On a relaxing model for the flow of a mixture of gas with porous particles

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A SUSPENSION of solid porous particles in a perfect gas is considered. The gas penetrates into the particles and its quantity contained in the pores may vary with the flow conditions. It is assumed that the flow within the particles obeys Darcy's law. The pressure of the gas in the particle pores does not follow immediately the outer conditions, and relaxation phenomena in the flow are taken into account. One-dimensional analysis has been applied to the case of a shock wave in the considered suspension.

Rozważono zawiesinę porowatych cząsteczek stałych w gazie doskonałym. Gaz przenika do cząsteczek, a jego zawartość w porach może zmieniać się wraz z warunkami przepływu. Zakłada się, że przepływ gazu wewnątrz cząsteczek jest zgodny z prawem Darcy'ego. Ciśnienie gazu w porach cząsteczek nie nadąża za warunkami zewnętrznymi i w przepływie bierze się pod uwagę zjawiska relaksacyjne. Przeprowadzono analizę jednowymiarową dla przypadku fali uderzeniowej rozprzestrzeniającej się w rozważanej zawiesinie.

Рассматривается взвесь твердых пористых частичек в идеальном газе. Газ проникает в частицы, а его содержание в порах может изменяться вместе с условиями течения. Предполагается, что течение газа внутри частичек происходит согласно закону Дарси. Давление газа в порах не поспевает за внешними условиями и в течение учитывается явление релаксации. Поведен одноразмерный анализ для ударной волны, распространяющейся в рассматриваемом взвеси.

WE INTRODUCE a mixture of solid porous particles and a gas. The solid porous particles may contain a quantity of gas, varying with the flow conditions. The possible mass exchange between the solid porous particles and the gas flow creates conditions for pressure relaxation when the pressure of the gas inside the porous particles is not following immediately the variations of pressure of the gas flow.

We intend to consider a mixture of a gas and of solid particles. The particles are supposed to be porous; they may contain some quantity of the gas in their pores. The pressure of the gas in the pore is not necessarily always the same as the pressure of the gas outside the solid particles (in the "free" gas flow). The time needed to establish the equilibrium between the two pressures which, at the end, must become the same inside and outside of the solid particles may, be assimilated to a relaxation time.

The solid porous particles considered are supposed to be all of the same spherical form and the same diameter. They are rigid, and neither their dimensions nor their porosity do change under the action of external forces.

The dimensions of the particles are supposed to be large enough for the pores to contain some amount of gas. In fact, the diameter D_m of a gas molecule being of the order of $1 \text{ Å} = 10^{-7}$ mm, and the diameter D_p of a smoke particle being of the order of $1 \mu = 10^{-3}$ mm, we have $D_p/D_m = 10^4$.

We suppose that collisions between particles do not take place; the effect of particles interaction is neglected.

One can discern three types of gas-particles interaction.

1) Supposing that the particles flow velocity is at some instant different from the gas velocity, the viscous interaction between the gas and a particle tends to bring the two velocities to their common equilibrium value. As arriving at this dynamic equilibrium needs some time, there exists a possibility of introducing the notion of the velocity relaxation time or velocity relaxation length [2, 4].

2) Supposing that at a given instant the particles temperature and the gas temperature are not the same, one may find the time necessary for these temperatures to arrive to equality. By the same, one introduces the temperature relaxation time and temperature relaxation length [2, 3, 4].

3) Supposing that the particles are porous, one may imagine that a mass exchange is realised between the particles and the external "free" gas. In fact if at a given instant the pressure of the gas inside the particle pores is different from the external gas pressure, a flow across the porous particle material will take place, and this till the two pressures, external and internal, become equal. In this way a pressure relaxation can exist in a mixture of gas and of porous particles.

To simplify this case of relaxation, we suppose that the mixture is otherwise in equilibrium, that is, there do not exist temperature or velocity differences between the gas and the particles, the effects of an acceleration or of a temperature change being immediately transmitted to both.

