Arch. Mech., 44, 3, pp. 331–344, Warszawa 1992

Small amplitude wave propagation in the Eimer's cracked material
(The instructive case of irremovable nonlinearity)

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LONGITUDINAL small amplitude plane wave propagation in the elastic nonlinear material behaving linearly under any proportional strain (with positive proportionality coefficient) is considered. Purely mechanical model of the material behaving under uniaxial strain as a linear elastic medium everywhere outside zero is assumed. It is shown, that purely mechanical positive dissipation principle together with the displacement continuity and momentum conservation conditions yields the same qualitative discontinuity propagation conditions as the complete thermomechanical approach. Several particular propagation schemes are considered. Some interesting qualitative effects are disclosed. Among them the reflection of a linear elastic wave from the moving strain discontinuity surface, severe dumping of periodic waves in purely elastic material and the qualitative change of the wave-motion scheme occurring in the compression and tension wave collisions for different amplitude ratios can be mentioned.

1. Introduction

DURING THE PAST two decades some attention was paid to very specific class of the elastic materials — to the piece-wise linear materials (some, possibly incomplete, references can be found e.g. in [1]). Particularly the papers dealing with the discontinuity propagation by Z. WESOŁOWSKI [2–4] and by Z. WESOŁOWSKI and A. SZWEYCER [5] can be mentioned.

The research interest in the subject was evoked by several causes, among them probably the efforts of the description of hypoelastic materials modeling the behavior of the elastic-plastic media under proportional loading should be mentioned. Among the mentioned materials the models of cracked materials proposed by C. EIMER [6–9] should be considered separately. The ideas standing behind the concept of Eimer's materials are very simple — he considered the material with perfectly two-dimensional (in unloaded state) plane stationary cracks with vanishing friction at their inner surfaces. Under some reasonable assumptions on the behavior of the cracks in initially linear elastic material, a nonlinear homogeneous elastic constitutive relation of degree one between the averaged values of stress and small strain can be obtained, the theorem on the uniqueness of the solution of the linear elasticity problems being involved in the course of derivation. The material behaves linearly under any proportional deformation with positive proportionality coefficient, its nonlinearity manifests itself under change of the direction in the strain space only, particularly when the sign of the deformation changes. For example, in the case of simple tension-compression test the Young modulus changes abruptly at zero point, while preserving constant values in these strain intervals which do not include this point. Thus, no matter how small is the one-dimensional deformation considered, the nonlinearity of the material behavior cannot be disregarded. Particularly, even for very small amplitude we can expect, in principle, different propagation velocities for tension and compression waves, this effect giving rise to several interesting qualitative effects occurring during plane longitudinal wave propagation.

The present paper is devoted to the discussion of these facts. In our considerations we shall have to do with a very simple situation, but it is author's hope that the results, which we shall be able to obtain, will prove to be not quite trivial. (1)

As it has already been mentioned, such nonlinearity as the one considered here is of innate nature and cannot be removed; moreover we can even call it drastic, because we have to do with an infinite curvature of stress-strain curve at point zero. On the other hand the linearity in constant sign regions makes it possible to apply some elementary concepts for the wave-motion scheme analysis and to obtain effective and sufficiently general results.

2. Preliminary considerations

Let us consider the following equation of one-dimensional motion

$$c^2 u_{,xx} = \ddot{u}, \tag{2.1}$$

where $u = u(x,t)$ is the displacement along x-axis, comma denotes partial differentiation with respect to the space variable and dot stands for the time derivative. Contrary to the linear theory c is not constant here, it is a function of the strain $\varepsilon = u_{,x}$

$$c(\varepsilon) = \begin{cases} c_1 & \text{for } \varepsilon < 0 \\ c_2 & \text{for } \varepsilon > 0 \quad (c_1 > c_2). \end{cases} \tag{2.2}$$

We assumed, in accordance with the common sense and with the physical interpretation of the material as a model of the cracked medium that the sound velocity is higher in the compressed regions. We shall consider here the propagation of the plane wave through the elastic space, however all the considerations will be valid also for the case of the longitudinal wave propagation in the rod.(2)

It is a proper place here to mention that we consider a purely mechanical model, disregarding the thermal expansion effects and the possible dependence of the elastic properties on the temperature.

The propagation of the wave in the regions of the constant strain sign does not differ from the well-known behavior of linear material, thus we shall assume in our considerations that at the left-hand side of some plane, whose position will be denoted as $x = Y(t)$, the material is compressed, while at the right-hand side it is extended. Of course, the opposite case is also possible in real life, but it can be simply obtained as a mirror reflection of the present situation. Let us notice that the effect of abrupt elastic properties change at zero strain can be interpreted according to the notions of the classical thermodynamics as a phase transformation, thus it is justified to refer to the plane $x = Y(t)$ as to the interface surface.

