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On coupling acceleration waves in a thermoviscoplastic medium I. Symmetry and hyperbolicity conditions

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IN THIS PAPER inelastic and thermal properties of the material are described by internal state variables. It is assumed that thermal disturbance can propagate with a finite wave speed. For a three-dimensional medium the sufficient and necessary conditions for the existence of real wave speeds are formulated. Thermal properties described lead to nonsymmetrycity of the waves.

W pracy wykorzystano teorię materiału z parametrami wewnętrznymi do opisu własności niesprężystych i termicznych. Proces przewodzenia ciepła ma charakter falowy i zaburzenia cieplne propagują się ze skończoną prędkością. Dla ośrodka trójwymiarowego wyprowadzono warunki konieczne i dostateczne na istnienie rzeczywistych fal przyspieszenia. Otrzymana niesymetria fali jest wynikiem własności termicznych materiału.

В работе использована теория материала с внутренними параметрами для описания неупругих и термических свойств. Процесс теплопроводности имеет волновый характер и тепловые возмущения распространяются с конечной скоростью. Для трехмерной среды выведены необходимые и достаточные условия существования вещественных волн ускорения. Полученная несимметрия волны является результатом термических свойств материала.

1. Introduction

THE PROPERTIES of materials, their theoretical description and initial conditions determine the possibilities of shock and acceleration waves propagation. In this paper propagation of acceleration waves for a heat conducting, dissipative and deformable material is analyzed. Internal dissipation is described by using internal state variables. It is assumed that thermal disturbance can propagate with a finite wave speed which is in contradiction to the behaviour of thermal disturbance governed by Fourier's law. To describe the internal dissipation two sets of internal state variables (internal parameters) are introduced: mechanical and thermal. One set is responsible for viscoplastic deformation and the other for finite velocity heat transport. To describe the viscous effect Perzyna's theory of viscoplasticity [15, 16, 17] is used. According to this theory the set of mechanical internal parameters contains the inelastic deformation α, the strain hardening parameter $\varkappa$ and the viscosity parameter γ.

There are two possibilities to obtain a finite speed of thermal disturbance. One is based on the assumption that the thermal conductivity coefficient K is a nonlinear function of temperature ϑ. This leads in the one-dimensional case to the Burger's parabolic differential equation for ϑ:

$$\frac{\partial\vartheta}{\partial t}+l(\vartheta)\frac{\partial\vartheta}{\partial x}=K\frac{\partial^2\vartheta}{\partial x^2}. \tag{1.1}$$

This kind of equation was used by SUVOROV [19, 20] and MARTINSON [12].

The other possibility modifies Fourier's law and postulates a new constitutive equation for the heat flux $\mathbf{q}$, which leads to the second-order hyperbolic equation

$$(1.2) \qquad \mu\frac{\partial\vartheta}{\partial t}+\frac{1}{\lambda_T^2}\frac{\partial^2\vartheta}{\partial t^2}=\nabla^2\vartheta$$

with a speed λ_T of the thermal wave. When $\lambda_T\to\infty$ Eq. (1.2) becomes a heat conduction equation of the parabolic type (describing diffusion).

First MAXWELL [13], then CATTANEO [2] VERNOTTE [21] used the modified Fourier's law of heat conduction

$$(1.3) \qquad \tau\dot{\mathbf{q}}+\mathbf{q}=K\nabla\vartheta.$$

The thermal relaxation time τ is the time required to obtain steady-state heat conduction after a temperature gradient is suddenly imposed. According to CHESTER [5], the relaxation time is of the order 10^{-10} s. For $\tau=0$ Eq. (1.3) represents exactly Fourier's law. An analogical modification of Fourier's law was introduced by GURTIN, PIPKIN [8], CHEN [3, 4] for materials with memory, BOGY, NAGHDI [1], FOX [7] for rate type materials and KOSIŃSKI [9], KUKUDZANOV [11] and SULICIU [18] for materials with internal state variables. Also MÜLLER in [14] obtained the hyperbolic equation for ϑ by introducing so-called coldness.

