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A thermomechanical sketch on solid state heat engine based on shape memory effect

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ENERGY BALANCE in the solid state heat engine based on the shape memory effect is studied from the point of view of Metallo-Thermomechanics, a common research field of thermomechanics and metallurgy (theory of transformation in metals). Employing a simple pulley shape memory engine as an example, a certain amount of energy stored at higher temperature state as the strain energy in the alloy wire is shown to be partly consumed for deformation and other purposes as the process develops between the hot and cold reservoirs. The remaining energy is finally converted to the kinetic energy which drives the pulleys-shape memory wire system. Change in the stress-strain curve due to transformation is shown to be an essential factor in constructing an efficient and powerful engine.

Przeanalizowano problem bilansu energetycznego silnika cieplnego opartego na wykorzystaniu własności pewnych ciał stałych polegających na pamięci kształtu; analizę przeprowadzono z punktu widzenia termomechaniki metali, dziedziny leżącej na pograniczu termomechaniki i metalurgii. Posłużono się dla przykładu modelem składającym się z dwóch krążków opasanych drutem wykonanym ze stopu zachowującego pamięć kształtu. Pokazano, że pewna ilość energii zmagazynowanej w materiale przy podwyższonej temperaturze w postaci energii odkształcenia zostaje wydatkowana na deformację drutu oraz na inne cele, podczas gdy cały proces przebiega między zbiornikami ciepła o różnych temperaturach. Pozostała część energii przekształca się ostatecznie w energię kinetyczną napędzającą układ krążków. Wykazano, że zmiany krzywej "naprężenie–odkształcenie" wywołane przemianą zachodzącą w metalu stanowią istotny czynnik w konstrukcji wydajnego silnika tego rodzaju.

Критические состояния двойного маятника, подвергнутого следящему нагружению, вполне исследованы совместно с учетом нелинейного динамического анализа так описывающего поведение материала, как и геометрическую нелинейность. Нелинейное поведение материала принято в виде квадратной функции; итак идеальная система теряет стабильность в асимметричной точке бифуркации. В работе показано, что в случае система с имперфекциями появляются существенные разницы в значении критическич нагружений, определенных из статического и динамического анализов. Таких разниц не набюдаетця в системах без имперфекций. Это ябление обясняется через физыческую интерпретацию вышеперечисленных критических нагружений.

1. Introduction

A SHAPE MEMORY alloy specimen is deformed to a new configuration by a small external force at a low temperature and recovers its original shape with substantially greater force at high temperature [1, 2]. Thus the specimen has the potential ability to perform a certain amount of work during a thermomechanical process. When the alloy is thermally cycled between hot and cold reservoirs, it plays a role of working substance in an engine which converts the thermal energy to the mechanical work [3–6]. This "solid state heat engine" is designed to use the low-grade thermal energy in the form of low-temperature water from such sources as industrial waste heat, geothermal hot water, oceanic and fresh water thermal gradient [2], and attracts wide interest due to its pollution-free quality fit for the surrounding environment as well.

In order to get higher efficiency and stable action, some types of shape memory engines have been invented and demonstrated [3–6]; offset crank engine, inclined plane engine, turbine engine are among them. From the mechanical point of view, the stress-strain relation of the alloy and its temperature dependence should be the fundamental factors to be investigated, while intensive study on the cycle-dependence of the characteristics as well as on the fracture, fatigue and corrosive properties during the thermomechanical loading process, together with on the formability and weldability, must be carried out in order to construct an engine operating efficiently in the practical situations.

Why the shape memory engine actually works between hot and cold reservoirs is not a simple question to answer since it is closely related not only to the thermomechanical behavior of the alloy but also to the transformation occurring in the alloy during the cycling; a typical problem to be discussed in "Metallo-Thermomechanics [6]". The first step is to understand the mechanism of energy conversion in the system consisting of the engine itself and the heat reservoirs.

In this study, employing a simple pulley engine [7, 8] as an example, the flow of energy from the heat in the reservoir to the final kinetic energy is examined. Which amount of energy is first stored in the alloy as a strain energy, for which purposes it is partly compensated, and how much energy can be finally transferred to rotate the pulleys-shape memory wire system are explained from the thermomechanical point of view. It is stressed that the change in the stress-strain curve due to transformation plays an essential role in realizing an effective and powerful engine.

