have been developed theoretically by Seeger [1954, 30, 31] and [1955, 32].

In Seeger's theory it was assumed that the forest is distributed uniformly, and that the activation energy de-

* Cf. Conrad [1964, 5].

** Common thermal obstacles or mechanisms in pure metals are the Peierls-Nabarro stress, forest dislocations, the motion of jogs in screw dislocations, cross-slip of screw dislocations, and climb of edge dislocations.

increases linearly with increasing applied stress over the whole process of intersection of dislocations.

Here, we shall not assume a priori the form of the variation of activation energy with stress, but rather derive relations by which it can be obtained from experimentally observable quantities /cf. Basinski [1959, 2]/.

When the deformation is controlled by a single thermally activated process we have

$$/3.2/ \quad V = V_0 \exp[-U/kT],$$

where U is the energy that must be supplied by a thermal fluctuation for each successful activation, k denotes the Boltzmann's constant, and

$$/3.3/ \quad V_0 = A\gamma,$$

if A is the area swept out per activation and γ the Debye's frequency.

The equations /3.1/ - /3.3/ give

$$/3.4/ \quad \dot{\epsilon} = \alpha N A b \gamma \exp[-U/kT].$$

Let us assume that

$$/3.5/ \quad U = \varphi[(\tau - \tau^*)Lb]$$

where τ^* is the back stress not surmountable by a thermal fluctuation and L is the mean cord distance between neighbouring points at which the dislocation is arrested.

Expansion of the function φ gives

$$/3.6/ \quad U = \varphi|_{\tau=\tau^*} + \varphi'|_{\tau=\tau^*} (\tau - \tau^*)Lb + \varphi''|_{\tau=\tau^*} \frac{(\tau - \tau^*)^2}{2!} L^2 b^2 + \dots$$

Let us denote by

$$/3.7/ \quad v^* = -\varphi' |_{\tau = \tau^*} Lb = vLb, \quad U_0 = \varphi |_{\tau = \tau^*},$$

the activation volume and the activation energy for intersection at zero stress, respectively.

The linear approximation of Eq./3.4/ gives the Seeger's relation

$$/3.8/ \quad \dot{\epsilon} = \alpha NAb\gamma \exp\left[-\frac{U_0}{kT} + \frac{(\tau - \tau^*)v^*}{kT}\right]$$

or

$$/3.9/ \quad \tau = \left(\tau^* + \frac{U_0}{v^*}\right) + \frac{kT}{v^*} \ln \frac{\dot{\epsilon}}{\alpha NAb\gamma}$$

In the case when the activation energy U is a nonlinear function of stress the relation /3.4/ yields

$$/3.10/ \quad \dot{\epsilon} = \alpha NAb\gamma \exp\left\{-\varphi[(\tau - \tau^*)Lb]/kT\right\}$$

or

$$/3.11/ \quad \tau = \tau^* + (1/Lb)\varphi^{-1}[kT \ln(\alpha NAb\gamma/\dot{\epsilon})].$$

4. Discussion of experimental results

The effect of stress and temperature on dislocation velocity has been determined by the etch-pit technique employed by Johnston and Gilman [1959, 13] on lithium fluoride and Stein and Low [1960, 33] on silicon-iron. The results for silicon-iron are given in Fig.1. It is noted that approximately straight lines result when the logarithm of the velo-

city V is plotted versus the logarithm of the applied stress τ . It suggests the power relationship between dislocation velocity and stress. In general, the velocity of dislocation motion was found to be a very sensitive function of stress. Different relationships between velocity and stress have been proposed by Johnston and Gilman [1959, 13], Gilman [1965, 9] and Stein and Low [1960, 33]. However, since there does not exist a theoretical or physical interpretation of these relationships, it seems more reasonable to start with Eq. 3.2/ /cf. Conrad [1964, 5]/.

Lindholm and Yeakley [1965, 18] investigated single crystal and polycrystalline specimens of high purity aluminium in compression at strain rates to 500 sec^{-1} using the split Hopkinson pressure bar method. They obtained average stress-strain curves for the six orientations of single crystal and similar curves for the polycrystalline material. Activation volume as a function of strain can be computed from the data obtained[‡]. These results for the single and polycrystalline specimens of high purity aluminium are given in Fig. 2. The most interesting feature of these curves is that the activation volume for the polycrystalline material falls within the bounds and near the average of the single crystal data. This implies that the same thermally activated mechanisms control the deformation in single and polycrystals and that the distribution of the activation barriers are essentially the same in both cases.

‡ From strain data at constant temperature the activation volume can be determined from the relation

$$v^n = k \gamma \frac{\partial \ln \dot{\epsilon}}{\partial \tau} \Big|_{\gamma = \text{const}} \approx k \gamma \frac{\ln \dot{\epsilon}_2 / \dot{\epsilon}_1}{\tau_2 - \tau_1} \Big|_{\gamma = 279^\circ \text{K}}$$

This is in agreement with the previous results presented for aluminium at low temperature* and those of Conrad [1934, 5] for iron and steel.

Hauser, Simmons and Dorn [1961, 12] presented the stress-strain-strain rate properties of high purity aluminium for 295, 194 and 77.4°K over strain rates from 2 to $12 \cdot 10^3 \text{sec}^{-1}$. These experimental results are recorded in Figs.3-5.

Marsh and Campbell [1963, 21] obtained results of constant-stress tests on mild steel specimens of different mean ferrite grain sizes /cf. Figs.6 and 7/.

Chiddister and Malvern [1963, 4] investigated aluminium specimens at strain rates from 300 to 2000 sec^{-1} at six temperatures from 30 to 550°C. The stress strain rate curves at each temperature were plotted on both semilogarithmic and log-log coordinate systems in order to evaluate the validity of the logarithmic and the power flow laws. The semilogarithmic curves are shown in Fig.8 and the log-log curves are shown in Fig.9.

Lindholm [1934, 17] investigated three annealed face centred cubic metals-lead, aluminium and copper, both dynamically and at lower strain rates to determine their rate sensitivity. His results are plotted in Figs.10 - 12.

In order to assess the thermal dependence of the constitutive relations Lindholm [1967, 19] performed elevated temperature tests for aluminium in tension and compression. These results are presented in plots of stress versus strain-rate in Fig.13 and stress versus temperature in Fig.14.

Campbell and Cooper [1967, 3] described experimental results obtained in tensile tests on annealed low-carbon steel at mean plastic strain rates in the range 10^{-3} to 10^2sec^{-1} /cf. Fig.15/.

* Cf. Mitra and Dorn [1963, 22].

Lindholm [1967, 19] presented experimental results on the plastic flow of aluminium subject to wide spectrum of loading conditions. This spectrum includes stress states of pure compression, tension and torsion as well as combined stress states. The rate of loading is varied to produce strain-rates within the range of 10^{-3} sec^{-1} to 10^3 sec^{-1} . For compression and tension elevated temperature data from approximately 300°K to 700°K is obtained.

Figure 16 presents stress-strain curves for the aluminium plotted in terms of the invariants^{*}. Each curve is obtained at constant temperature / 294°K / and strain rate.

In Figure 17, the data of Fig.16 and a large number of additional tests is replotted to show the relation between the stress and strain-rate invariants at constant strain amplitude and temperature. According to the linearized theory /Seeger [1955, 32] /, this should be a linear relationship on the semi-logarithmic plot. The straight line through the experimental points at each strain amplitude is a best least-squares fit. The standard estimate of error of the data about the mean is 5.4% for the lowest strain and less than 3% for the two higher strain amplitudes.

All the preceding Lindholm's experimental data was obtained with the stress ratio, shear stress/tension stress, and the rate of plastic deformation nearly constant. Several tests of an exploratory nature were performed by Lindholm [1967, 19] to determine the material response under sudden changes in the direction or the rate of loading. These tests indicate that within the normal scatter of the data, there does not appear to be any marked evidence of the effect of previous loading history, either rate or direction of loading, on the subsequent material response.

