Arch. Mech., 43, 1, pp. 29–39, Warszawa 1991

The linearized theory for submerged thin hydrofoil of infinite span

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IN THE FRAMEWORK of the linearized theory we study the plane-parallel uniform flow of an incompressible, ponderable, inviscid fluid in the presence of a submerged thin hydrofoil. We represent the complex velocity potential by a continuous distribution of sources and vortices potentials. From the boundary conditions we derive a singular integral equation for γ (the vortex circulation). A numerical method is used in order to solve the integral equation and to calculate the lift, drag and moment coefficients. Numerical results are given in the case of an oblique flat plate of small inclination.

W ramach zlinearyzowanej teorii przepływu rozważono opływ płata zanurzonego w ważkiej, nielepkiej i nieściśliwej cieczy. Zespolony potencjał prędkości przedstawiono w postaci ciągłych rozkładów potencjałów źródeł i wirów. Z warunków brzegowych wyprowadzono osobliwe równanie całkowe dla γ. Równanie to rozwiązano numerycznie w przypadku opływu płaskiej płytki o małym kącie nachylenia względem kierunku przepływu.

В рамках линеаризованной теории течения рассмотрено обтекание крыла, погруженного в веской, невязкой и несжимаемой жидкости. Комплексный потенциал скорости представлен в виде непрерывных распределений потенциалов источников и вихрей. Из граничных условий выведено сингулярное интегральное уравнение для γ. Это уравнение решено численно в случае обтекания плоской пластинки с малым углом наклона по отношению к направлению течения.

Notations

$2L_0$	length of the hydrofoil,
(x_1, y_1)	Cartesian variables,
(x, y)	dimensionless Cartesian variables
$z = x + iy$	complex variable in the (x, y)-plane,
$\mathbf{i}$	versor of the Ox-axis,
$\mathbf{j}$	versor of the Oy-axis,
$\mathbf{k} = \mathbf{i} \times \mathbf{j}$,	
$y = \eta(x)$	equation of the free surface (in dimensionless variables),
$y_1 = -h_1 + h_1^{\pm}(x)$	equations of the upper surface and inner surface of the hydrofoil,
$y = -h + h^{\pm}(x)$	equations of the outer surface and inner surface of the hydrofoil in dimensionless variables,
$\mathbf{n}_1, \mathbf{n}$	inward normal of the hydrofoil surface,
$\mathbf{v}_1$	fluid velocity,
U	speed of the unperturbed flow,
$\mathbf{v} = (u, v)$	dimensionless perturbation velocity,
p_1	pressure,
p_{atm}	atmospheric pressure,
ϱ	fluid density,
g	gravitation constant,
$p_1^* = p_1 + \varrho g y$	reduced pressure,
p^*	dimensionless perturbation pressure,
φ	dimensionless potential of the perturbation velocity,
f	dimensionless complex potential of the perturbation velocity,
$df/dz = w = u - iv$	complex perturbation velocity,

$k_0 = gL_0/U^2$ ($1/k_0$ is the Froude number),
$[p^*]$ jump of the perturbation pressure over the hydrofoil,
C_D drag coefficient,
C_L lift coeficient,
C_M moment coeficient.

In the incompressible irrotational hydrodynamics the potential of motion is represented by a continuous distribution of sources and vortices. We denote by:

$q(z)$ source intensity at point z,
$\gamma(z)$ vortex circulation at point z.

1. Introduction

IN THE FRAMEWORK of the linearized theory, a numerical method of calculation of the lift, drag and moment coefficients for arbitrary submerged thin hydrofoils of infinite span is given. Numerical results are obtained in the case of an oblique flat plate of small incidence.

2. The velocity potential

Suppose that the uniform motion of an incompressible, ponderable inviscid fluid bounded by a horizontal free surface is perturbed by the presence of a submerged hydrofoil of infinite span.

The motion is plane-parallel and the hydrofoil is considered to be thin, i.e.

(2.1) $$\left|\frac{h_1^{\pm}(x_1)}{L_0}\right| < \varepsilon, \quad \varepsilon \ll 1, \quad h_1^+(\pm L_0) = h_1^-(\pm L_0).$$

Let the Ox_1-axis be in the plane of the unperturbed free surface, parallel to the unperturbed velocity (Fig. 1).