In what follows we establish the equations of motion of such a mixture.

As in most problems of classical gas-dynamics the one-dimensional treatment of a mixture of gas and porous particles may furnish the characteristic features of the flow.

The fundamental equations may be written as follows:

The continuity equation

$$\varrho_{ge} u_{ge} S_{ge} = \dot{m}_{ge},$$

$$\varrho_{gi} u_{gi} S_{gi} = \dot{m}_{gi}, \quad \text{with} \quad u_{gi} = u_{ps},$$

$$\varrho_{ps} u_{ps} S_{ps} = \dot{m}_{ps},$$

where the indices ge, gi, ps stand for external gas, internal gas, solid particle, respectively.

There must be:

$$S_{ge} + S_{ps} = S,$$

where S is the section surface of the flow, and

$$\frac{S_{gi}}{S_{gi} + S_{ps}} = K = \left(\frac{V_{gi}}{V_{gi} + V_{gs}} \right)^{2/3},$$

where K is characterizing the porous material.

Finally:

$$\dot{m}_{gi} + \dot{m}_{ge} = \dot{m}_g \quad \text{and} \quad \frac{\dot{m}_{ps}}{\dot{m}_g + \dot{m}_{ps}} = \alpha,$$

where α — solid mass fraction of the mixture.

The momentum equations are

$$\varrho_{ge} u_{ge} \frac{du_{ge}}{dx} + \frac{dp_{ge}}{dx} = F_p,$$

$$\left(\frac{\varrho_{gi} V_{gi} + \varrho_{ps} V_{ps}}{V_{gi} + V_{ps}} \right) \frac{du_{ps}}{dx} = -F_p,$$

F_p represents the forces of interaction between the gas on the outside and in the inside of the pores:

$$F_p = \int_{S_p} (p_{ge} - p_{gi}) dS,$$

where S_p represents the surface section of the pores.

Energy equation:

$$\varrho_{ge} u_{ge} \left[C_p \frac{dT_{ge}}{dx} + \frac{d}{dx} \left(\frac{u_{ge}^2}{2} \right) \right] + p_{gi} u_{ps} \left[C_p \frac{dT_{gi}}{dx} + \frac{d}{dx} \left(\frac{u_{ps}^2}{2} \right) \right]$$

$$+ \varrho_{ps} u_{ps} \left[C_s \frac{dT_{ps}}{dx} + \frac{d}{dx} \left(\frac{u_{ps}^2}{2} \right) \right] = 0$$

with C_s — specific heat of the solid.

Supposing, as we do, that there is no slip between the particles and the gas ($u_{ge} = u_{ps}$) and no temperature difference ($T_{ge} = T_{gi} = T_{ps}$), we get

$$\varrho_{ge} u S_{ge} = \dot{m}_{ge} \quad \text{with} \quad \dot{m}_{ge} + \dot{m}_{gi} = \dot{m}_g,$$

$$\varrho_{gi} u S_{gi} = \dot{m}_{gi}$$

$$\varrho_{ps} u S_{ps} = \dot{m}_{ps} \quad \text{and} \quad \frac{\dot{m}_{ps}}{\dot{m}_{ps} + \dot{m}_g} = \text{const},$$

$$\varrho_{ge} u \frac{du}{dx} + \frac{dp\, ge}{dx} = F,$$

$$[\varrho_{gi} \lambda + \varrho_{ps}(1 - \lambda)] \frac{du}{dx} = -F,$$

$$(\varrho_{ge} + \varrho_{gi}) \left[C_p \frac{dT}{dx} + \frac{d}{dx} \left(\frac{u^2}{2} \right) \right] + \varrho_{ps} \left[C_s \frac{dT}{dx} + \frac{d}{dx} \left(\frac{u^2}{2} \right) \right] = 0,$$

where

$$\lambda = \frac{V_{gi}}{V_{ps} + V_{gi}}$$

is a characteristic coefficient relating the volume of the pores to the volume of a whole porous solid particle.