2. Stress-strain-temperature relation in shape memory alloys

Before going directly into the present theme, let us start here from a general explanation of the uniaxial thermomechanical behavior of the shape memory alloys [9].

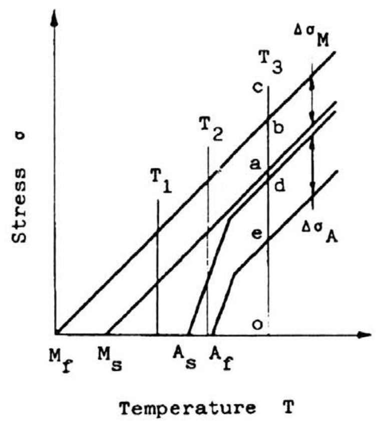

FIG. 1. Transformation lines and zones.

The stable metallurgical structure of the shape memory alloy at higher temperature level is an austenite (parent) phase. When the alloy is cooled under stress-free condition, the martensite phase starts to form at temperature M_s and the transformation completes at temperature M_f. On subsequent heating, initial full martensite phase transforms back to the austenite phase at the temperature range between temperatures A_s and A_f. According to the intensive experimental observations [1, 2, 10, 11], these transformation temperatures become higher, with the stress applied to the specimen, as shown schematically in Fig. 1. The lines intersecting the abscissa (the stress-free state) at the transformation temperatures may be called the martensitic or reverse transformation starting line and the martensitic or reverse transformation completing line. Each transformation progresses only in the range between the starting and completing lines, which is called the transformation zone.

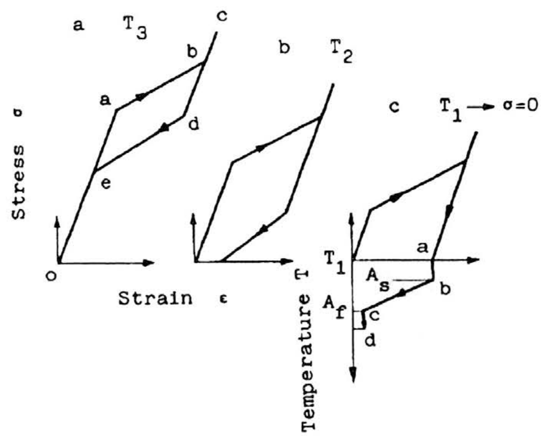

FIG. 2. Stress-strain curves.

Stress-strain curves obtained at the temperature levels T_1, T_2 and T_3 in Fig. 1 are illustrated schematically in Fig. 2. At T_3 above A_f, as explained above, the initial phase is the austenite. The elastic deformation of the austenite phase begins from point o, and then the martensitic transformation starts at point a (Fig. 2 a), where the stress-strain curve "yields". As the transformation develops between the points a and b, the alloy, which is now a mixture of the austenite and martensite phases when observed microscopically, behaves "inelastically". At point b the transformation is completed and the elastic deformation of the martensite phase follows in loading. In the unloading process, after the elastic unloading of the martensite phase to the point d, the reverse transformation takes place from the point d to the point e. Then the elastic unloading of the austenite phase follows. At this temperature level, the "inelastic" strain induced in the loading process fully recovers in the subsequent unloading process. The transformation pseudoelasticity or the superelasticity is thus observed.

At T_1 between M_s and A_s, the alloy behaves in a quite different manner. As Fig. 1 shows, the generic point never meets the reverse transformation zone in the unloading process. The stress-strain curve, therefore, ends up with the elastic unloading of the martensite phase (cf. Fig. 2c). The residual strain oa in the figure recovers when the alloy is heated up to a temperature above A_f, because the generic point moves to the higher temperature side on the abscissa and passes the reverse transformation zero, promoting the reverse transformation. The strain recovery in this heating process is shown in the lower part of Fig. 2c. The phenomenon is called the shape memory effect.

At T_2 between A_s and A_f, the reverse transformation progresses during unloading, but only partly. The residual strain in the figure also recovers in the heating process. Figure 2b clearly shows that the behavior is a transition state from the pseudoelasticity to the shape memory effect.

Some points to be noted for the latter discussion: Quite contrary to the ordinary metallic materials, the "yield stress" increases with the temperature, at least within the temperature range considered here.