We do not know at the moment if the velocity and strain values are continuous or not at $x = Y(t)$, we assume however, that the displacement field is continuous, i.e. that

$$u^R = u^L, \tag{2.3}$$

(1) We shall not use here the results obtained by Z. WESOŁOWSKI [2–5] for more general cases. Our subject stands rather aside from his considerations on the geometry of wavefronts, thus it is simpler to re-derive some particular relations than to "translate" them from the very general notation.

(2) In the last case the following relation is valid: $c^2 = E/\rho$, where E denotes the Young modulus and ρ is the material density.

where u^R and u^L denote, respectivewly, right and left-hand side limit of the function $u(x,t)$ at $x = Y(t)$. We have not employed here the usual notation u^+ and u^- because we do not know (at least as yet) the sign of the interface velocity $U = dY(t)/dt$, thus we cannot point out what is "in front of" and what is "behind" the moving interface (possibly being a discontinuity surface). For practical application we shall need condition (2.3) expressed in terms of strains and velocities.

If any quantity $f(x,t)$ is given, then the rate of its change $\overset{*}{f}(x,t)$ at the point $x = Y(t)$ can be expressed as follows:

$$\overset{*}{f}(x,t) = \partial f(x,t)/\partial t + U \partial f(x,t)/\partial x, \tag{2.4}$$

thus the displacement continuity condition (2.3) yields the following relation:

$$v^R + U\varepsilon^R = v^L + U\varepsilon^L. \tag{2.5}$$

Linear relation (2.5) is valid also for finite strains; on the other hand, it is not difficult to show that it is a linear expression for the mass conservation condition at arbitrary surface (compare [10]).

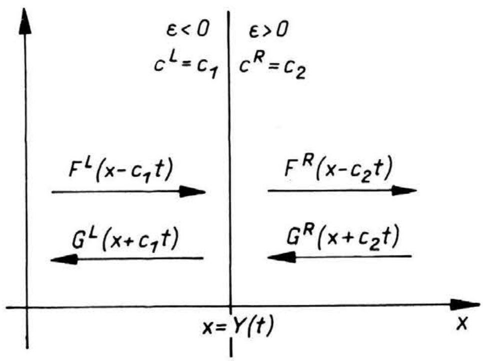

FIG. 1. Incoming and outgoing waves at the interface position $Y(x)$.

For the sake of brevity we shall also re-derive here the linear form of the second fundamental principle — the momentum conservation at the interface. Let us choose two material surfaces localized at the current time instant at $x = a$ and $x = b$, where $a < Y(t) < b$ (Fig. 1). The integral momentum balance equation for the domain contained between these two surfaces can be expressed as follows:

$$-\sigma(a) + \sigma(b) = \frac{d}{dt}\left[\int_a^{Y(t)} \rho v\, dx + \int_{Y(t)}^b \rho v\, dx\right], \tag{2.6}$$

where σ denotes the normal stress: $\sigma = \rho c^2 \varepsilon$. Applying well-known differentiation rule to the variable limit integral and taking limiting values for $a \to Y$, $b \to Y$ we are able to express (2.6) as follows:

$$-\rho^L c_1^2 \varepsilon^L + \rho^R c_2^2 \varepsilon^R = U\rho^L v^L - U\rho^R v^R, \tag{2.7}$$

where c_1 and c_2 replace c^L and c^R, according to our assumption about positive sign of ε at the left side and negative at the right-hand side. Taking into account that, for one-dimensional deformation, $\rho = \rho_0(1-\varepsilon)$ (or $\rho = \rho_0[1-(1-2\nu)\varepsilon]$ for the case of rod) and preserving only linear terms we can finally write:

$$(c_2^2\varepsilon^R - c_1^2\varepsilon^L) + U(v^R - v^L) = 0. \tag{2.8}$$

Re-deriving relation (2.8) we have not plainly saved some efforts, we have also shown a useful approach, which will be applied to the next item in this paper.

Let us recall that we deal with a purely mechanical model, thus the standard thermodynamic entropy principle (Clausius–Duhem inequality) cannot be directly applied here. We have to formulate some purely mechanical dissipation principle instead. We plainly demand the dissipation rate to be positive, i.e. we demand the mechanical energy growth rate not to exceed, at any instant, the power of external forces:

$$-\sigma(a)v(a) + \sigma(b)v(b) - \frac{d}{dt}\left[\int\limits_a^{Y(t)} \frac{1}{2}(\rho c^2\varepsilon^2 + \rho v^2)dx + \int\limits_{Y(t)}^{b} \frac{1}{2}(\rho c^2\varepsilon^2 + \rho v^2)dx\right] \geq 0. \tag{2.9}$$

Using the same reasoning as in the case of formula (2.6) and preserving only quadratic terms we obtain the following inequality:

$$-c_1^2\varepsilon^L v^L + c_2^2\varepsilon^R v^R - \frac{1}{2}U[(c_1^2(\varepsilon^L)^2 + (v^L)^2) - (c_2^2(\varepsilon^R)^2 + (v^R)^2)] \geq 0. \tag{2.10}$$

At the first glance this quadratic inequality does not fit the previous linear considerations and, within the framework of the approximation adopted, all the left-hand terms of (2.10) should be plainly considered as equal to zero. In fact, the situation here is not so simple, it is very similar to that in the thermomechanical theory of the small amplitude shock wave [10], where the Clausius–Duhem inequality having also quadratic form plays, nevertheless, a significant role.