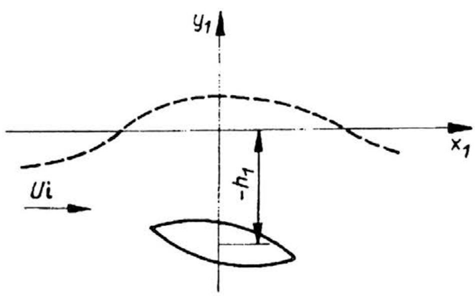

FIG. 1.

Introduce the dimensionless Cartesian variables x, y by the relation:

(2.2) $$(x_1, y_1) = L_0(x, y),$$

hence

(2.3) $$h_1^{\pm} = L_0 h^{\pm}.$$

The dimensionless perturbation velocity and pressure $\mathbf{v}$, p are given by the relations:

(2.4) $$\mathbf{v}_1 = U(\mathbf{i} + \mathbf{v}),$$

$$p_1^* = p_{\text{atm}} + \varrho U^2 p^* . \tag{2.5}$$

In the linear approximation the perturbation is determined by the system

$$\frac{\partial \mathbf{v}}{\partial x} + \operatorname{grad} p^* = 0, \quad \operatorname{div} \mathbf{v} = 0, \quad \lim_{x \to -\infty} (p^*, \mathbf{v}) = 0 . \tag{2.6}$$

From Eqs. (2.6) we deduce that rot $\mathbf{v} = 0$. Introduce $\varphi(x, y)$, the potential of the perturbation velocity given by the relation

$$\mathbf{v} = \operatorname{grad} \varphi(x, y) . \tag{2.7}$$

The linearized conditions imposed on the free surface are [3, 4].

$$\frac{\partial \varphi}{\partial y}(x, 0) = \frac{d\eta(x)}{dx} , \tag{2.8}$$

$$\frac{\partial \varphi}{\partial x}(x, 0) = -k_0 \eta . \tag{2.9}$$

From Eqs.(2.9) and (2.9) we get

$$\frac{\partial^2 \varphi}{\partial x^2} + k_0 \frac{\partial \varphi}{\partial y} = 0 \quad \text{for} \quad y = 0 . \tag{2.10}$$

In order to determine the perturbation produced by the hydrofoil, we represent it by the perturbation produced by an a priori unknown continuous distribution of sources and vortices.

As it is known from [3, 4], the complex velocity potential of the flow around a singularity consisting of a source of intensity q and a vortex of circulation γ at point z_0 is

$$f(z) = \frac{q - i\gamma}{2\pi} \ln(z - z_0) + \frac{q + i\gamma}{2\pi} \ln(z - \overline{z}_0) - \frac{q + i\gamma}{\pi} \exp(-ik_0 z) \int_{-\infty}^{z} \frac{\exp(ik_0 t)}{t - \overline{z}_0} \, dt . \tag{2.11}$$

The limits in the integral result from the requirement for the perturbation to vanish at $x \to -\infty$.

We can easily prove that $\varphi(x, y) = \operatorname{Re} f(z)$ satisfies Eq. (2.10).

Let

$$\begin{aligned} f(z) = \frac{1}{2\pi} \int_{-1}^{1} [q(\xi) - i\gamma(\xi)] \ln(z + ih - \xi) \, d\xi &+ \frac{1}{2\pi} \int_{-1}^{1} [q(\xi) + i\gamma(\xi)] \ln(z - ih - \xi) \, d\xi \\ &- \frac{\exp(-ik_0 z)}{\pi} \int_{-1}^{1} \left\{ [q(\xi) + i\gamma(\xi)] \int_{-1}^{1} \frac{\exp(ik_0 t)}{t - ih - \xi} \, dt \right\} d\xi \end{aligned} \tag{2.12}$$

be the complex velocity potential of a continuous distribution of sources and vortices on the segment $x \in [-1, 1]$, $y = -h$.