* The flags on the points plotted in this and subsequent figures indicate the direction of the principal stress axes. The horizontal axis is tension to the right and compression to the left, with the vertical axis being pure shear. Combined stress states lie between the tension and shear axes.

5. Comparisons and conclusions

Let us compare the theoretical dynamical yield criterion /2.9/ with the physical predictions based on thermally activated process /3.9/ and /3.11/. From this comparison it is seen that the phenomenological yield criterion /2.9/ may be treated as a simple generalization to general stress state and finite strains the physically justified relation /3.11/. In this generalization it has been assumed that the influence of the strain rate and the temperature on the yield limit is described by the nonlinear function $\phi(\dot{F})$.

The comparisons of theoretical predictions with experimental results showed that a hypothesis that the strain rate and temperature dependence in metals is due primarily to a thermally activated process described by the linearized relation /3.9/ is valid for certain metals and for certain ranges of strain rate only*. On the other hand the nonlinear relation /3.11/ may describe the strain rate and temperature phenomenon in the entire range of strain rate**.

Thus, this analysis indicated that the introduction of the nonlinear function $\phi(\dot{F})$ in the constitutive equations /2.1/ - /2.5/ and its choice on the basis of experimental results may be treated as a very well founded hypothesis.

The differential equation /2.5/ shows that the present theory of a rate sensitive plastic material takes into account the history of deformation and temperature. This is

* Cf. Alder and Phillips [1954, 1], Chiddister and Malvern [1963, 4], Krafft, Sullivan and Tipper [1954, 14], Lindholm [1964, 17], [1967, 19], and Trozera, Sherby and Dorn [1957, 35].

** Cf. Campbell and Cooper [1967, 3], Hauser, Simmons and Dorn [1961, 12], and Marsh and Campbell [1963, 21].

implied by the fact that to integrate the differential equation /2.5/ and to determine the actual value of the inelastic deformation tensor $\underline{\zeta}(t)$ at \mathbf{X} in \mathcal{B} , we must know the initial value $\underline{\zeta}^0$ and full histories of the inelastic deformation tensor $\underline{\zeta}$, the deformation tensor \underline{C} and temperature \mathcal{T} at \mathbf{X} .

The constitutive equations /2.16/ - /2.20/ for an elastic-plastic material were obtained as a limit case of the strain rate sensitive constitutive equations /2.1/ - /2.5/ and therefore they describe the quasi-static behaviour of a material.

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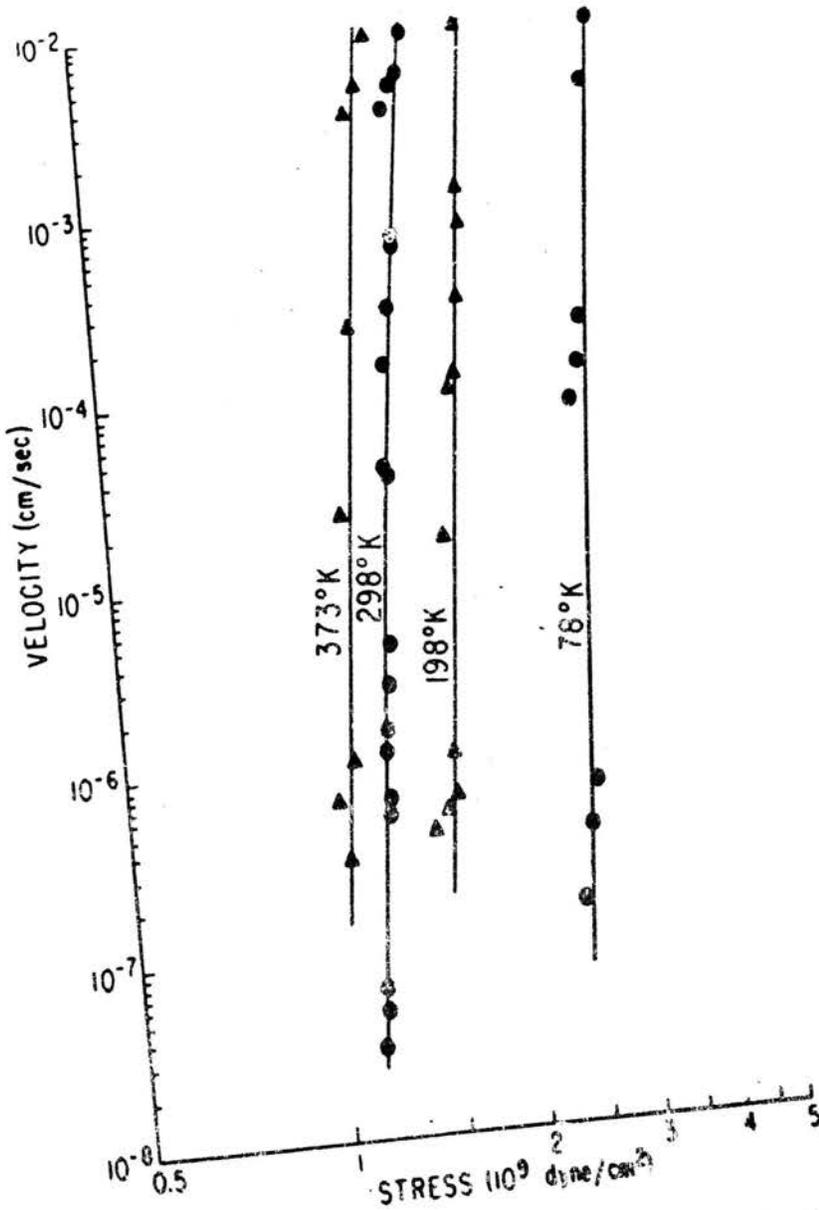


Fig. 1. Stress dependence of edge dislocation velocity at four different temperatures. After Stein and Low [1960, 33].

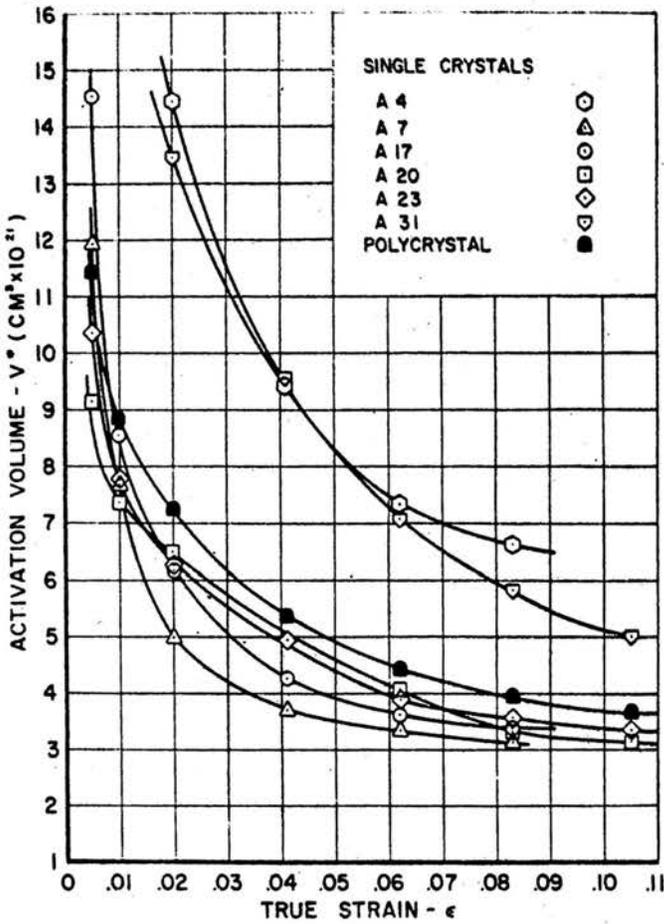


Fig. 2. Activation volumes for single and polycrystalline aluminium /99.995%/. After Lindholm and Yeakley [1965, 18].