To complete this system we need a state equation. Or if the mixture is in equilibrium, the state equation is exactly the same as the state equation of a simple gas-solid particles mixture [3, 4]. In a general case, however, the gas in the interior of the pores may stay,

for some time at least, in conditions different from the conditions of the gas outside. It is convenient then to use two equations of state:

$$\frac{p_{ge}}{\varrho_{ge}} = RT_{ge},$$

$$\frac{p_{gi}}{\varrho_{gi}} = RT_{gi}.$$

The two pressures p_{ge} and p_{gi}, different as they may be, are not independent. In fact, the exchange of the gas between the pores and the exterior "free" gas modifies the pressures inside and outside the particles.

A flow across a porous material obeys Darcy's law [5],

$$\nabla \mathbf{v} = -C\nabla p,$$

where C is a coefficient characteristic of a given material. In our case we assume a similar law:

$$v = C(p_{ge} - p_{gi})$$

and the mass exchanged between the "free" gas and the particles is

$$\dot{m} = -\int_S C\varrho_{ge}(p_{ge} - p_{gi})ds,$$

where the integration is extended on the porous surface of the particle S_p.

For an isothermal process, when $p \sim \varrho$, we have

$$\dot{m} = -S_p C\varrho_{ge}^2 \left(1 - \frac{p_{gi}}{p_{ge}}\right)RT.$$

But the variations of the pressure with time must obey the equation

$$\frac{dp_{gi}}{dt} = \frac{RT}{V_{gi}}\frac{dm}{dt} = -\frac{RT}{V_{gi}}\dot{m}$$

$$= (S_p C\varrho_{gi}^2 RT)\frac{RT}{V_{gi}}\left(1 - \frac{p_{gi}}{p_{ge}}\right) = \frac{S_p C\varrho_{gi}RT}{V_{gi}}(p_{ge} - p_{gi}).$$

For pressure differences which are not too large, one may write

$$\frac{dp}{dt} = -\frac{1}{\tau}(p - p_{ge}),$$

where

$$\tau = \frac{V_{gi}}{S_p C\varrho_{ge}RT} = \frac{V_{gi}}{S_p Cp_{ge}}$$

as well inside as outside the particles.

We do not search here for the dependence between C and τ, but it is obvious that they are interdependent. Integrating, we have for the external pressure

$$p_{ge} = (p_{0ge} - p_{eq})e^{-\frac{t}{\tau}} + p_{eq}$$

and for the internal pressure

$$p_{gi} = (p_{0gi} - p_{eq}) e^{-\frac{t}{\tau}} + p_{eq}$$

where p_{0ge} — initial external pressure, p_{0gi} — initial internal pressure, p_{eq} — equilibrium pressure, the equilibrium pressure p_{eq} being the same for the two.

Introducing these expressions into the preceding equation (and exchanging p and ϱ for isothermal transformations) we get

$$\dot{m} = -S_p C R T \left[(\varrho_{0ge} - \varrho_{eq}) e^{-\frac{t}{\tau}} + \varrho_{eq} \right]^2 \left[1 - \frac{(\varrho_{0gi} - \varrho_{eq}) e^{-\frac{t}{\tau}} + \varrho_{eq}}{(p_{0ge} - \varrho_{eq}) e^{-\frac{t}{\tau}} + \varrho_{eq}} \right].$$

For $t = 0$, one comes back to Darcy's law with

$$\dot{m}_{t=0} = -S_p C R T \varrho_{0ge} (\varrho_{0ge} - \varrho_{0gi}).$$

For $t \to \infty$, one arrives at equilibrium

$$\dot{m}_{t \to \infty} = 0$$

the exchange between the gas outside the particles and inside ceases.

For $\tau = 0$, the equilibrium is instantaneous:

$$\dot{m}_{\tau = 0} = 0$$

and the gas, internal and external, behaves like one and the same.