3. Distribution of velocity on the hydrofoil

As in the thin airfoil theory [1], we linearize the boundary condition $\mathbf{v}_1\mathbf{n}_1 = 0$ on the hydrofoil and impose it on the segment $x \in [-1, 1]$, $y = -h$

$$v^{\pm}(x, -h) = \frac{\partial h^{\pm}}{\partial x}(x), \quad x \in [-1, 1], \tag{3.1}$$

with

$$v^{+}(x, -h) = \lim_{\substack{y \to -h \\ y > -h}} v(x, y), \quad v^{-}(x, -h) = \lim_{\substack{y \to -h \\ y < -h}} v(x, y).$$

From Eq. (2.12) we get the complex velocity

$$w(z) = \frac{1}{2\pi} \int_{-1}^{1} \left\{ [q(\xi) - i\gamma(\xi)] \frac{1}{z + ih - \xi} - [q(\xi) + i\gamma(\xi)] \frac{1}{z - ih - \xi} + 2ik_0 [q(\xi) + i\gamma(\xi)] \int_{-\infty}^{z} \frac{\exp[ik_0(t - z)]}{t - ih - \xi} \, dt \right\} d\xi. \tag{3.2}$$

From Eq. (3.2), according to the Plemelj formulae, we have

$$w^{\pm}(x - ih) = \frac{1}{2\pi} \int_{-1}^{1} \left\{ [q(\xi) - i\gamma(\xi)] \frac{1}{x - \xi} - i\gamma(\xi)] \frac{1}{x - \xi - 2ih} + 2ik_0 [q(\xi) + i\gamma(\xi)] \int_{-\infty}^{x - ih} \frac{\exp[ik_0(t + ih - x)]}{t - ih - \xi} \, dt \right\} d\xi \mp \frac{i}{2}(q(x) - i\gamma(x)), \tag{3.3}$$

where

$$w^{+}(x - ih) = \lim_{\substack{y \to -h \\ y > -h}} w(x + iy), \quad w^{-}(x - ih) = \lim_{\substack{y \to -h \\ y < -h}} w(x + iy).$$

The first of the integrals is taken in the sense of Cauchy's principal value. In order to calculate the integral

$$\int_{-\infty}^{x - ih} \frac{\exp[ik_0(t + ih - x)]}{t - ih - \xi} \, dt,$$

the relation

$$\int_{-\infty}^{x - ih} \frac{\exp[ik_0(t + ih - x)]}{t - ih - \xi} \, dt = \int_{-\infty}^{\xi - ih} \frac{\exp[ik_0(t + ih - x)]}{t - ih - \xi} \, dt + \int_{\xi - ih}^{x - ih} \frac{\exp[ik_0(t + ih - x)]}{t - ih - \xi} \, dt \tag{3.4}$$

is used, where

$$(3.5)\qquad \int\limits_{-\infty}^{\xi-ih} \frac{\exp[ik_0(t+ih-x)]}{t-ih-\xi}\,dt = \lim_{a\to\infty} \int\limits_{-a-ih}^{\xi-ih} \frac{\exp[ik_0(t+ih-x)]}{t-ih-\xi}\,dt\,.$$

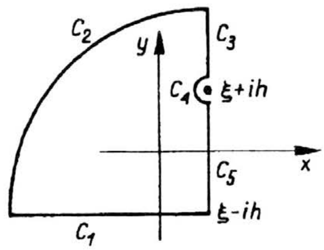

FIG. 2.

To calculate the last integral, the relation

$$(3.6)\qquad 0 = \int\limits_{C} \frac{\exp[ik_0(t+ih-x)]}{t-ih-\xi}\,dt\,,$$

is used, where

$$C = C_1(a)\cup C_2(a)\cup C_3(a,\varepsilon)\cup C_4(\varepsilon)\cup C_5(\varepsilon)$$

is the path of integration consisting of (Fig. 2) the following segments:

$$C_1(a) = \{t = t_1 + it_2; t_1 \in [-a,\xi], t_2 = -h\}\,,$$

$$C_2(a) = \{t = t_1 + it_2; t_1 + it_2 = \xi - ih + (a+\xi)\exp(i\theta); \theta \in [\frac{\pi}{2},\pi]\}\,,$$

$$C_3(a,\varepsilon) = \{t = t_1 + it_2; t_1 = \xi, t_2 \in [h+\varepsilon, a]\}\,,$$

$$C_4(\varepsilon) = \{t = t_1 + it_2; t_1 + it_2 = \xi + ih + \varepsilon\exp(i\theta); t \in [\frac{\pi}{2},\pi]\}\,,$$

$$C_5(\varepsilon) = \{t = t_1 + it_2; t_1 = \xi; t_2 \in [-h, h-\varepsilon]\}\,.$$