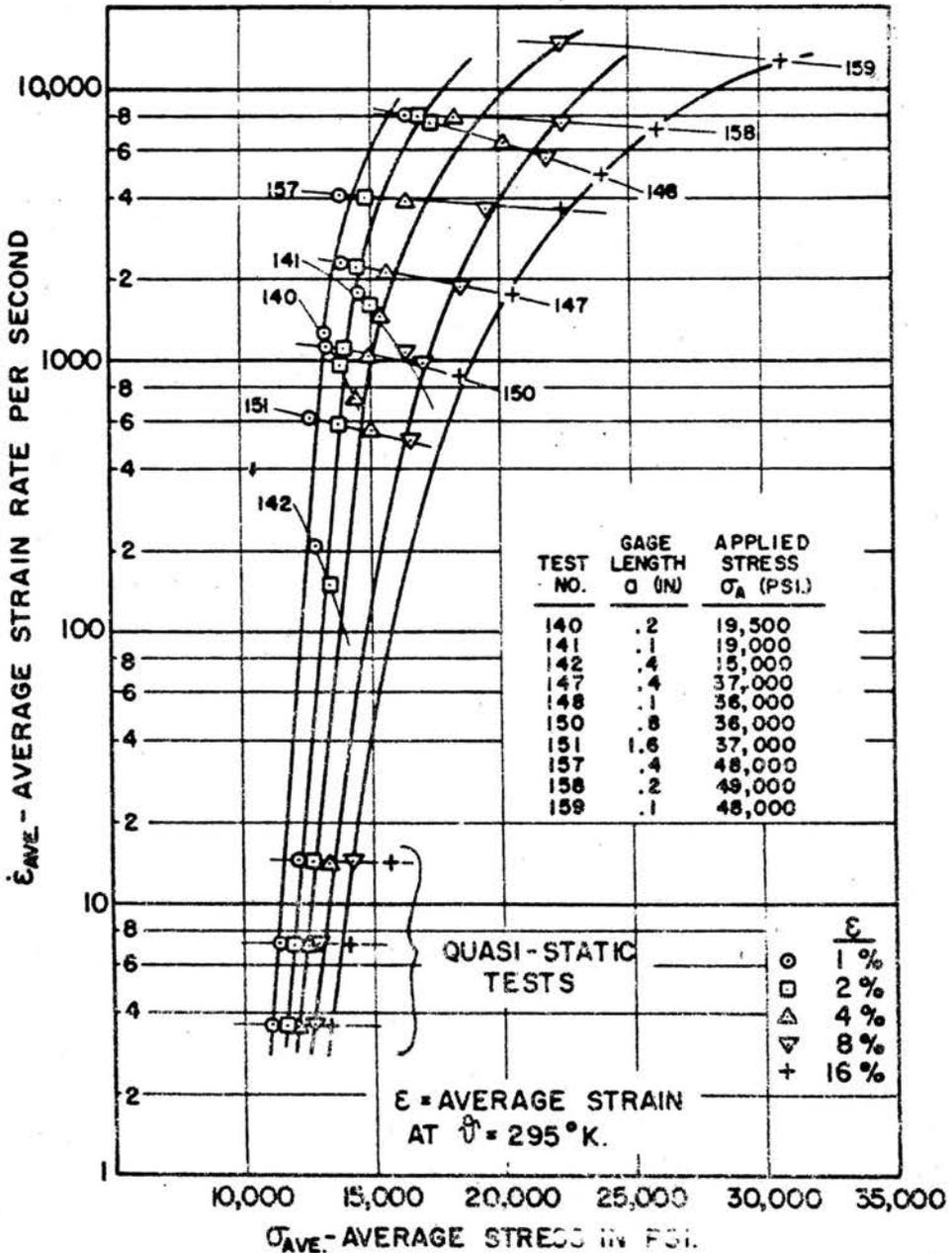


Fig. 3. Effect of stress on strain rate at constant strain and temperature $\dot{\nu} = 295^\circ\text{K.}$ After Hauser, Simmons and Dorn [1961, 12].

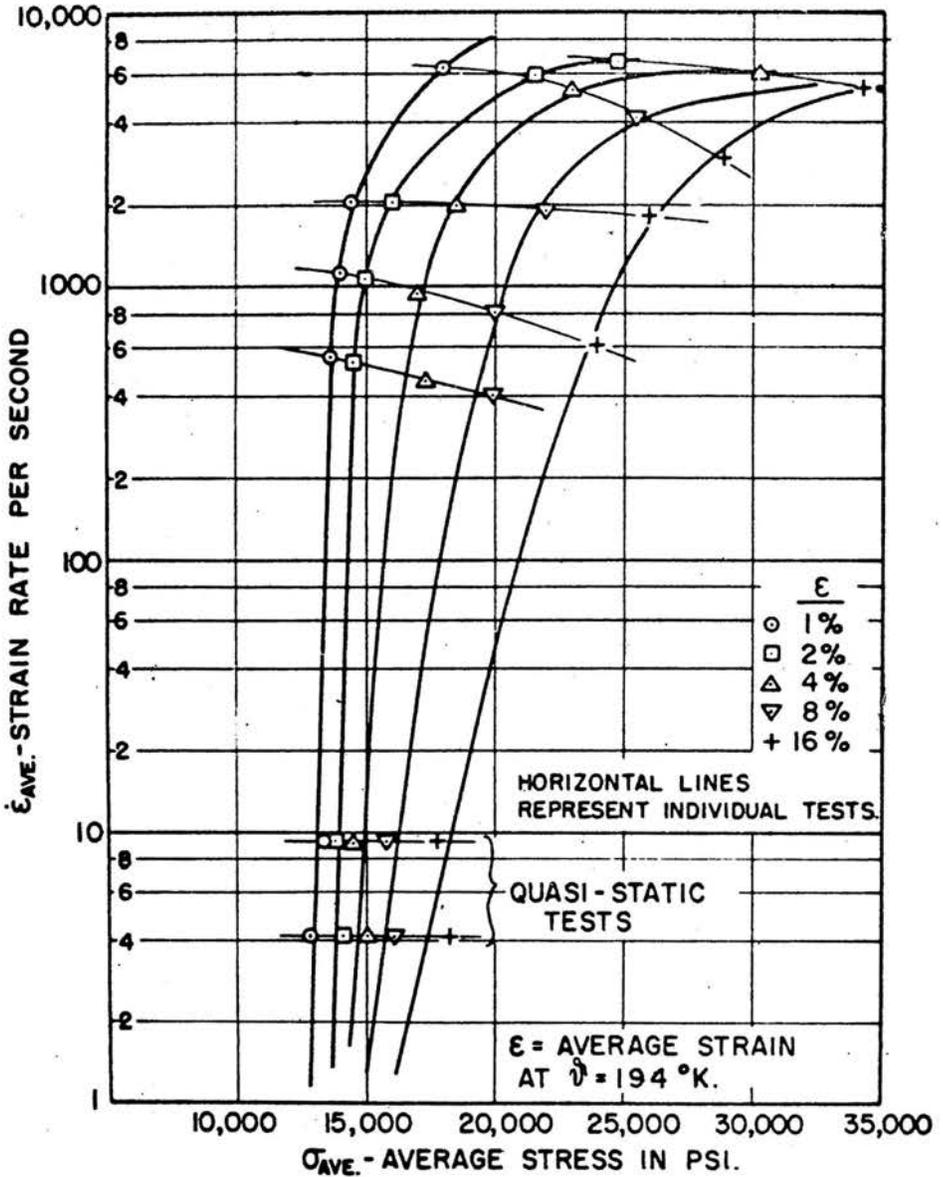


Fig. 4. Effect of stress on strain rate at constant strain and temperature $\hat{T} = 194^{\circ}\text{K}$. After Hauser, Simmons and Dorn [1961, 12].