For $\tau \to \infty$, the gas in the interior of solid particles is no more in contact with the external gas; the porous surface $S_p = 0$ and the flow is frozen,

$$\dot{m}_{\tau \to \infty} = 0 \quad \text{with} \quad S_p = 0.$$

The only parameter we still have to calculate is the equilibrium density ϱ_{eq}. We have

$$\varrho_{eq} = \frac{\varrho_{gi} V_{ge} + \varrho_{gi} V_{gi}}{V_{ge} + V_{gi}}.$$

The single particle volume V_{p0}, the solid contained in the particle V_{ps0}, and the void V_{gs0} are constant for given particles,

$$V_{p0} = V_{ps0} + V_{gs0} = V_{ps0} + V_{gi0}.$$

The mass of a particle being m_{s0}, the number of particles crossing a given flow section per second is

$$n = \frac{\dot{m}_{ps}}{m_{s0}}$$

and, as

$$\varrho_{ge} V_{ge} + \varrho_{gi} V_{gi} = m_g,$$

we have

$$\varrho_{eq} = \frac{m_g}{V_{ge} + V_{gi}} = \frac{\dot{m}_g}{Q_{ge} + Q_{gi}},$$

where Q_{ge} and Q_{gi} are the volume flows of gas (exterior and interior).

Finally, putting

$$Q_{ge} = \frac{\dot{m}_g - Q_{gi} x \varrho_{gi}}{\varrho_{ge}}$$

one gets:

$$\varrho_{eq} = \frac{\dot{m}_g \varrho_{ge}}{\dot{m}_g + n V_{gi0}(\varrho_{ge} - \varrho_{gi})}.$$

In this way we have established the equations describing the one-dimensional flow of a mixture of a gas with solid porous particles, taking into account the possible mass exchange between the external gas and the gas in the pores.

These equations show that in such a mixture the existence of porous particles introduces a pressure (or density) relaxation of the flow which does not appear in mixtures of gas and solid, non-porous particles [1].

The most simple manifestations of this capacity of relaxation due to the porosity of the particles participating in the flow will be a shock-wave flow and a flow through channels with a variable cross-sectional area. The case of a shock wave is particularly simple: the pressure inside the porous particles just after traversing the shock wave is the equilibrium pressure of the flow before the shock-wave. The gas in the interior of the pores having not been compressed by the shock-wave passage, there will exist a pressure relaxation behind a shock-wave, and the final equilibrium gas pressure will be different from the theoretically previsible pressure in a classical case.

Before proceeding with this example, it is necessary to be sure that the mass of the solid material of the porous particles does not contribute to the pressure of the mixture [3]. To obtain an estimate of this contribution, the particles are considered as large molecules of a second gas; it is then possible to determine the conditions for which the particle pressure of this "gas" reaches some established level, say x percent of the pressure of the whole mixture. We have

$$\frac{p_g}{p} > x$$

and, proceeding after Rudinger [3], treating the porous particles as regular spheres of a constant diameter D, we get

$$D^3 > \frac{x \dfrac{\alpha}{1-\alpha} \dfrac{\varrho_g}{\varrho_{ps}} \cdot C}{1-x},$$

where C is a numerical constant.

One remarks, by the way, that for fixed conditions of the pressure and of the mass fraction of the mixture, the diameter of the porous particles acceptable is superior to the acceptable diameter of the solid, non-porous particles, as the mass density of the porous particles is inferior to the mass density of the corresponding solid non-porous particles.

Consider now a shock-wave in a mixture of a perfect gas and of solid, porous particles.

As we wish to make evident the effect of the porosity of the particles alone, we suppose that the temperature of the gas, external and internal to the porous particles, and the

temperature of the particles themselves are the same and are submitted to the same variations. We suppose also that the velocity of the gas remains always the same as the velocity of the particles (in fact, the problem of slip and of the velocity relaxation was treated by CARRIER [1], and by MARBLE [2]); the variation of the velocity caused by the shock-wave is assumed to be the same for both.