Eliminating the velocity difference from Eqs. (2.5) and (2.8) and recalling that the sign of ε is different at different sides of the interface, we obtain the following useful relation:

$$\frac{\varepsilon^R}{\varepsilon^L} = \frac{c_1^2 - U^2}{c_2^2 - U^2} \leq 0. \tag{2.11}$$

This implies that the absolute value of the interface velocity must be contained between c_1 and c_2.

$$c_1 \geq |U| > c_2. \tag{2.12}$$

Important case when $|U| = c_2$ and $\varepsilon^L = 0$ will be discussed later.

Rewriting (2.8) as follows:

$$c_2^2\varepsilon^R + Uv^R = c_1^2\varepsilon^L + Uv^L \tag{2.13}$$

and then multiplying one by one both the left and right-hand sides of Eqs. (2.5) and (2.13), one obtains:

$$U\{[c_1^2(\varepsilon^L)^2 + (v^L)^2] - [c_2^2(\varepsilon^R)^2 + (v^R)^2]\} = -(c_1^2 + U^2)\varepsilon^L v^L + (c_2^2 + U^2)\varepsilon^R v^R. \tag{2.14}$$

Substituting the last relation into energy inequality (2.10) we obtain:

$$(c_2^2 - U^2)\varepsilon^R v^R - (c_1^2 - U^2)\varepsilon^L v^L \geq 0. \tag{2.15}$$

Combining (2.15) with (2.11) and recalling that $\varepsilon^L < 0 \leq \varepsilon^R$, we easily arrive at the following linear inequality:

$$v^R - v^L \leq 0, \tag{2.16}$$

and finally, coming back to (2.5), we obtain an elegant result

$$U > 0. \tag{2.17}$$

The last result is a counterpart of well-known fact of nonexistence of a rarefaction shock wave in gases, thus all our results obtained up to this point are closely related to the classical theory of moderate amplitude shock waves. Let us recall here that all the considerations beginning with eq. (2.11) are valid provided $\varepsilon^L \neq 0$.

3. General formulation of the problem and some useful relations

We shall consider moving interface between the compressed (at the left side) and extended (at the right side) regions. Both in the left and in the right-hand neighborhood of the interface, the usual equations of motion of linear elasticity should be fulfilled; thus, together with the interface motion, we have to consider (in the most general case) two incoming and two outgoing displacement waves at the vicinity of the interface:

$F^L(x - c_1 t)$ the wave incoming from the left-hand side,

$F^R(x - c_2 t)$ the wave outgoing towards the right-hand side,

$G^R(x + c_2 t)$ the wave incoming from the right-hand side,

$G^L(x + c_1 t)$ the wave outgoing towards the left-hand side.

At this point we do not apply to any of the mentioned waves any particular interpretation such as "primary" or "secondary", "incident", "reflected" or "transmitted". Denoting by f^R, f^L, g^R, g^L the limiting values (taken at the interface) of the spatial derivatives of F^R, F^L, G^R, G^L, respectively, we obtain at once the following values of time derivatives at the interface:

$$\begin{aligned} \partial F^L/\partial t\Big|_{x=Y(t)} &= -c_1 f^L, & \partial F^R/\partial t\Big|_{x=Y(t)} &= -c_2 f^R, \\ \partial G^L/\partial t\Big|_{x=Y(t)} &= c_1 g^L, & \partial G^R/\partial t\Big|_{x=Y(t)} &= c_2 g^R, \end{aligned} \tag{3.1}$$

thus for $\varepsilon^R, \varepsilon^L$ and v^R, v^L we get the following expressions

$$\begin{aligned} \varepsilon^R &= f^R + g^R, & \varepsilon^L &= f^L + g^L, \\ v^R &= -c_2(f^R - g^R), & v^L &= -c_1(f^L - g^L). \end{aligned} \tag{3.2}$$

Substituting relations (3.2) into (2.5) and (2.8) we obtain, after some rearrangement, the following relations between the incoming wave strain amplitudes, interface velocity and the total strain values at both sides of the interface:

$$\begin{aligned} \varepsilon^R &= f^R + g^R = \frac{2(c_2 g^R + c_1 f^L)}{c_1 + c_2}\,\frac{c_1 - U}{c_2 - U}, \\ \varepsilon^L &= f^L + g^L = \frac{2(c_2 g^R + c_1 f^L)}{c_1 + c_2}\,\frac{c_2 + U}{c_1 + U}. \end{aligned} \tag{3.3}$$