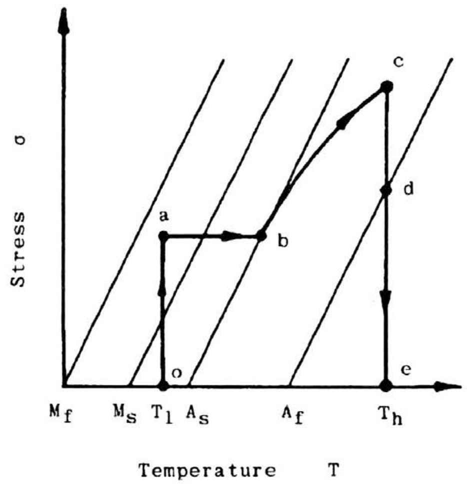

FIG. 3. Stress-temperature diagram during subsequent loading-unloading-heating process.

When the specimen is constrained and heated after being loaded to a certain amount of strain (to point a in Fig. 3, say) at a temperature T_1 between M_s and A_s, nothing is observed until the generic point reaches the point b in Fig. 3 on the reverse transformation starting line, if the effect of the thermal expansion is not considered. The stress-strain curve during this process is illustrated in Fig. 4. Due to the macroscopic transformation contraction a tensile stress is induced in the specimen as the reverse transformation progresses along the path bc in Figs. 3 and 4. Since the reverse transformation is not complete at point c, the residual martensite transforms to the austenite along the path cd when the constrained strain is released at point c. Nonlinear behavior cd in the stress-strain history is due to this transformation. It should be noted that the section cd is a part of the pseudoelastic unloading curve, de in Fig. 2a. The elastic unloading of the austenite phase is then followed from point d to the final unloading state e. The stress induced during the heating process is called the recovery stress [12], which plays a role of the working force in the actual shape memory devices.

3. Model of solid state engine

Geometrical configuration of a solid-state engine to be investigated here is shown in Fig. 5.

A uniform shape memory alloy wire, with circular cross-section of diameter d is shape-memorized as a straight wire at higher temperature and is looped between the two, large and small, pulleys, the radii of which are R and r, respectively. The whole length of the wire is L. We assume that the wire is straight between the pulleys, i.e. between the contact points A and D, and B and C. Part of the smaller pulley is dipped in the hot heat bath.

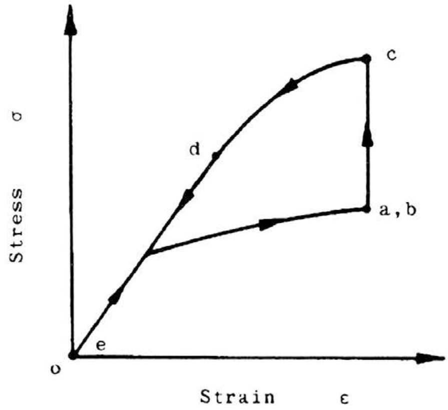

FIG. 4. Hysteresis during the process.

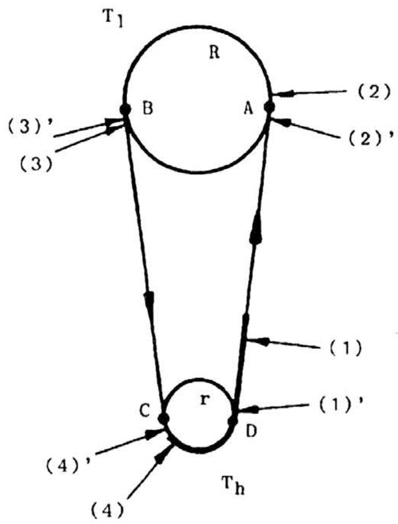

FIG. 5. Simple pulley engine.

Relative motion between the wire and the pulleys is neglected. The friction between the pulleys and their axes is also not considered. The frictional energy loss of pulleys should not, however, be simply disregarded, since for the smaller pulley we have to consider the friction with the hot ambient substance, and for the larger pulley the work output could be estimated as a frictional loss.

Let us now consider a steady state, in which the wire runs at a constant speed. The straight wire deforms suddenly at point A to a circular ring with the curvature $1/R$. The ring deforms back to the straight state at point B. The straight wire again deforms to the circular ring, but this time, with a larger value of the curvature $1/r$ at point C. No change is observed in the strain state between the points C and D.