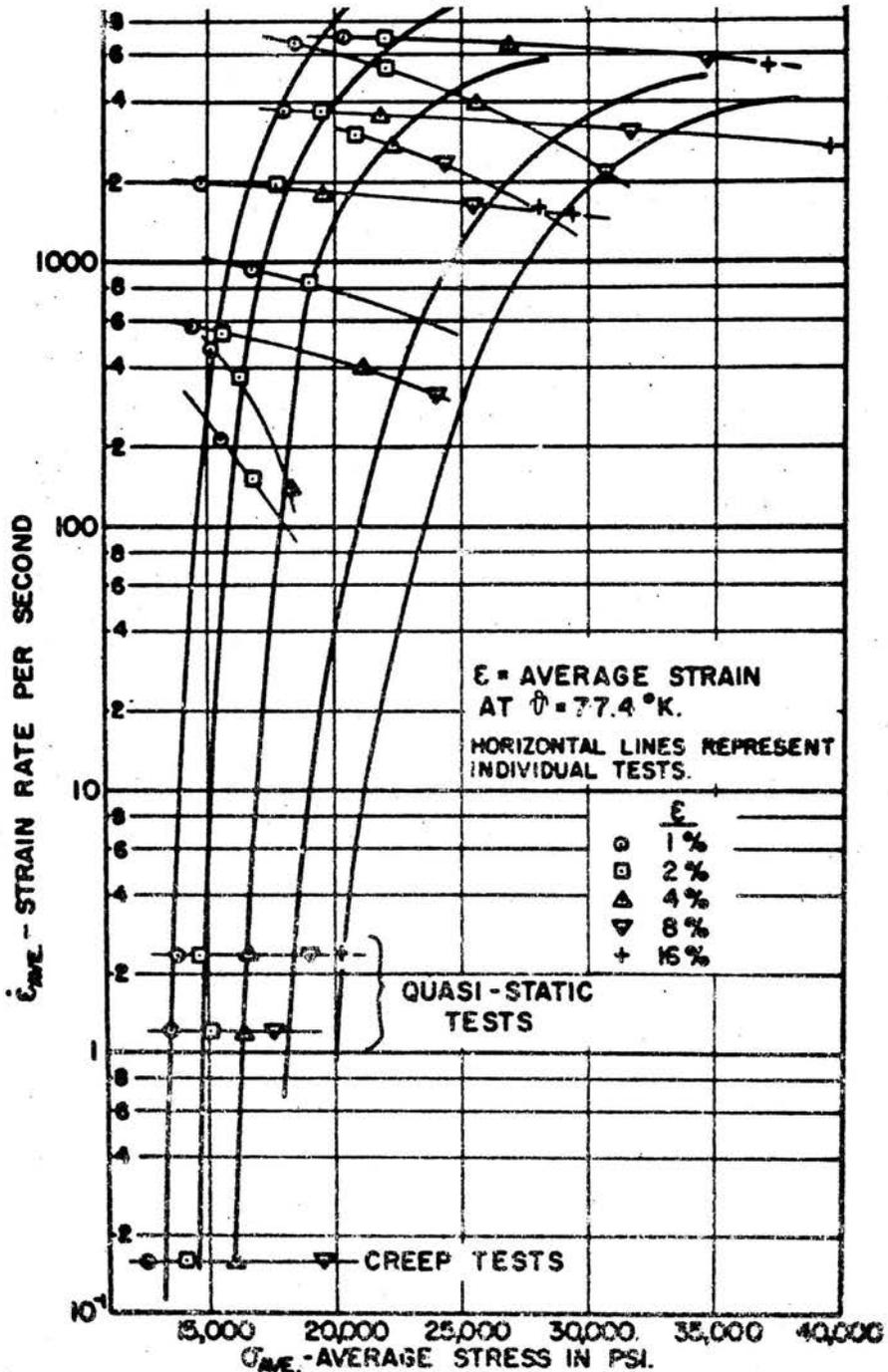


Fig. 5. Effect of stress on strain rate at constant strain and temperature $\bar{\sigma} = 77.4^\circ K$. After Hauser, Simmons and Dorn [1961, 12].

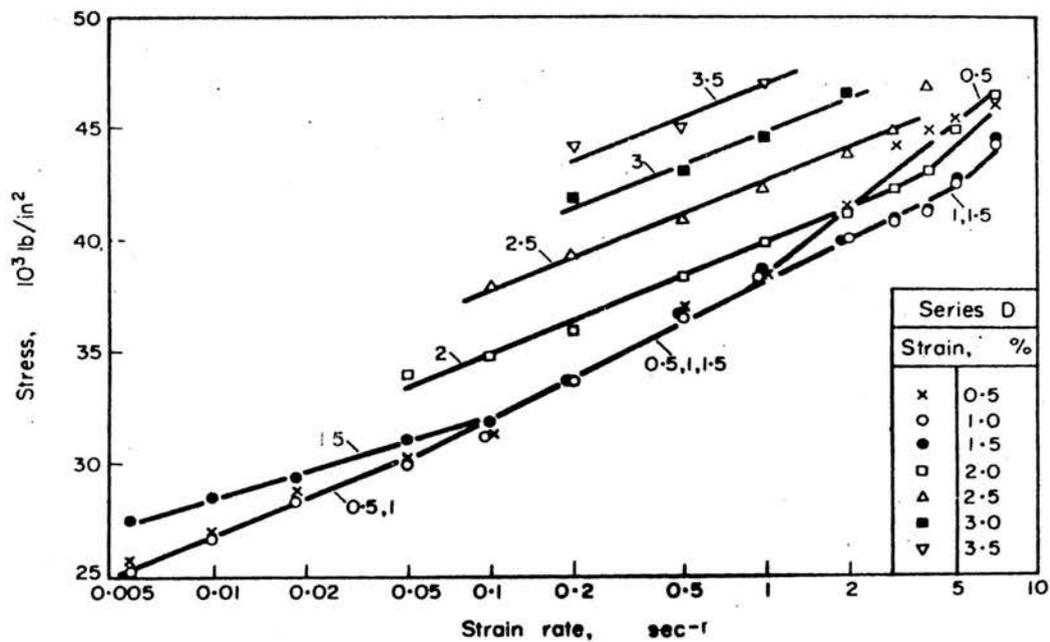


Fig. 6. Stress against strain rate /logarithmic scale/ at constant strain /mean grain density, 2033 grains per mm^2 /. After Marsh and Campbell [1963, 21].

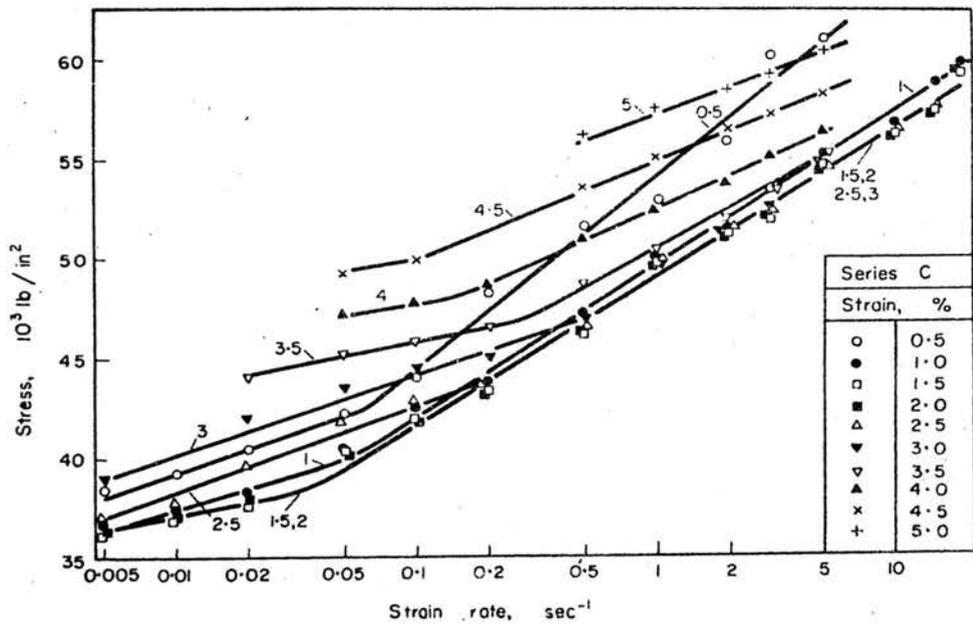


Fig. 7. Stress against strain rate /logarithmic scale/ at constant strain /mean grain density, 346 grains per mm²/. After Marsh and Campbell [1963, 21].