Relations (3.3) will serve in the next section as a convenient starting point for the investigations of the particular cases of the wave-motion schemes. There is however another fact, which can be established at this level of generality: it is the possibility of strain continuity at the interface. Let us notice that this is possible only if on both sides $\varepsilon = 0$; this, in turn, implies the stress continuity at zero level and, by virtue of (2.5), also the velocity continuity. One can see readily from Eq. (3.3) that this peculiar case needs a special relation between the strain amplitudes of the two incoming waves:

$$c_1 f^L + c_2 g^R = 0. \tag{3.4}$$

We shall discuss this particular case in the next section when considering collision of two waves of opposite signs and various strain amplitude ratios; just now we can say only that the strain, stress and velocity fields continuity at the interface can be expected to occur as an exception rather than as a rule.

4. Some important particular examples

4.1. The overtaking wave

We shall begin our considerations with the case depicted at Fig. 2, i.e. the extension wave running towards the right side and the faster compression wave running behind it, and, eventually, overtaking it. When the compression wave reaches the slower extension wave, the discontinuity arises (see Fig. 2b([3]), ([4])). According to (2.13) and (2.17), discontinuity moves towards the right side and its velocity exceeds the sound velocity in the extended zone, thus we do not observe any emitted wave at the right-hand side of the interface, i.e. we have, according to our assumptions, $g^R = 0$ and the outgoing wave strain amplitude is equal to that of the primary extension wave. Thus in relations (3.3) we have two unknowns only, namely U and g^L. Solving Eqs. (3.3) we obtain the following values:

$$U = \frac{\alpha(1+\alpha)f^R - 2f^L}{(1+\alpha)f^R - 2f^L} c_1, \tag{4.1}$$

where α is the sound velocity ratio $\alpha = c_2/c_1$, and

$$g^L = -\frac{(1-\alpha)^2 f^L}{(1+\alpha)^2 - 4(f^L/f^R)}. \tag{4.2}$$

The wave G^L outgoing at the left side with strain amplitude g^L can be considered as the F^L-wave partly reflected from the interface. The absolute value of g^L is always smaller than the absolute value of F^L([5]), thus there is no contradiction with the starting assumptions: negative sign of the strain at the left side from the interface is preserved and the propagation velocity of G^L-wave equals c_1, despite the fact that this wave is, obviously, an extension wave.

([3]) All the pictures of wave-motion demonstrated here, beginning with Fig. 2, were obtained directly by finite element method computations with no relation to the results of the present considerations, thus one can treat these pictures as some kind of the experimental (in the sense of numerical experiment) verification of the results. As far as the author knows, the instabilities visible behind the discontinuity front are of computational origin and probably do not reflect any physical reality.

([4]) We do not use in this paper the usual pictures of the wavefront motion in $\{x, t\}$ coordinates in order to show not only the wavefronts but also the wave profiles.

([5]) In fact it is usually much smaller, for example if $\alpha = 0.5$ and $|f^L| = |f^R|$, then $g^L = -0.04 f^L$.

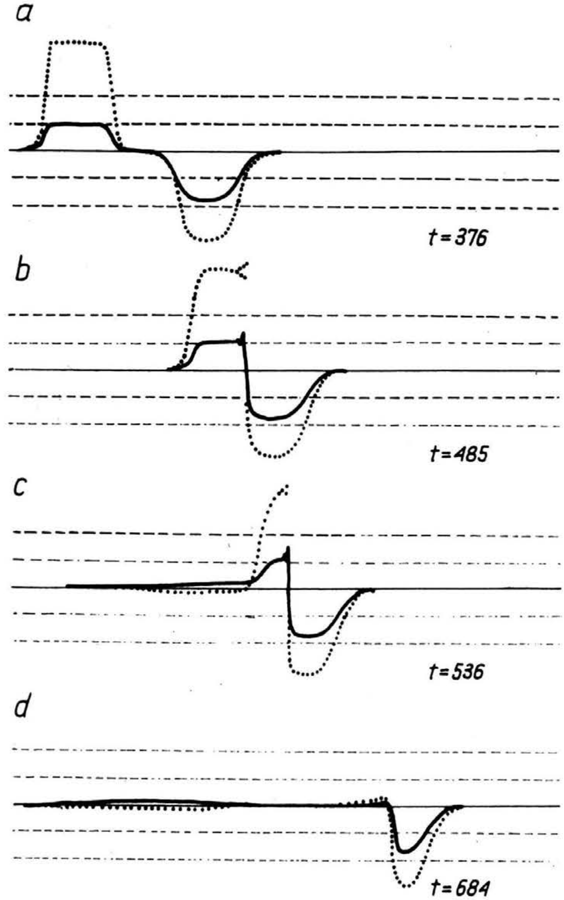

FIG. 2. The compression wave overtaking the extension wave. Four subsequent stages are shown. Dots depict the negative values of normal stress ($-\sigma_n$), solid line shows velocity distribution in an arbitrarily taken scale. The backward running reflected extension wave is visible in Fig. 2c and 2d.