As long as the porous particles are big enough, as compared with the gas particles, the shock-wave structure may be considered conventional. In our case the shock-wave will be followed by a relaxation zone where the gas parameters inside and outside the porous particles will tend to the equilibrium. Let us denote by 1 — the conditions before the shock-wave; by 2 — the conditions directly behind the shock-wave; by 3 — the conditions at some distance behind the shock-wave, where the mixture arrives at equilibrium.

Treating this kind of flow, one may discern three different possible cases, depending on the value of the relaxation time τ. If $\tau \to 0$, the equilibrium is instantaneous, and we have to treat a mixture of perfect gas and solid particles, the mass of a particle being equal to the mass of solid from which it is formed. If $\tau \to \infty$, the mixture is frozen, there is no gas exchange between the interior and the exterior of the porous particles; in fact, we have then to treat a mixture of a perfect gas and of particles, whose mass is equal to the mass of the solid, forming the particle, and the mass of the gas, contained inside a particle, taken together. The case which is interesting for us is the intermediary one, when $0 < \tau < < \infty$, the gas inside the porous particles remaining in contact with the external gas, though the equilibrium is not instantaneous.

Given the pressure p_1, the density ϱ_1, the velocity u_1, the Mach number M_1 of the equilibrium flow before the shock-wave, the conditions immediately after the passage of the shock are the following: for the "free" gas, exterior to the porous particles, the conventional formulas give

$$\frac{\varrho_2}{\varrho_1} = \frac{M_1^2}{1 + \dfrac{\gamma-1}{\gamma+1}(M_1^2-1)},$$

$$\frac{p_2}{p_1} = 1 + \frac{2\gamma}{\gamma+1}(M_1^2-1),$$

$$\frac{T_2}{T_1} = \frac{1}{M_1^2}\left[1 + \frac{2\gamma}{\gamma+1}(M_1^2-1)\right]\left[1 + \frac{\gamma-1}{\gamma+1}(M_1^2-1)\right],$$

$$\frac{u_2}{u_1} = \frac{\varrho_1}{\varrho_2}.$$

For the gas which passed through the shock-wave in the interior of the pores of the particles, we have

$$T_{2i} = T_{2e} = T_2.$$

For simplicity, we have supposed here that the temperature of the whole mixture is established instantaneously behind the shock and is the same for the external gas, for the

internal gas and for the particles themselves. As, during the passage of the shock, there was no mass transfer, the density of the gas in the pores did not change:

$$\varrho_{2i} = \varrho_{1i} = \varrho_{1e} = \varrho_1.$$

The pressure in the interior of the pores changed only because of the variation of the temperature:

$$\frac{p_{2i}}{p_1} = \frac{\varrho_{2i} T_2}{\varrho_1 T_1} = \frac{T_2}{T_1},$$

$$\frac{p_{2i}}{p_1} = \frac{M_1^2}{1 + \dfrac{\gamma-1}{\gamma+1}(M_1^2-1)}.$$

It is easy to verify that in this way, immediately behind the shock wave, the external pressure and density are superior to the internal pressure and density:

$$\frac{p_{2e}}{p_1} > \frac{p_{2i}}{p_e} \quad \text{and} \quad \frac{\varrho_{2e}}{\varrho_1} > \frac{\varrho_{2i}}{\varrho_1}.$$

To bring back the flow to equilibrium, a relaxation phenomenon will take place, due exclusively to the porosity of the solid particles.