In this paper the sufficient and necessary conditions for the existence of real wave speeds are formulated. The thermal properties described lead to nonsymmetricity of the waves.

2. Constitutive equation

The thermomechanical state of a particle $\mathbf{X}$ at time t of the deformable body $\mathscr{B}$ is described by the value of the following function: $G(\mathbf{X},t)=\{\mathbf{E}(\mathbf{X},t),\vartheta(\mathbf{X},t),\operatorname{Grad}\vartheta(\mathbf{X},t),\mathbf{a}(\mathbf{X},t)\}$ where $\mathbf{E}=\operatorname{Grad}\mathbf{u}$ — strain gradient (displacement $\mathbf{u}=\mathbf{X}-\mathbf{x}$), $\vartheta>0$ — absolute temperature, $\operatorname{Grad}\vartheta(\mathbf{X},t)=\mathbf{g}(\mathbf{X},t)$ — material gradient of the temperature, $\mathbf{a}$ — internal parameters.

The selection internal parameters and their physical interpretation depends on the inelastic properties of a material. Additionally, internal parameters are described by the initial-value problem for a differential equation. We postulate the following equations for $\boldsymbol{\alpha}$:

$$(2.1) \qquad \dot{\boldsymbol{\alpha}}=\mathbf{A}(\mathbf{E},\vartheta,\boldsymbol{\alpha},\gamma,\varkappa)$$

for the thermal parameter $\boldsymbol{\beta}$:

$$(2.2) \qquad \dot{\boldsymbol{\beta}}=\mathbf{B}_1(\mathbf{E},\vartheta,\boldsymbol{\alpha},\boldsymbol{\beta},\gamma,\varkappa)\operatorname{Grad}\vartheta+\mathbf{B}_2(\mathbf{E},\vartheta,\boldsymbol{\alpha},\gamma,\varkappa),$$

for γ and $\varkappa$:

$$(2.3) \qquad \dot{\gamma}=\boldsymbol{\Gamma}(\boldsymbol{\alpha},\vartheta)\cdot\boldsymbol{\alpha}=\boldsymbol{\Gamma}(\boldsymbol{\alpha},\vartheta)\cdot\mathbf{A}(\mathbf{E},\vartheta,\boldsymbol{\alpha},\gamma,\varkappa),$$

$$(2.4) \qquad \dot{\varkappa}=\mathscr{K}(\boldsymbol{\alpha},\vartheta)\cdot\boldsymbol{\alpha}=\mathscr{K}(\boldsymbol{\alpha},\vartheta)\cdot\mathbf{A}(\mathbf{E},\vartheta,\boldsymbol{\alpha},\gamma,\varkappa).$$

Here we assume that the evolution equations for the internal parameters α, γ and $\varkappa$ do not depend on $\mathbf{g}$ and $\boldsymbol{\beta}$.

The equations for γ and $\varkappa$ are particular cases of evolution equations introduced by PERZYNA [17]. The function **A** for a viscoplastic material contains:

$$\mathbf{A} = \gamma\Phi\left(\frac{\mathrm{II}_T}{\varkappa}-1\right)\frac{\mathbf{T}}{\sqrt{\mathrm{II}_T}}+C \quad \text{for } \mathrm{II}_T > \varkappa, \tag{2.5}$$

where Φ is a function of $\left(\frac{\mathrm{II}_T}{\varkappa}-1\right)$, T — stress, II_T — the second invariant of **T**.