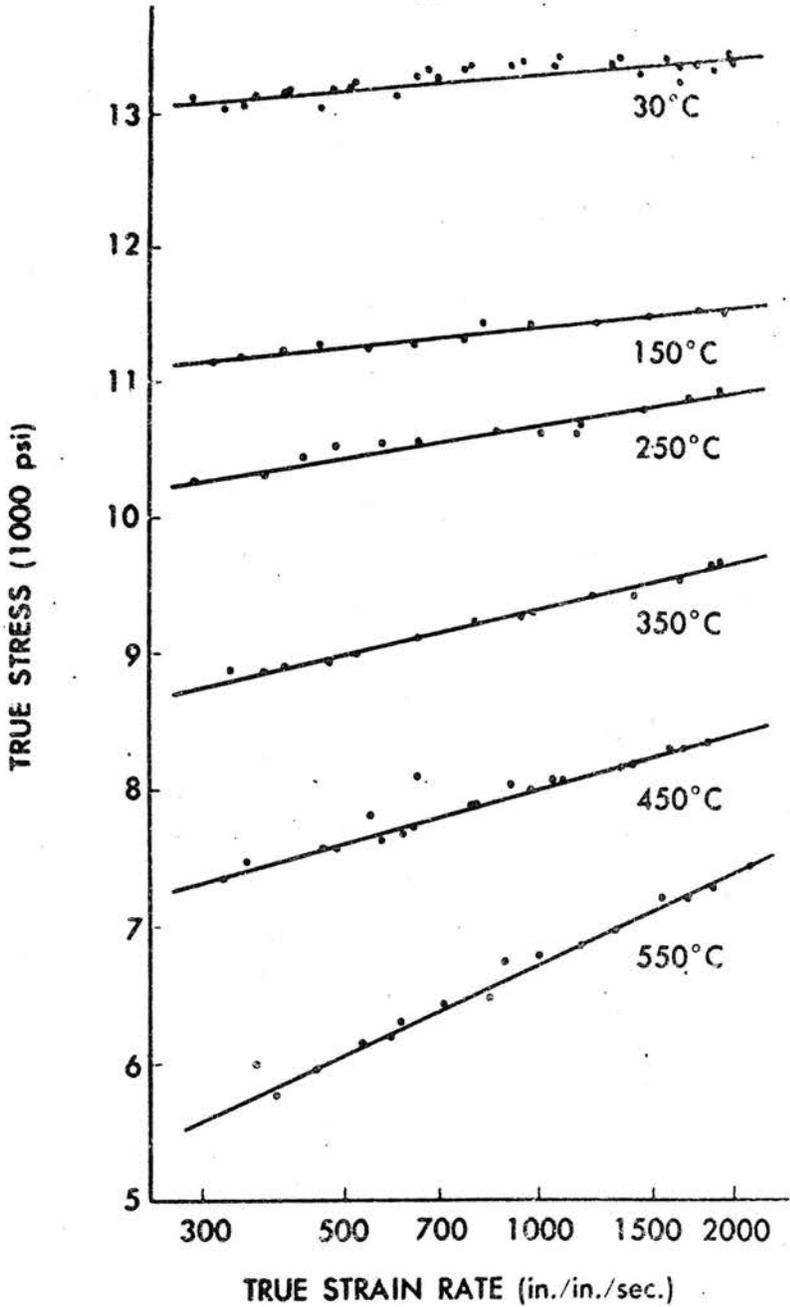


Fig. 8. Semilogarithmic plot of stress vs. true strain rate. After Chiddister and Malvern [1963, 4].

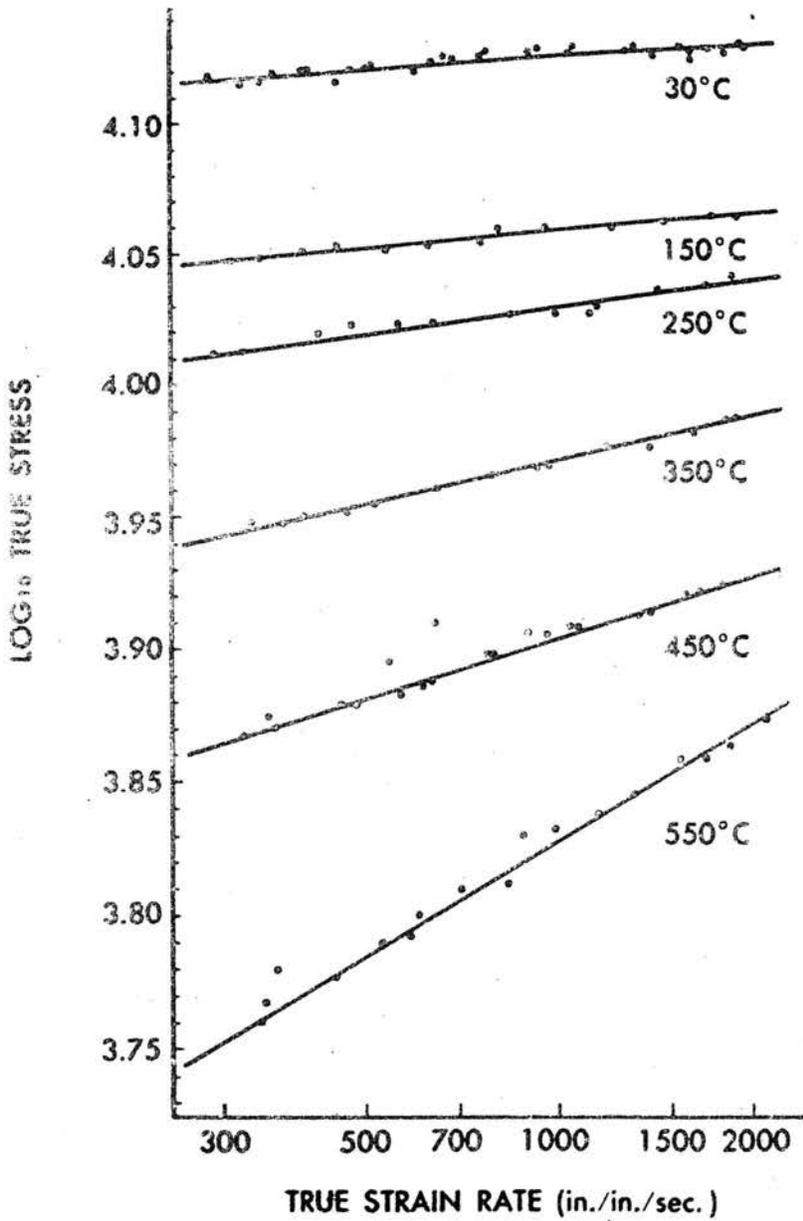


Fig. 9. Log-log plot of stress vs. true strain rate. After Chiddister and Malvern [1963, 4].