If the domain of F^L-wave is bounded from the left side, then G^L would emerge eventually at some point, as a rule, close to the left edge of the mentioned domain where the total strain changes its sign, i.e. $f^L + g^L = 0$. Beginning from this point, G^L-wave becomes a real extension wave propagating with velocity c_2, its strain amplitude however changes. To find this new amplitude we have to come back to relations (3.3) and (3.4); we should remember, however, that this time the situation is reversed in comparison to the one considered in Sec. 2 and 3, i.e. we have to do with compresssion at the right-hand side and extension at the left and therefore quantities c_1 and c_2 in (3.3) and (3.4) should be interchanged. Denoting $F^L(x-ct)$ by $\widetilde{F}^R(x-ct)$, $G^L(x+ct)$ by $\widetilde{G}^R(x+ct)$ etc. we obtain at once from the modified relations (3.3) and (3.4) the following simple results:

$$c_1\widetilde{g}^R + c_2\widetilde{f}^L = 0, \qquad \widetilde{f}^L + \widetilde{g}^L = 0. \tag{4.3}$$

Remembering that total strain amplitude at the right-hand side is equal to zero, we obtain at once:

$$\tilde{g}^L = \frac{1}{\alpha}\tilde{g}^R, \tag{4.4}$$

i.e. the extension wave emerging from the compression region increases its strain amplitude $1/\alpha$ times. The considered wave — first reflected, and then transformed at the next interface — travelling to the left with velocity c_2 is clearly visible in Fig. 2c.

4.2. Colliding waves (first case)

As a next example we shall consider the case of the compression wave running from left to right and an extension wave travelling in the opposite direction. Here, depending on the amplitude relation between the two colliding waves, two cases are possible and should be analyzed separately. Thus we consider first the case of prevailing compression wave, i.e. we shall assume that

$$c_1 f^L + c_2 g^R < 0. \tag{4.5}$$

The discontinuity, as in the previous case, moves to the right with a velocity exceeding the sound velocity in the extended medium, thus if there is no primary wave travelling to the right at the right-hand side of the interface, then all the time f^R is equal to zero, and we again have only two unknown functions U and g^L in relations (3.3). From these relations we obtain

$$U = \frac{2(f^L + \alpha g^R) - \alpha(1+\alpha)g^R}{2(f^L + \alpha g^R) - (1+\alpha)g^R} c_1, \tag{4.6}$$

$$g^L = \frac{(1+\alpha)^2 g^R f^L}{4(f^L + \alpha g^R) - (1+\alpha)^2 g^R}. \tag{4.7}$$

One can easily verify that there is no contradiction in our results, i.e. that at the left side we indeed obtain negative ε^L $(|g^L| < |f^L|)$.

It should also be noticed that if $c_1 f^L + c_2 g^R \to 0$, then ε^L also tends to zero and U tends to c_2. In the previous example we have decided to consider G^L-wave as the F^L-wave reflected at the interface; it was justified by the fact of absence of the incident wave at the right-hand side of the interface. In the present case we rather withdraw from such interpretation since it would make little sense to divide G^L-wave into two parts — one due to reflection of a F^L-wave, and another one due to transmission of a G^R-wave. Four subsequent stages of the wave-motion in the considered case are depicted at Fig. 3 a, b, c.

4.3. Colliding waves (second case)

Lack of symmetry with respect to the strain sign expressed in our model by the relation (2.17) becomes clearly visible if one reverses the sign in inequality (4.5) of the previous example, switching from the case of prevailing compression wave to the case of prevailing extension wave. If we try to make use of relations (3.3) in the case of $c_1 f^L + c_2 g^R < 0$ then, taking into account that ε^L should be negative, we would obtain $-c_1 < U < -c_2$, but this result is in contradiction with condition (2.17).