Taking into account that

$$(3.7)\qquad \lim_{a\to\infty} \int\limits_{C_2(a)} \frac{\exp[ik_0(t+ih-x)]}{t-ih-\xi}\,dt = 0\,.$$

and (from the semi-residue theorem) that

$$(3.8)\qquad \lim_{\varepsilon\to 0} \int\limits_{C_4(\varepsilon)} \frac{\exp[ik_0(t+ih-x)]}{t-ih-\xi}\,dt = \pi i \exp[ik_0(\xi-x) - 2hk_0]\,,$$

we obtain

$$(3.9)\qquad \int\limits_{-\infty}^{\xi-ih} \frac{\exp[ik_0(t+ih-x)]}{t-ih-\xi}\,dt = -i\pi\exp[ik_0(\xi-x)-2hk_0] - \lim_{\substack{a\to\infty\\ \varepsilon\to 0}} \int\limits_{C_3(a,\varepsilon)\cup C_5(\varepsilon)} \frac{\exp[ik_0(t+ih-x)]}{t-ih-\xi}\,dt\,,$$

whence

$$(3.10)\qquad \int_{-\infty}^{\xi-ih} \frac{\exp[ik_0(t+ih-x)]}{t-ih-\xi}\,dt = [\pi i + Ei^*(2hk_0)]\exp[ik_0(\xi-x)-2hk_0]\,,$$

where

$$(3.11)\qquad Ei^*(2hk_0) = \lim_{\varepsilon\to\infty}\left[\int_{-\infty}^{-\varepsilon}\frac{\exp s}{s}\,ds + \int_{\varepsilon}^{2hk_0}\frac{\exp s}{s}\,ds\right]$$

is the exponential integral function.

On the other hand

$$(3.12)\qquad \int_{z-ih}^{x-ih}\frac{\exp[ik_0(t+ih-x)]}{t-ih-\xi}\,dt$$

$$= \int_{\xi}^{x}\frac{\{(s-\xi)\cos[k_0(x-s)]+2h\sin[k_0(x-s)]\}}{(s-\xi)^2+4h^2} --- $$

$$--- \frac{+i\{2h\cos[k_0(x-s)]-(s-\xi)\sin[k_0(x-s)]\}}{+(s-\xi)^2+4h^2}\,ds\,.$$

From Eqs. (3.4), (3.10) and (3.12) we obtain

$$(3.13)\quad w^{\pm}(x-ih) = \frac{1}{2\pi}\int_{-1}^{1}\Big\{[q(\xi)-i\gamma(\xi)]\frac{1}{x-\xi} - [q(\xi)+i\gamma(\xi)]$$

$$\cdot\frac{1}{x-\xi-2ih} + 2ik_0[q(\xi)+i\gamma(\xi)]\cdot[Ei^*(2kh_0)+\pi i]\cdot\exp[-ik_0(x-\xi)-2hk_0]$$

$$+2ik_0[q(\xi)+i\gamma(\xi)]$$

$$\cdot\int_{\xi}^{x}\frac{\{(s-\xi)\cos[k_0(x-s)]+2h\sin[k_0(x-s)]\}}{(s-\xi)^2+4h^2} ---$$

$$--- \frac{+i\{2h\cos[k_0(x-s)]-(s-\xi)\sin[k_0(x-s)]\}}{+(s-\xi)^2+4h^2}\,ds\Big\}\,d\xi$$

$$\mp\frac{i}{2}[q(x)-i\gamma(x)]\,.$$

By separating in (3.13) the real part from the imaginary one, we get

$$(3.14)\quad u^{\pm}(x-ih) = \frac{1}{2\pi}\int_{-1}^{1}\frac{q(\xi)}{x-\xi}\,d\xi + \frac{1}{2\pi}\int_{-1}^{1}\left\{\frac{-q(\xi)(x-\xi)+2h\gamma(\xi)}{(x-\xi)^2+4h^2}\right.$$

$$+2k_0\exp(-2hk_0)[q(\xi)\sin[k_0(x-\xi)]-\gamma(\xi)\cos[k_0(x-\xi)]]\cdot Ei^*(2hk_0)$$

$$-[\pi\gamma(\xi)\sin[k_0(x-\xi)]+\pi q(\xi)\cos[k_0(x-\xi)]]$$

$$-2k_0q(\xi)\int_{\xi}^{x}\frac{2h\cos[k_0(x-s)]-(s-\xi)\sin[k_0(x-s)]}{(s-\xi)^2+4h^2}\,ds$$

(3.14) [cont.]

$$- 2k_0\gamma(\xi) \int_{\xi}^{x} \frac{(s-\xi)\cos[k_0(x-s) + 2h\sin[k_0(x-s)]}{(s-\xi)^2 + 4h^2} ds \Bigg\} d\xi \mp \frac{\gamma(x)}{2};$$