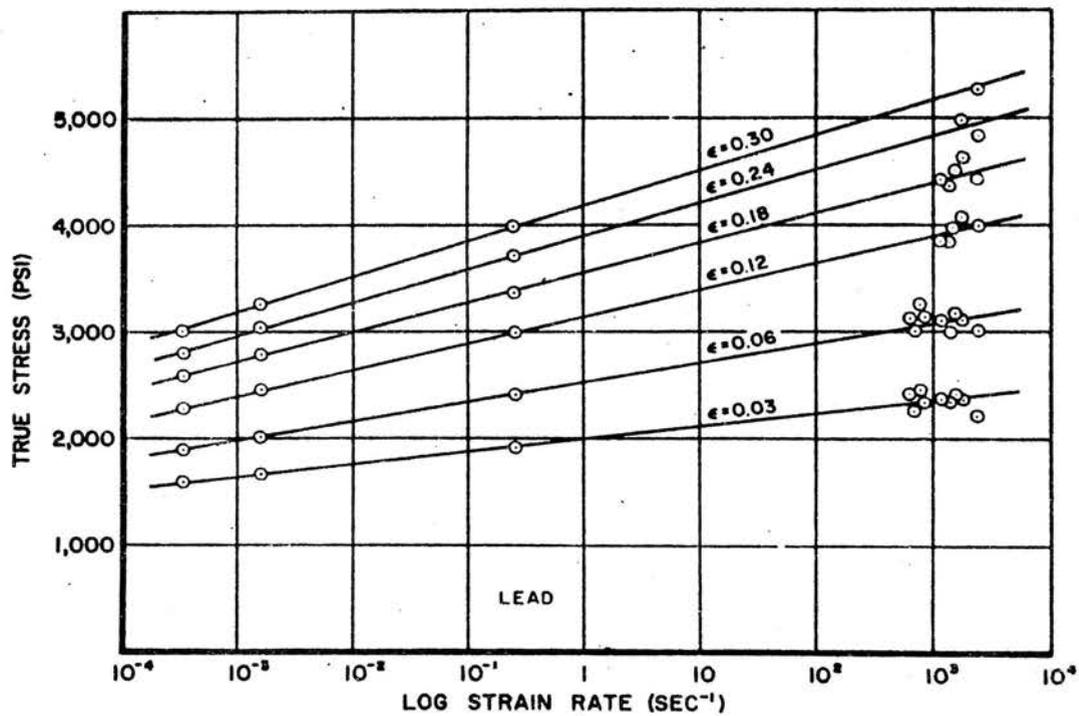


Fig. 10. Flow stress as a function of strain rate for lead. After Lindholm [1964, 17].

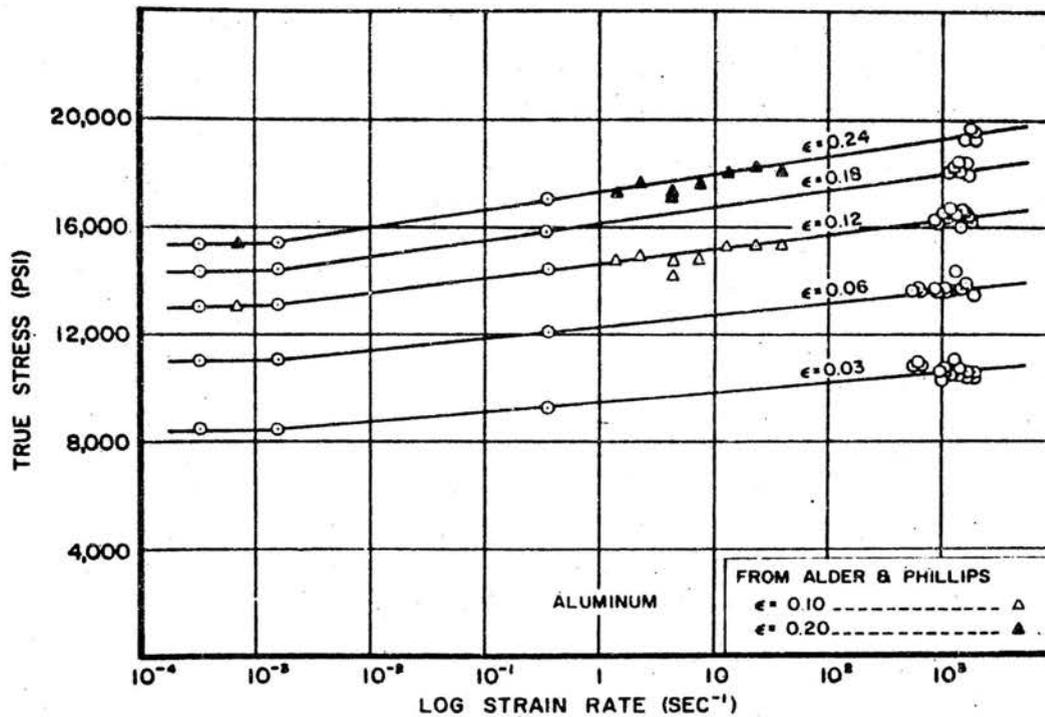


Fig. 11. Flow stress as a function of strain rate for aluminium. After Lindholm [1964, 17].

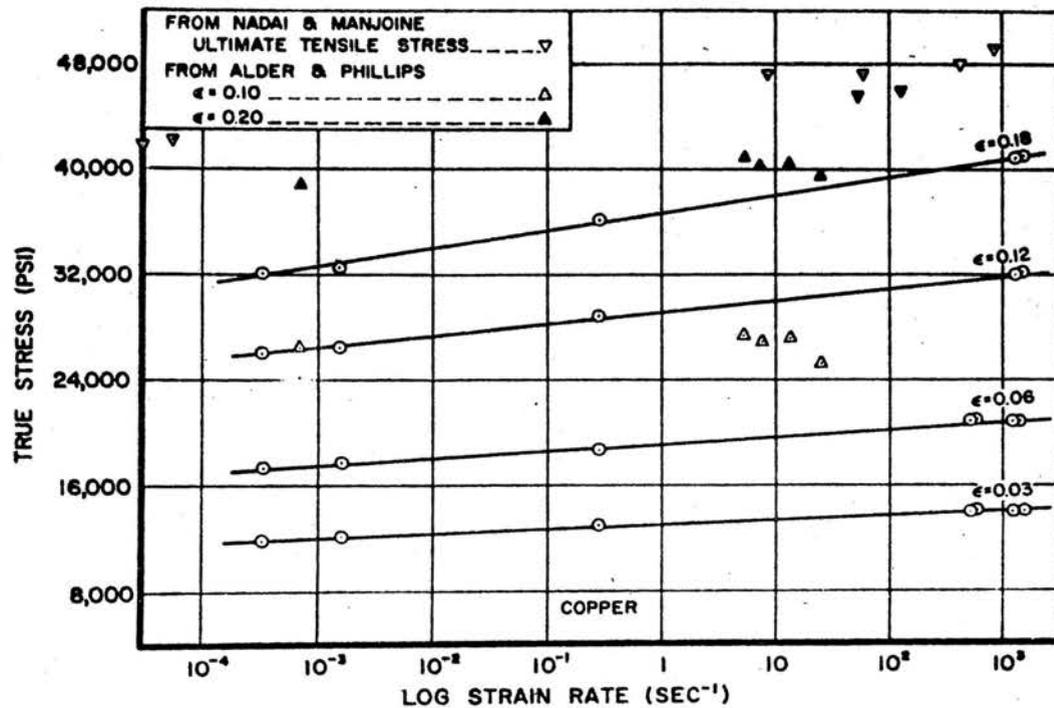


Fig. 12. Flow stress as a function of strain rate for copper. After Lindholm [1964, 17].