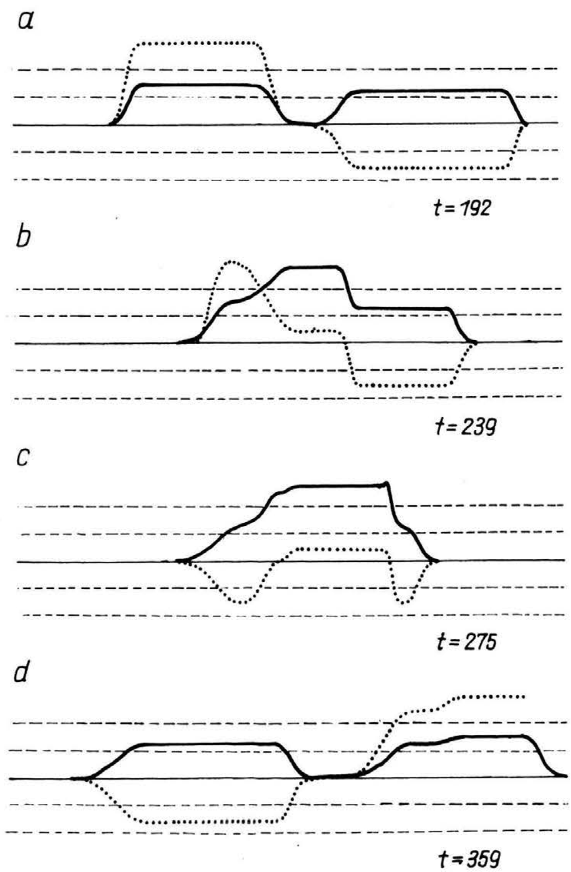

FIG. 3. Scheme of the collision of compression wave (left) and the extension wave running in the opposite direction, case I: prevailing compression amplitude. Legend — the same as in Fig. 2. The amplitude change of the compression wave passed across the extension region can be pointed out in Fig. 3d.

Thus, we have obtained the conclusion of nonexistence of a discontinuity. On the other hand, the continuity condition (3.4) is evidently not fulfilled. The common sense tells us, however, that the situation which we are trying to describe is quite feasible in reality and the real process should occur some way. Where is the answer then?

The only solution of this dilemma is to assume that there exist more discontinuities. Any attempt to consider a set of discontinuities containing a discontinuity tied to the interface between the compressed and extended regions brings us back to the same point which we came from. The cue can be found in the concluding remark in Sec. 2, claiming that the condition (2.17) is not valid if ε^L is equal to zero(6). The last condition, however, can be met only if $g^L = -f^L$, i.e. if F^L-wave behaves at the interface as if it arrived at a free surface — it performs complete reflection without amplitude change, but with opposite strain sign. The reflected wave runs to the left with velocity c_1. Substituting this

(6) Frankly speaking, this is quite obvious: there is no such a restriction for linear elastic material, thus we cannot expect any if the strain does not change its sign.

result into relations (3.3)([7]), we obtain at once

(4.8) $$U = -c_2,$$

(4.9) $$f^R = \frac{1}{\alpha} f^L.$$

Strictly speaking, U does not denote here the interface velocity, but we preserved the same symbol to stress the point that it retains the same value as in the limit case described in Sec. 4.2.

Let us mention at last that similar reasoning as the one shown in the Sec. 4.1 applied to the extension wave emerging from the compressed region, readily tells us that, in turn, the compression wave emerging from the extended zone decreases its strain amplitude α times. Thus if the moving extended zone is bounded, then a low intensity compression wave moving in the opposite direction passes it without any amplitude change, which is not true neither in the case of an overtaking wave in Sec. 4.1, nor in the previous case of wave collision when the compression wave was prevailing, as in the Sec. 4.2.

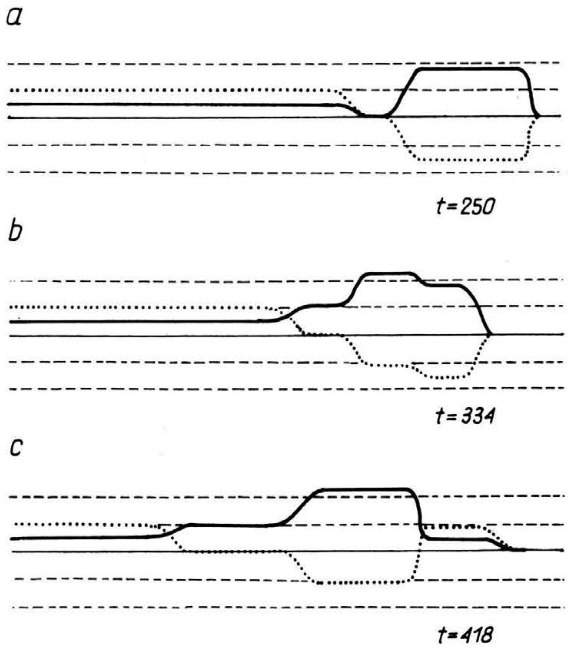

FIG. 4. The same collision scheme as in Fig. 3, case II: prevailing extension wave amplitude. Legend — the same as in Fig. 2. Complete reflection of the compression wave at the interface can be pointed out. The amplitude of the compression wave emerging from the extension region is almost the same (slightly lower due to viscosity effect) as that of the primary incident compression wave.