Response of the material is defined as a family of functions:

$$S(\mathbf{X}, t) = \{\mathbf{T}(\mathbf{X}, t), \psi(\mathbf{X}, t), \eta(\mathbf{X}, t), \mathbf{q}(\mathbf{X}, t)\}$$

for time t and the particle **X**, where **T** is the first Piola-Kirchhoff stress tensor, ψ denotes the free energy, η — the entropy and **q** — the heat flux. There is unique relation between the functions $G(\mathbf{X}, t)$ and $S(\mathbf{X}, t)$ which is expressed by the constitutive relations $\mathscr{R} = \{\mathscr{T}_\varkappa, \Psi, \mathscr{N}, \mathbf{Q}\}$(1)

$$\begin{aligned} \psi &= \Psi(\mathbf{E}, \vartheta, \boldsymbol{\alpha}, \boldsymbol{\beta}, \gamma, \varkappa),\\ \mathbf{T} &= \mathscr{T}_\varkappa(\mathbf{E}, \vartheta, \boldsymbol{\alpha}, \boldsymbol{\beta}, \gamma, \varkappa) = \varrho_0\,\partial_{\mathbf{E}}\Psi,\\ \eta &= \mathscr{N}(\mathbf{E}, \vartheta, \boldsymbol{\alpha}, \boldsymbol{\beta}, \gamma, \varkappa) = -\partial_\vartheta\Psi,\\ \mathbf{q} &= \mathbf{Q}(\mathbf{E}, \vartheta, \boldsymbol{\alpha}, \boldsymbol{\beta}, \gamma, \varkappa) = -\varrho_0\vartheta\mathbf{B}_1\,\partial_{\boldsymbol{\beta}}\Psi. \end{aligned} \tag{2.6}$$

The relations (2.6) are compatible with the second law of thermodynamics:

$$-\dot{\psi}-\eta\dot{\vartheta}+\frac{1}{\varrho_0}\mathbf{T}\cdot\dot{\mathbf{E}}-\frac{1}{\varrho_0\vartheta}\mathbf{q}\cdot\mathrm{Grad}\,\vartheta \geqslant 0, \quad \varrho_0 \text{ — mass density}. \tag{2.7}$$

It was assumed that the heat flux **q** does not depend on an actual value of the Grad ϑ but only on the past history of Grad ϑ till the actual time t. This influence is taken into consideration by the parameter $\boldsymbol{\beta}$ which depends on the history Grad ϑ as the solution of Eq. (2.2) with the initial condition $\boldsymbol{\beta}(\mathbf{X}, t_0) = \boldsymbol{\beta}_0$.

Now, the dissipation inequality under Eq. (2.6) takes the form

$$-\Psi_{,\boldsymbol{\alpha}} \cdot\cdot\, \mathbf{A}-\Psi_{,\boldsymbol{\beta}}\cdot\mathbf{B}_2-\Psi_{,\gamma}\boldsymbol{\Gamma}\cdot\mathbf{A}-\Psi_{,\varkappa}\mathscr{K}\cdot\mathbf{A} \geqslant 0. \tag{2.8}$$

Let us note that the Maxwell-Cattaneo equation (1.3) results from the linearization of Eqs. (2.2) and $(2.6)_4$:

$$\dot{\boldsymbol{\beta}} = \frac{b}{\tau}\mathrm{Grad}\,\vartheta-\frac{1}{\tau}\boldsymbol{\beta}, \quad b = \mathrm{const}, \tag{2.9}$$

$$\partial_{\boldsymbol{\beta}}\Psi = \frac{\tau f(\vartheta)}{\varrho_0}\boldsymbol{\beta}, \tag{2.10}$$

where f is an arbitrary function of ϑ.

Hence using Eq. $(2.6)_4$

$$\mathbf{q} = -\vartheta b f(\vartheta)\boldsymbol{\beta}. \tag{2.11}$$

Differentiating **q** with respect to time and eliminating $\boldsymbol{\beta}$ by using Eq. (2.11), we have

$$\tau\dot{\mathbf{q}} = -\vartheta b^2 f(\vartheta)\mathrm{Grad}\,\vartheta-\mathbf{q}-\tau b\big(\boldsymbol{\beta}f(\vartheta)+\vartheta f_{,\vartheta}(\vartheta)\boldsymbol{\beta}\big)\dot{\vartheta}. \tag{2.12}$$

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(1) GURTIN and PIPKIN [8] introduced a similar dependence **q** on Ψ for a material with memory; also KOSIŃSKI [9] and SULICIU [18] for a material with internal parameters.