(3.15)

$$v^{\pm}(x - ih) = \frac{1}{2\pi} \int_{2\pi}^{'1} \Bigg\{ \frac{\gamma(\xi)}{x-\xi} + \frac{2hq(\xi) + (x-\xi)\gamma(\xi)}{(x-\xi)^2 + 4h^2}$$

$$-2k_0 \Bigg[q(\xi) \int_{\xi}^{x} \frac{(s-\xi)\cos[k_0(x-s)] + 2h\sin[k_0(x-s)]}{(s-\xi)^2 + 4h^2} ds$$

$$-\gamma(\xi) \int_{\xi}^{x} \frac{2h\cos[k_0(x-s)] - (s-\xi)\sin[k_0(x-s)]}{(s-\xi)^2 + 4h^2} ds \Bigg]$$

$$-2k_0 \exp(-2hk_0)[[\gamma(\xi)\sin[k_0(x-\xi)] + q(\xi)\cos[k_0(x-\xi)]] \cdot Ei^*(2hk_0)$$

$$+\pi q(\xi)\sin[k_0(x-\xi)] - \pi\gamma(\xi)\cos[k_0(x-\xi)]] \Bigg\} d\xi \pm \frac{q(x)}{2}.$$

From Eq. (3.15) and from the boundary conditions (3.1) we obtain the relation

(3.16)

$$q(x) = \frac{dh^+(x)}{dx} - \frac{dh^-(x)}{dx}$$

and the singular integral equation for $\gamma(\xi)$

(3.17)

$$\frac{1}{\pi} \int_{-1}^{1} \frac{\gamma(\xi)}{\xi - x} d\xi + \frac{1}{\pi} \int_{-1}^{1} \gamma(\xi) K(\xi, x)\, d\xi = \tilde{f}(x),$$

where

(3.18)

$$K(\xi, x) = \frac{\xi - x}{(x-\xi)^2 + 4h^2} - 2k_0 \int_{\xi}^{x} \frac{2h\cos[k_0(x-s)] - (s-\xi)\sin[k_0(x-s)]}{(s-\xi)^2 + 4h^2} ds$$

$$+2k_0 Ei^*(2hk_0)\sin[k_0(x-\xi)]\exp(-2hk_0) - 2k_0\pi\cos[k_0(x-\xi)]\exp(-2hk_0)$$

and

(3.19)

$$\tilde{f}(z) = -\left(\frac{dh^+(x)}{dx} + \frac{dh^-(x)}{dx} \right) + \frac{1}{\pi} \int_{-1}^{1} \Bigg\{ \frac{2hq(\xi)}{(x-\xi)^2 + 4h^2}$$

$$-2k_0 q(\xi) \int_{\xi}^{x} \frac{(s-\xi)\cos[k_0(x-s)] + 2h\sin[k_0(x-s)]}{(s-\xi)^2 + 4h^2} ds$$

$$+2k_0 \exp(-2hk_0) q(\xi)\cos[k_0(x-\xi)] Ei^*(2hk_0) - 2k_0\exp(-2hk_0)\pi q(\xi)\sin[k_0(x-\xi)] \Bigg\} d\xi.$$

4. The lift, moment and drag coefficients

The lift and drag coefficients are

(4.1)

$$C_L = \mathbf{j} \int_{\partial\mathcal{D}} p^* \mathbf{n}\, ds,$$

$$C_D = \mathbf{i} \int_{\partial \mathcal{D}} p^* \mathbf{n}\, ds\,. \tag{4.2}$$

The moment coefficient with respect to the point $M(0, h)$ is

$$C_M = \frac{\mathbf{k}}{2} \int_{\partial \mathcal{D}} \mathbf{x} \times \mathbf{p}^* n\, ds\,, \tag{4.3}$$

where

$$\mathcal{D} = \{x + iy; x \in [-1, 1], -h + h^-(x) \le y \le -h + h^+(x)\} \tag{4.4}$$

and $\mathbf{n}$, the inward normal, is

$$\mathbf{n}^+ = \frac{dh^+}{dx}\mathbf{i} - \mathbf{j}, \quad \mathbf{n}^- = -\frac{dh^-}{dx}\mathbf{i} + \mathbf{j}\,. \tag{4.5}$$