The example of a wave-motion case described in this section is shown in Fig. 4. Aside from the effects discussed earlier, one interesting feature can be observed: the existence of the unstressed region between two wavefronts moving in positive direction as a rigid block (in the particular case of the constant amplitude incident wave).

([7]) These relations are valid for any value of ε^L.

4.4. Harmonic wave excitation

It is evident that no harmonic wave can propagate in the medium under consideration; it can be interesting, however, to look what happens if a harmonic([8]) excitation is applied at some plane in the medium. The complete theoretical analysis of this case is of course possible (as far as the transcendental equations can be considered solvable) but it would be rather boring and not very informative; we should rather look at the results of computations (Fig. 5 and 6) performed both for the case of harmonic stress oscillation and for the harmonic displacement of the boundary (at $x = 0$ in our pictures).

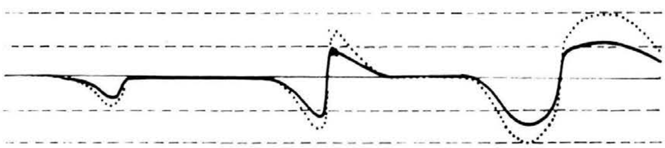

FIG. 5. Periodic wave propagation, the case of the symmetric stress cycle at the source. Legend — see Fig. 2.

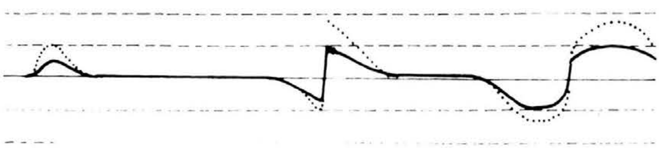

FIG. 6. Periodic wave propagation, the case of the symmetric displacement cycle at the source. Legend — see Fig. 2.

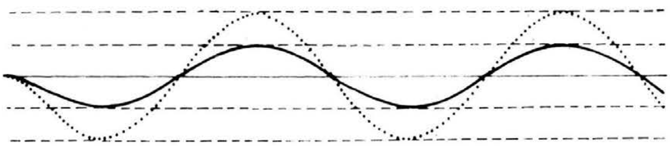

FIG. 7. Periodic wave in the linear material ($c_1 = c_2$). Negligible dumping effect due to the same viscosity as that involved in the schemes in Fig. 5 and 6 is visible.

The presented results need no comments but one: to obtain such a severe dumping in a viscoelastic material with symmetric characteristics, enormous viscosity should be involved. For reference, a viscoelastic wave propagation is shown at Fig. 7 for the same viscosity as that involved in the wave-motion pictures in Fig. 5 and 6 (for all computation procedures used here some material viscosity was assumed for the sake of suppressing the instability behind the discontinuity surface).

([8]) Strictly speaking it should be specified what exactly should be harmonic here: applied stress or displacement, since the both cases yield different wave motion pictures.

5. Final remarks

The author would be very glad if the presented results could bring their, even very modest, contribution to the understanding of some dynamic effects in damaged materials like the reflection of a compression wave from the extension wave, or the qualitative distinction between two cases of wave collision (as in Sec. 4.2 and 4.3), provided these effects can be observed in real experiments. The author hopes also that the described specific effects can be applied in material testing techniques, e.g. in damage diagnostics, particularly for the estimation of the broken fibres ratio in unidirectional fibre reinforced composites. For example, probably, the scheme of a modified split Hopkinson bar sketched in Fig. 8 enabling the generation of overtaking wave scheme (compare Sec. 4.1) for the investigation of the wave reflection at the discontinuity surface can be used. It is not clear, however, at least to the author, what should be the necessary signal level for the experimental detection of the effects discussed. For the time being the question of the order of magnitude of the expected phenomena in real materials seems to be open for further theoretical and experimental investigations.

Appendix. On the dissipation rate at the interface

Within the framework of the presented purely mechanic model, the problem of mechanical energy dissipation at the discontinuity, which is usually partly obscured by the thermodynamic considerations, arises and is more clearly visible. We cannot assume of course that any purely elastic process takes place at the interface. In such a case we would have in relation (2.10) and, therefore, also in (2.16), an equality sign forbidding any velocity discontinuity. This result would contradict both the common sense (real process should occur some way) and the results of numerical experiments.

In all cases quoted in Sec. 4 (except 4.3 where is no dissipation) the dissipation rate can be readily calculated despite the fact, that the dissipation mechanisms have been not specified earlier. In our considerations we have started with the fundamental principle of mechanics—the momentum balance law, and we have obtained the result that certain energy should be dissipated if the displacement continuity condition has to be preserved. Thus we have tacitly assumed that the real dissipative processes would adapt in such a way, that the proper dissipation rate would be secured and exactly such amount of energy, which should be dissipated, would be really dissipated. Without this assumption our model contains intrinsic contradiction.