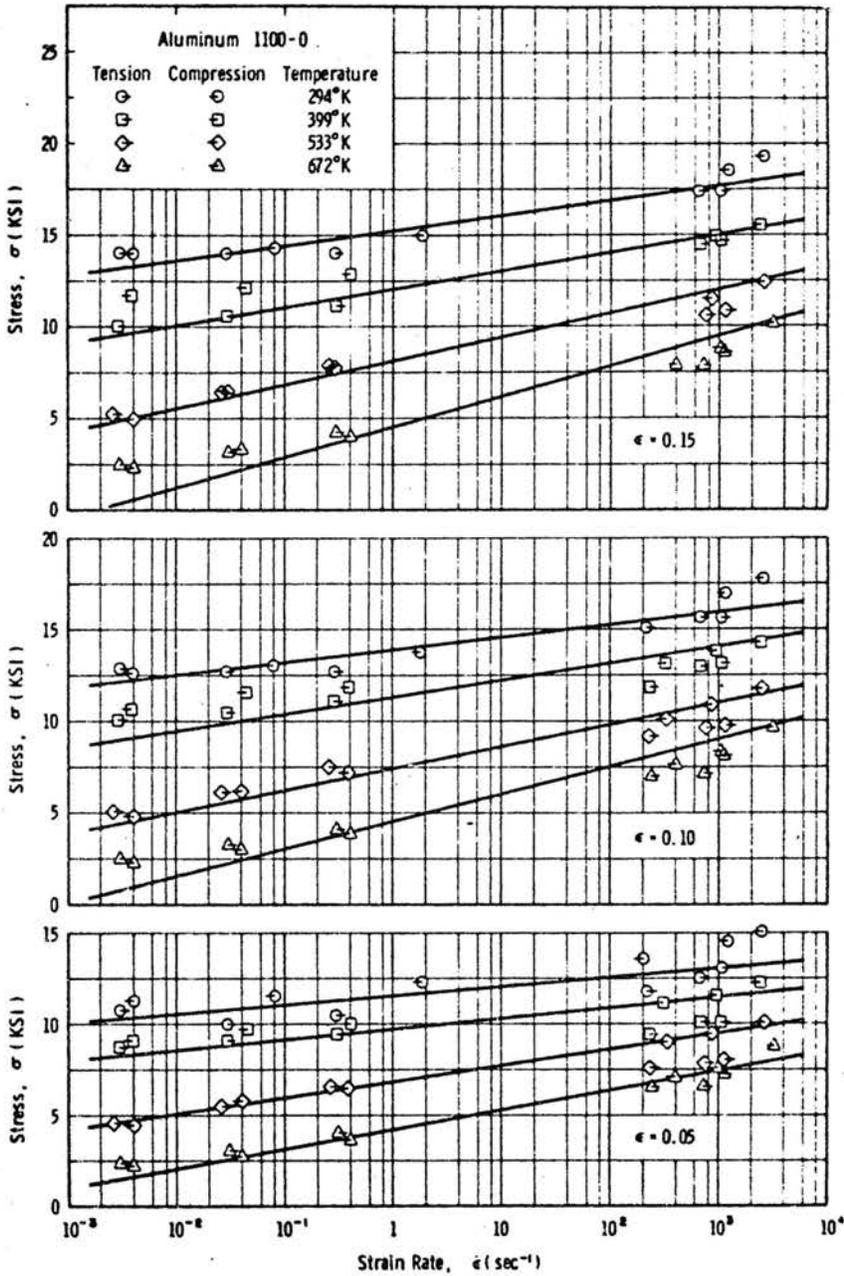


Fig. 13. Semilogarithmic plot of stress vs. true strain rate for aluminium. After Lindholm [1967, 19].

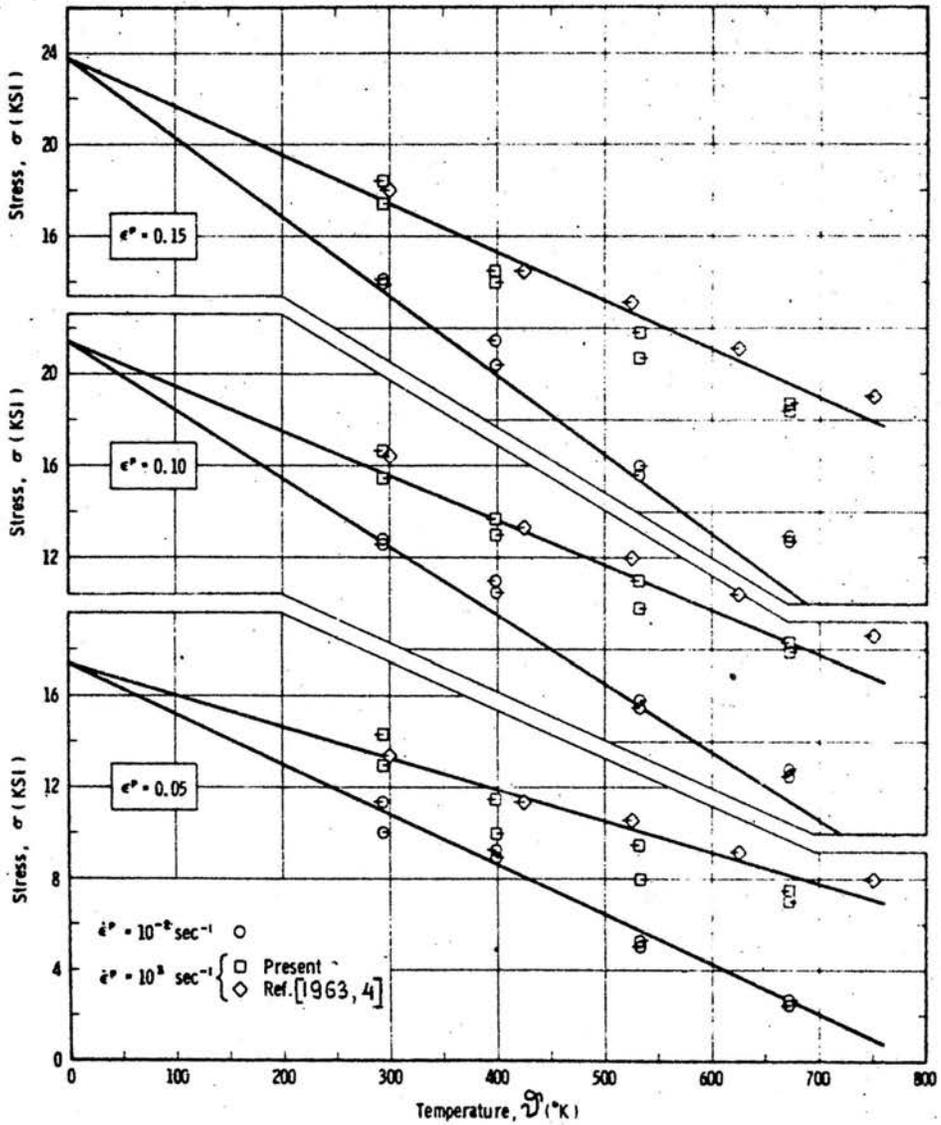


Fig. 14. Plot of stress vs. temperature for aluminium.
After Lindholm [1967, 19].