To find the final equilibrium state, we use the formulas introduced earlier and, for the equilibrium density, we get

$$\varrho_3 = \frac{\dot{m}_g \varrho_{2e}}{\dot{m}_g + n V_{gio}(\varrho_{2e} - \varrho_{2i})} = \frac{\varrho_{2e}}{1 + \dfrac{n V_{gio}}{\dot{m}_g}\varrho_{2e}\left(1 - \dfrac{\varrho_{2i}}{\varrho_{2e}}\right)}.$$

Figures 1 and 2 represent the variation of ϱ_3/ϱ_{2e} for a flow with the volume concentration of solid particles equal 0.01, for $M = $ const and λ variable, and for $\lambda = $ const and M variable, respectively.
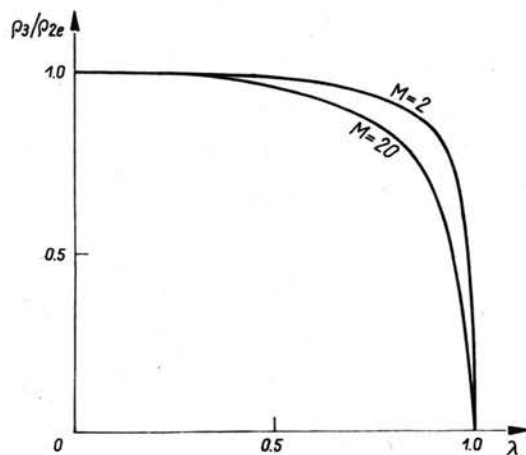

FIG. 1.

As we supposed, the transformation at constant temperature, the equilibrium pressure is readily obtained when the equilibrium density is known:

$$p_3 = \frac{p_{2e}}{\varrho_{2e}} \, \varrho_3.$$
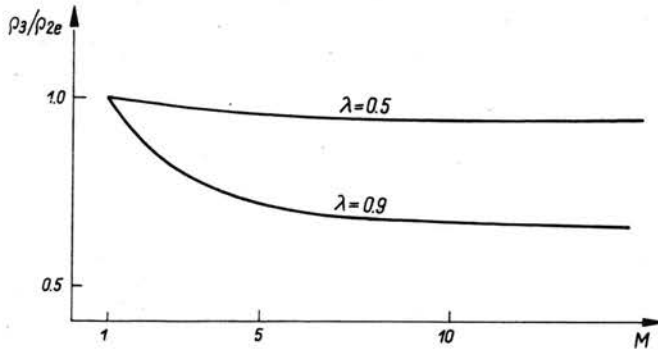

FIG. 2.

With these results in mind, we get the following formulas for the shock-wave flow:

$$\frac{\varrho_3}{\varrho_1} = \frac{\dfrac{M_1^2}{1 + \dfrac{\gamma-1}{\gamma+1}(M_1^2-1)}}{1 + \varrho_1 \dfrac{nV_{g10}}{m_g}\left[\dfrac{M_1^2}{1+\dfrac{\gamma-1}{\gamma+1}(M_1^2-1)}-1\right]},$$

$$\frac{p_3}{p_1} = \frac{p_{2e}}{p_1} \cdot \frac{\varrho_3}{\varrho_1} \cdot \frac{\varrho_1}{\varrho_{re}} = \frac{1 + \dfrac{2\gamma}{\gamma+1}(M_1^2-1)}{1 + \varrho_1 \dfrac{nV_{g10}}{m_g}\left[\dfrac{M_1^2}{1+\dfrac{\gamma-1}{\gamma+1}(M_1^2-1)}-1\right]},$$

$$\frac{T_3}{T_1} = \frac{T_2}{T_1},$$

$$\frac{u_3}{u_1} = \frac{\varrho_1}{\varrho_3}.$$

In the case when the porous particles disappear ($n = 0$) or when the parameters of the gas in the pores are, without a time lag, the same, as the parameters of the gas outside the pores ($p_i = p_e$; $\varrho_i = \varrho_e$), we get back the classical formulas of the shock-wave flow.

In this way it is shown that the introduced model of a gas-particles mixture when applied to a shock-wave flow provokes a variation of the characteristic parameters of this flow:

$$\frac{\varrho_3}{\varrho_1} \leqslant \frac{\varrho_2}{\varrho_1}, \qquad \frac{p_3}{p_1} \leqslant \frac{p_2}{p_1}, \qquad \frac{u_3}{u_1} \geqslant \frac{u_2}{u_1}.$$

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