From Eqs. (4.1)-(4.5) we obtain (taking into account that the order of magnitude of the terms we have in view is ε)

$$C_L = -\int_{-1}^{1} [p^*]\, dx + O(\varepsilon^2)\,, \tag{4.6}$$

with

$$[p^*] = p^*_+(x, -h) - p^*_-(x, -h), \quad p^*_\pm(x, -h) = p^*(x, -h + h^\pm(x))\,,$$

$$C_D = \int_{-1}^{1} \left(p^*_+ \frac{dh^+}{dx} - p^*_- \frac{dh^-}{dx}\right) dx + O(\varepsilon^3)\,, \tag{4.7}$$

$$C_M = -\frac{1}{2} \int_{-1}^{1} [p^*] x dx + O(\varepsilon^2)\,. \tag{4.8}$$

From Eq. (2.6) we deduce that $p^* = -u$; therefore, taking into account Eq. (3.14), we obtain

$$[p^*] = \gamma\,. \tag{4.9}$$

Hence

$$C_L = -\int_{-1}^{1} \gamma(x)\, dx\,, \tag{4.10}$$

$$C_D = \int_{-1}^{1} \left(u^- \frac{dh^-}{dx} - u^+ \frac{dh^+}{dx}\right) dx\,, \tag{4.11}$$

$$C_M = -\frac{1}{2} \int_{-1}^{1} x\gamma(x)\, dx\,, \tag{4.12}$$

where

$$u^\pm(x, -h) = u(x, -h + h^\pm(x))\,.$$

The drag, the lift and the moment of the hydrodynamical forces are

$$R_D = \mathbf{i} \int_{\partial\mathcal{D}_1} p_1 \mathbf{n}_1 \, ds = \mathbf{i} \int_{\partial\mathcal{D}_1} (p_1^* - \varrho g y_1)\mathbf{n}_1 \, ds, \tag{4.13}$$

$$R_L = \mathbf{j} \int_{\partial\mathcal{D}_1} p_1 \mathbf{n}_1 \, ds = \mathbf{j} \int_{\partial\mathcal{D}_1} (p_1^* - \varrho g y_1)\mathbf{n}_1 \, ds, \tag{4.14}$$

$$M = +\mathbf{k} \int_{\partial\mathcal{D}_1} \mathbf{x}_1 \times p_1 \mathbf{n}_1 \, ds = -\mathbf{k} \int_{\partial\mathcal{D}_1} (p_1^* - \varrho g y_1)\mathbf{n}_1 \times \mathbf{x}_1 \, ds. \tag{4.15}$$

$\mathcal{D}_1$ denotes the transversal section of the hydrofoil. If we know the drag, lift and moment coefficients, we get from Eqs. (2.2), (2.4), (2.5) and (4.1)–(4.3).

$$R_D = \frac{1}{2}\varrho U^2 (2L_0) C_D, \tag{4.16}$$

$$R_L = \frac{1}{2}\varrho U^2 (2L_0)\left[C_L + k_0 \int_{-1}^{1} (h^+(x) - h^-(x))\, dx\right], \tag{4.17}$$

$$M = \frac{1}{2}\varrho U^2 (2L_0)^2\left[C_M + \frac{k_0}{2} \int_{-1}^{1} x(h^+(x) - h^-(x))\, dx\right]. \tag{4.18}$$

5. Approximate evaluation of the lift and moment coefficients

First of all we seek for an approximate solution of the singular integral equation (3.17). From the Kutta–Jukovski condition we deduce that the solution must be finite at the trailing edge (i.e. for $\xi = 1$). Therefore we represent it as

$$\gamma(\xi) = \sqrt{\frac{1-\xi}{1+\xi}}\,\Gamma(\xi). \tag{5.1}$$

The Gauss-type quadrature formulae [2] are used,

$$\frac{1}{\pi}\int_{-1}^{1} \sqrt{\frac{1-\xi}{1+\xi}}\,F(\xi)\, d\xi = \frac{2}{2n+1}\sum_{\alpha=1}^{n}(1-\xi_\alpha)F(\xi_\alpha), \tag{5.2}$$

$$\frac{1}{\pi}\int_{-1}^{1} \sqrt{\frac{1-\xi}{1+\xi}}\,\frac{G(\xi)}{\xi - x_j}\, d\xi = \frac{2}{2n+1}\sum_{\alpha=1}^{n}\frac{1-\xi_\alpha}{\xi_\alpha - x_j}G(\xi_\alpha), \tag{5.3}$$