A very simple temperature distribution along the wire is assumed: A part of the wire, which is marked by heavy line in the figure, is at a higher uniform temperature T_h above

A_f, and the remaining part at a lower uniform temperature T_1 between M_s and A_s. Specification of the exact points of the temperature jump is not necessary in the present qualitative study. An important thing to be noted here is that point C should be on the lower temperature side, the reason of which will be discussed later.Effects of the thermal expansion is neglected. Dynamic effects which might come from the sudden change in temperature is also not considered here. A smooth temperature distribution along the wire could of course be assumed, or be estimated by means of the numerical calculation in the quantitative study. Any essential change in conclusion would not, however, be produced even by posing such a realistic temperature distribution.

4. Change in stress and strain during a cycle

The change in stress and strain during a cycle or, better to say, their distribution along the wire is studied here from the macroscopic point of view. No axial force is assumed to be produced along the wire. We also propose, though there might be some objections from the metallurgical point of view [13], that the transformation phenomenon is exactly the same, both in the tensile and compressive stress state or, in other words, both on the tension and compression sides of the bent wire. A point on the outermost surface of wire could, therefore, be selected to examine the change in stress and strain in a cycle.

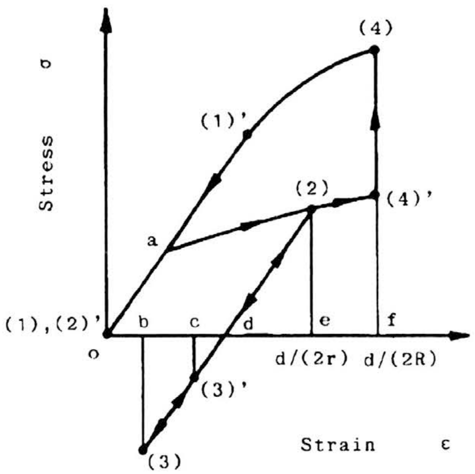

FIG. 6. Hysteresis in a cycle.

The stress-strain-temperature state at the position (1)–(4)′ in Fig. 5 is schematically shown as a stress-strain hysteresis in a cycle in Fig. 6, and as a locus in the stress-temperature plane in Fig. 7. At point (1), the temperature is T_h and the metallurgical state of the alloy is austenite. The wire is in the reference configuration (straight line). Neither stress nor strain, therefore, exist. No change of state is observed from point (1) to (2)′ just behind the point A except that the temperature lowers to T_1.

At point A a finite increase of strain, from 0 to $d/(2R)$, is observed due to sudden bending from a straight line to a circular ring with the curvature $1/R$. The associated stress increase promotes the martensitic transformation and the generic point reaches (2) in the stress-strain hysteresis in Fig. 6. The state does not change from point A to B in

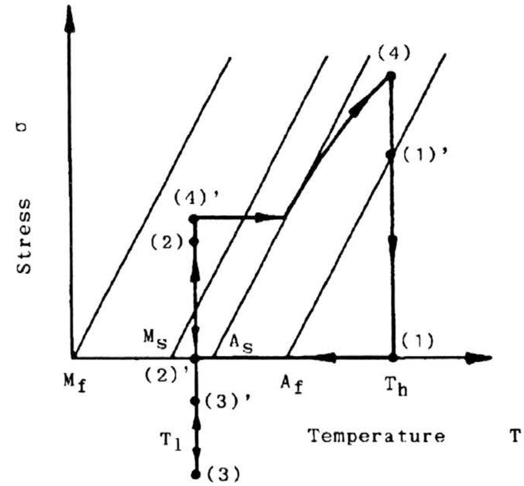

FIG. 7. Stress-temperature diagram in a cycle.

Fig. 5. Elastic unloading takes place at point B, which brings the state to (3)′. During this process the stress distribution over the cross-section of wire varies as illustrated in Fig. 8. In the state shown in Fig. 8b the resultant moment is zero, but a certain amount of curvature might still exist. To reach the state at (3), in which the macroscopic geometry of wire is straight, certain bending moment is necessary and it slightly changes the stress distribution illustrated in Fig. 8b.

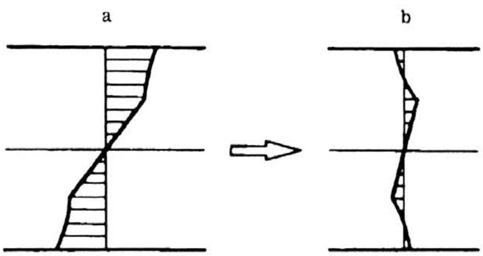

FIG. 8. Elastic unloading process in wire.