The situation met can be compared with the problem of rotating coin in an empty space. Let at any instant a coin of radius R and mass m rotate with the angular velocity ω around its diameter having the moment of momentum equal to $k_1 = mR^2\omega/4$ and the kinetic energy $E_1 = mR^2\omega^2/8$. After some centuries one can expect to find the coin rotating around the normal to the plane with the angular velocity equal to $\omega_2 = \omega_1/2$ having the same moment of momentum $k_2 = k_1$ and two times lower energy $E_2 = E_1/2$([9]). This result is valid of course independently of the details of the dissipation mechanisms, what is a direct result of the most fundamental principle of moment of momentum con-

([9]) For reasonable values of the angular velocity, the elastic strain energy of the rotating coin is by several orders of magnitude lower than the kinetic energy and has no influence on the qualitative effects.

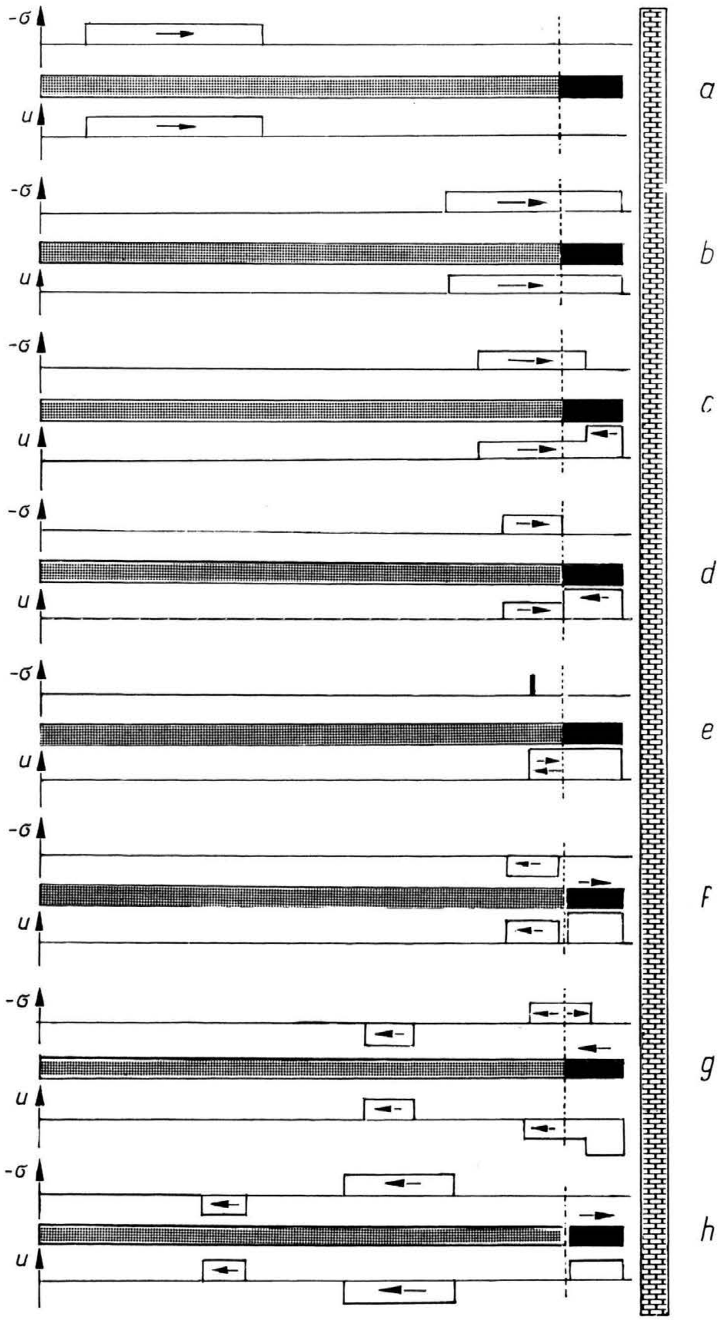

FIG. 8. Idealized scheme of the generation of the overtaking wave sequence. In the system, consisting of the split rod and a rigid obstacle, the long compression pulse generated at the left end, after a series of reflections generates the extension wave travelling from right to left and the compression wave behind it, running in the same direction.

servation and the very existence of the dissipation of any kind([10]) (a purely mathematical fact, that the moment of inertia of a thin uniform disc with respect to the normal axis is two times larger than that with respect to the diameter should be taken into consideration).

These two entirely different cases have the only common but very important feature: the overhelming role of the fundamental principles of mechanics enabling sometimes to obtain quantitative results with incomplete information on the material dissipative properties.

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POLISH ACADEMY OF SCIENCES
INSTITUTE OF FUNDAMENTAL TECHNOLOGICAL RESEARCH.

Received June 4, 1992.

([10]) The author does not know if the last example is widely known; if not then he wishes to inform, that he learnt it from V. V. Krotov [11] as the "pyatak" (five kopecks) case.