where ξ_α are the roots of the orthogonal polynomial of degree n, associated with the weight function $\sqrt{(1-\xi)/(1+\xi)}$, namely

$$\xi_\alpha = \cos\frac{2a\pi}{2n+1}, \quad \alpha = 1, \ldots, n, \tag{5.4}$$

and

$$x_j = \cos\frac{2j-1}{2n+1}, \quad j = 1, \ldots, n. \tag{5.5}$$

Formulae (5.2) and (5.3) make it possible to replace Eq. (3.17) by a system of linear algebraic equations for the approximate values of the unknown function Γ in the nodal points

(5.6) $$\sum_{\alpha=1}^{n}(A_{j\alpha} + B_{j\alpha})\Gamma = \tilde{f}_j, \quad j = 1, \ldots, n,$$

where

(5.7) $$A_{j\alpha} = \frac{2}{2n+1}\frac{1-\xi_\alpha}{x_j - \xi_\alpha}, \quad B_{j\alpha} = \frac{2}{2n+1}(1-\xi_\alpha)K(\xi_\alpha, x_j),$$

and

(5.8) $$\Gamma_\alpha = \Gamma(\xi_\alpha), \quad \tilde{f}_j = \tilde{f}(x_j).$$

From Eqs. (4.10), (4.12), (5.1) and (5.2) we obtain the approximate values of the lift and moment coefficients

(5.9) $$C_L = -\frac{2\pi}{2n+1}\sum_{\alpha=1}^{n}(1-\xi_\alpha)\Gamma_\alpha,$$

(5.10) $$C_M = -\frac{\pi}{2n+1}\sum_{\alpha=1}^{n}(1-\xi_\alpha)\xi_\alpha\Gamma_\alpha.$$

6. The oblique flat plate. Numerical results

Let

(6.1) $$y = -h + h^{\pm}(x), \quad h^+(x) = h^-(x) = -\varepsilon x, \quad \varepsilon \ll 1$$

be the equations of the surface of obstacle (Fig. 3). From Eqs. (3.16) and (3.19) we obtain

(6.2) $$q(x) = 0,$$

(6.3) $$\tilde{f}(x) = 2\varepsilon.$$

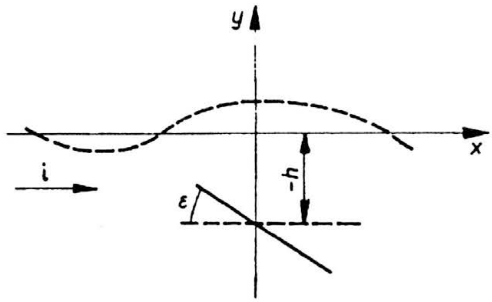

FIG. 3.

The numerical results obtained by means of the relations (3.18) and (5.1)–(5.10) for various values of k_0 and h are presented in Table 1. For $k_0 = 0$ the results obtained in this paper are very close to those obtained in [2].

Table 1.

h	k_0	$N_L = C_L/2\pi\varepsilon$	$N_M = C_M/\pi\varepsilon$
1	0	1.19082	1.10320
2	0.25	2.73542	2.66303
2	0.50	1.43311	1.52497
2	1	0.89607	0.99266
2	2	0.91086	0.95187
2	4	0.93400	0.96462
2	8	0.94319	0.96908
2	16	0.92465	0.97629
4	0.10	1.44423	1.42608
4	0.25	1.24306	1.25392
4	0.50	1.10079	1.03257
4	0.75	0.97271	0.99151
4	1	0.97329	0.98724
4	1.50	0.97855	0.98905
4	2	0.98066	0.99013
4.5	0.10	1.37773	1.36492
4.5	0.25	1.17175	1.18300
4.5	0.50	0.99712	1.01569
4.5	0.75	0.97810	0.99143
4.5	1	0.98003	0.99016
4.5	1.50	0.98363	0.99168
4.5	2	0.98502	0.99239
5	0.10	1.32444	1.31535
5	0.25	1.12332	1.13413
5	0.50	0.99235	1.00648
5	0.75	0.98253	0.99235
5	1	0.98461	0.99231
5	1.50	0.98710	0.99346
5	2	0.98806	0.99396
10	0	1.00249	1.00124
10	0.25	1.00378	1.00719
10	0.50	0.99609	0.99811

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Received February 25,1991