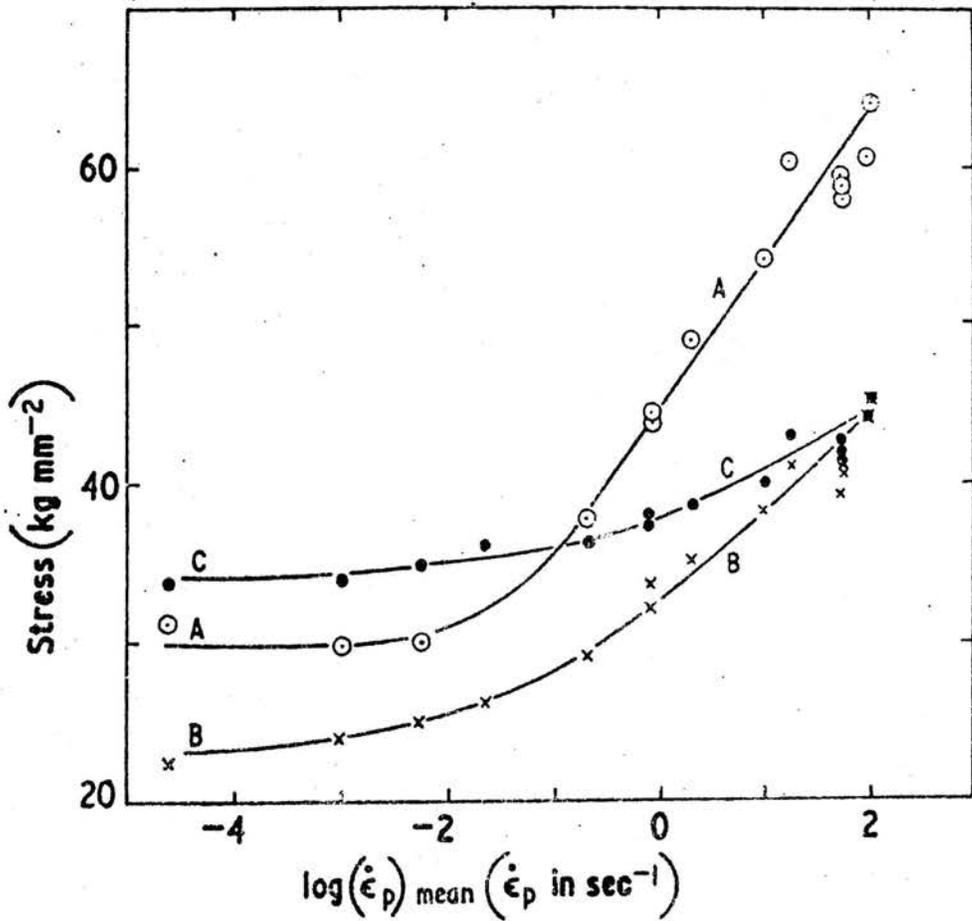


Fig. 15. Variation of strength with strain rate. A. Upper yield stress; B. Lower yield stress; C. Ultimate tensile stress. After Campbell and Cooper [1967, 3].

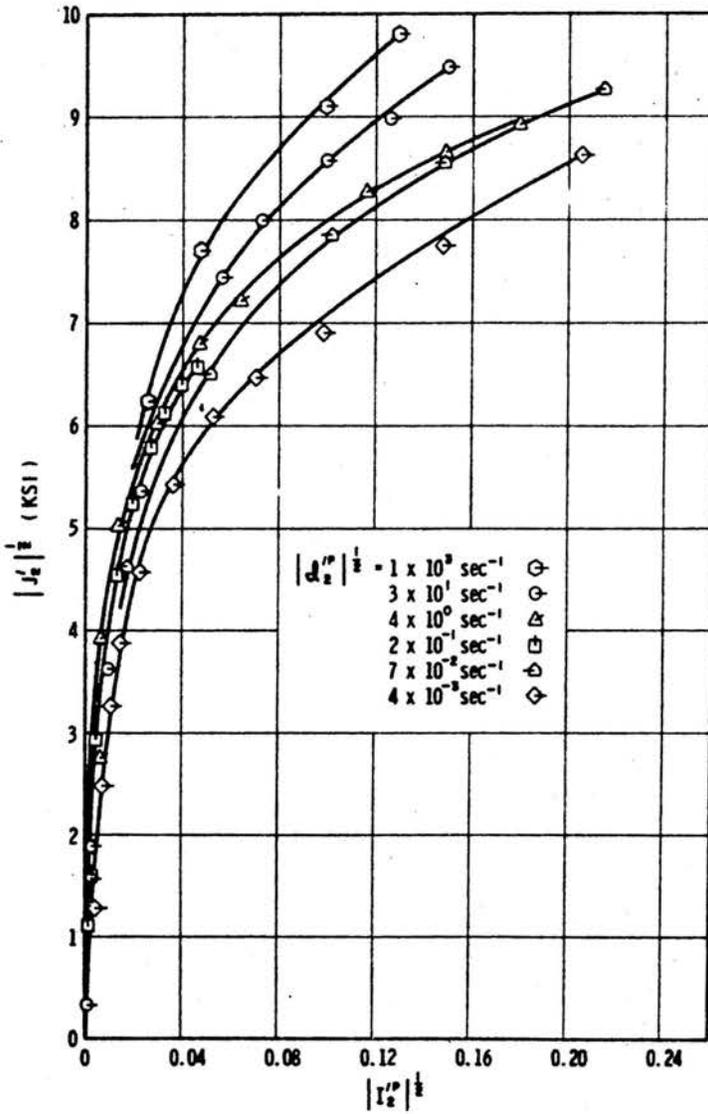


Fig. 16. Stress-strain curves for the aluminium plotted in terms of the invariants at constant temperature $\dot{\theta} = 294^\circ\text{K}$ and strain rate. After Lindholm [1967, 19].

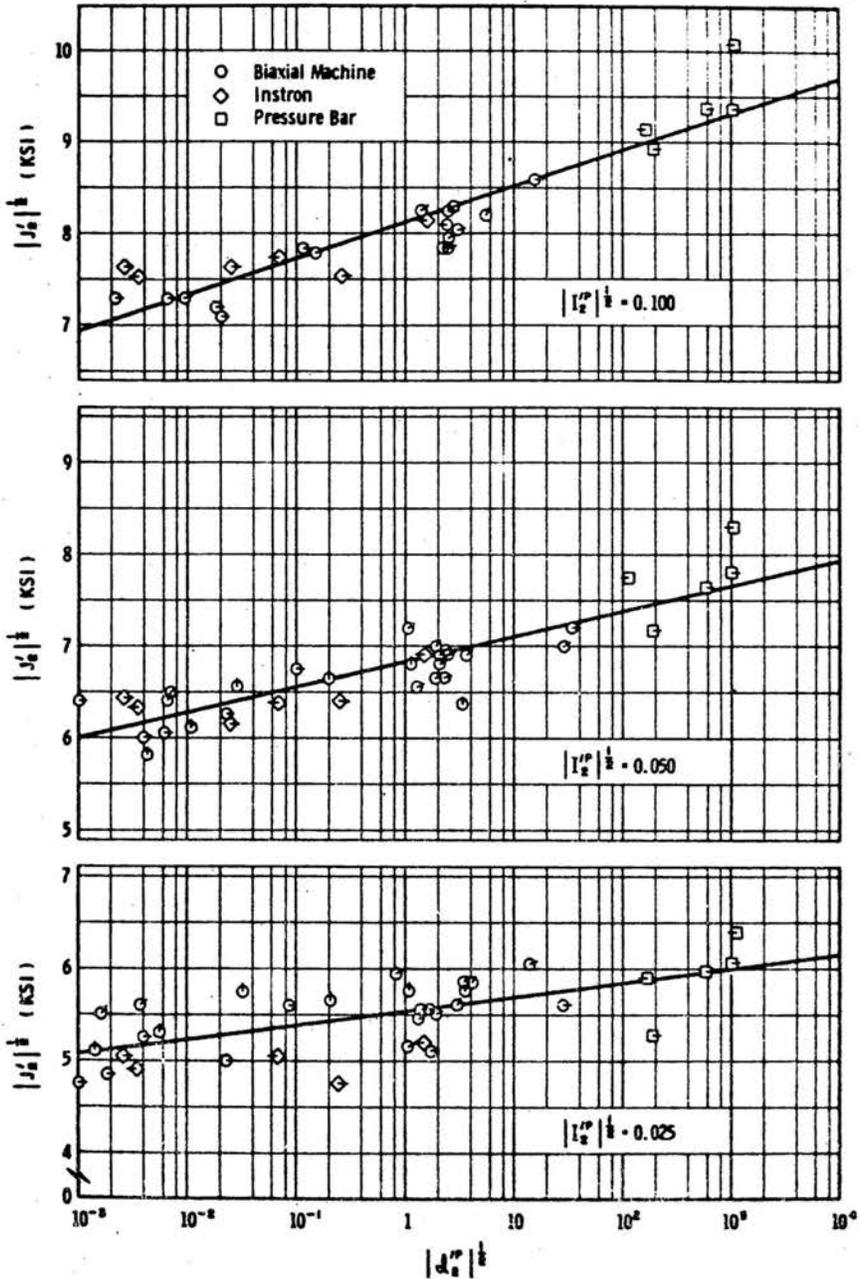


Fig. 17. Semilogarithmic stress-strain plots for the aluminum in terms of the invariants at constant temperature $\dot{\nu} = 294^{\circ}\text{K}$ and strain rate. After Lindholm [1967, 19].