Since the temperature is still low at the point C, the value of strain jumps to $d/(2r)$ due to bending deformation along the smaller pulley. In this purely mechanical process the generic point moves first along the elastic reloading line, and then along the "plastic" loading curve to reach (4)′ in Fig. 6. The change from (4)′ to (4) during the hysteresis (cf. Fig. 6) can be well explained by the temperature rise to T_h under the constrained strain discussed in Sect. 3. During this strain-constrained (sudden) heating process the reverse transformation progresses as shown in Fig. 3. The stress at the outermost surface increases. When the reverse transformation is not completed at point D, the transformation continues in the subsequent strain-releasing process up to (1)′ just ahead of the point D. Then the elastic unloading takes place over the whole cross-section of the wire to reach the initial reference (straight) state at (1).

5. Energy balance

Roughly speaking, the energy is transferred from the heat reservoir to the system of pulleys and wire as follows: The shape memory wire extracts the heat from the heat bath of temperature T_h and stores it within itself as a strain energy. A part of the energy is transferred to the kinetic energy of the system. The mechanism of rotation of the system is closely related to the fact that the wire is straightly shape-memorized at a higher temperature. This point will be discussed in the next section.

Before going into the detailed discussion of the process, it should be noted here how the total amount of strain energy stored in the wire can be estimated. As explained before, for example, in Fig. 8, the stress and strain are distributed over a cross-section of wire and the distribution changes along the wire. Let the stress and strain state be measured $\sigma(x,y)$ and $\varepsilon(x,y)$, respectively, where x stands for the coordinate directed along the outward normal to the neutral plane of the wire, and y is the spatial coordinate measured from a fixed point along the wire in the direction of rotation. With each point x, a stress-strain history, the same as that shown in Fig. 6 (which is for $x = d/2$), is associated. The strain energy stored in a unit length of the wire at y can be calculated from

$$E(y) = \left| \int_{-d/2}^{d/2} \sigma(x,y)[\partial\varepsilon(x,y)/\partial y](d^2/4 - x^2)^{1/2}\,dx \right|, \tag{5.1}$$

and the total amount of energy extracted from the heat bath is given by

$$E = \int_0^l E(y)\,dy, \tag{5.2}$$

where l denotes the length of the part of wire at temperature T_h (heavy line in Fig. 5).

We are now ready to examine the energy balance of the system referring mainly to the stress-strain history at the outermost point of the wire, Fig. 6. Let us start from the point (4) in the figures. At this moment the alloy becomes stronger and the stress-strain curve is shifted from $o(2)(4)'$ to $o(1)'(4)$. The strain energy stored corresponds to the area of $(4)(1)'$. Note that this amount refers only to the outermost point, and the total amount of energy must be calculated by means of Eqs. (5.1) and (5.2).

Mechanical work $oe(2)a$ is then extracted from the energy stored in the wire and spend on bending the wire at the point A in Fig. 5. The energy corresponding to the areas $de(2)$, $cd(3)$ and $bc(3)'(3)$ is brought out and in during the following successive processes between just behind the point B and the point C; unloading from (2) to $(3)'$, loading from $(3)'$ to (3) and reloading from (3) to (2). The energy $ef(4)'(2)$ is then consumed for the bending deformation at the point C in Fig. 5.

In the whole process the energy corresponding to the area $a(2)(4)'(4)(1)'$ remains as a net energy available as the kinetic energy of the system.

Once again we stress here that Figs. 6 and 7 and the discussion above are referred to a point at the outermost surface of wire. The total energy balance in the system should, of course, be estimated by means of Eqs. (5.1) and (5.2). The conclusion obtained above remains valid for that case.

Points to be noted in constructing an efficient solid-state engine are now clear: The stress-strain curve of the shape memory alloy wire at the higher temperature should be, as much as possible, on the higher stress side, in order to gain a larger amount of the strain energy, area of (4)(1)′. For the same reason, to get a larger bending strain, the pulley on the higher temperature side should be possibly small. The stress-strain curve at the lower temperature should, to the contrary, be as much as possible on the lower-stress side to save the energy during bending deformation at the points A and C in Fig. 5. It is again apparent that the pulley must be larger on the lower temperature side in order to lower the bending strain. The point C, where the bending deformation increases, should be at the lower temperature range, so that smaller amount of energy consumption is expected.

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