The last term is responsible for the influence $\dot{\vartheta}$ on a change of $\mathbf{q}$. For $\tau = 0$, Eq. (2.12) established the nonlinear Fourier's law:

$$\mathbf{q} = -\vartheta b^2 f(\vartheta) \operatorname{Grad} \vartheta = -K(\vartheta) \operatorname{Grad} \vartheta, \tag{2.13}$$

where K is the scalar function because f was assumed as a scalar function.

For subsequent considerations we introduce an equilibrium state $G^\# = (\mathbf{E}^\#, \vartheta^\#, \mathbf{0}, \mathbf{a}^\#)$. From the definition of the state the evolution function at point $G^\#$ is equal to zero, in our case it means

$$\mathbf{A}(\mathbf{E}^\#, \vartheta^\#, \mathbf{a}^\#) \equiv \mathbf{0} \tag{2.14}$$

and

$$\mathbf{B}_2(\mathbf{E}^\#, \vartheta^\#, \mathbf{a}^\#) \equiv \mathbf{0}. \tag{2.15}$$

State $G^\#$ is an asymptotical state when the solution of

$$\begin{aligned} \dot{\mathbf{a}}(t) &= \mathbf{A}(\mathbf{E}^\#, \vartheta^\#, \mathbf{0}, \mathbf{a}(t)), \\ \mathbf{a}(0) &= \mathbf{a}_0 \end{aligned} \tag{2.16}$$

has the following property:

$$\bigvee_{\delta>0} \bigwedge_{\mathbf{a}_0} |\mathbf{a}_0 - \mathbf{a}^\#| < \delta \Rightarrow \lim_{t\to\infty} \mathbf{a}(t) = \mathbf{a}^\#. \tag{2.17}$$

For the asymptotical state $h^\# = (\mathbf{E}^\#, \vartheta^\#, \mathbf{a}^\#)$ and from the dissipation inequality (2.8) we have (cf. COLEMAN and GURTIN [6])

$$\Psi(\mathbf{E}^\#, \vartheta^\#, \mathbf{a}) \geqslant \Psi(\mathbf{E}^\#, \vartheta^\#, \mathbf{a}^\#)$$

hence

$$\Psi_{,\mathbf{a}}(\mathbf{E}^\#, \vartheta^\#, \mathbf{a}^\#) = 0. \tag{2.18}$$

Form this property it results that the heat flux vanishes at the equilibrium state

$$\mathbf{q}^\# = -\varrho_0 \vartheta \mathbf{B}_1^\# \partial_{\boldsymbol{\beta}} \Psi^\# = \mathbf{0}.$$

Returning to the equation (2.12) we can see that under the assumptions

$$\begin{aligned} \partial_{\boldsymbol{\beta}} \Psi(\boldsymbol{\beta}, \vartheta) &= \frac{a\tau}{\varrho_0} \boldsymbol{\beta}, \\ \mathbf{q} &= = b a \vartheta^\# \boldsymbol{\beta}. \end{aligned} \tag{2.19}$$

Equation (2.12) is exactly the Maxwell-Cattaneo relation

$$\tau \dot{\mathbf{q}} = K \operatorname{Grad} \vartheta - \mathbf{q},$$

where $K = b^2 a \vartheta^\#$.

Further nonconductors ($\mathbf{q} \equiv \mathbf{0}$) imply

$$\mathbf{B}_1 \partial_\beta \Psi = \mathbf{0}. \tag{2.20}$$

This, together with Eqs. (2.9) and (2.10), means that the conductivity coefficient $K(\vartheta)$ vanishes.

When $\mathbf{q} \neq \mathbf{0}$, then from the dissipation inequality (2.8) we conclude:

$$\frac{f(\vartheta)}{\varrho_0} \boldsymbol{\beta} \cdot \boldsymbol{\beta} \geqslant 0 \tag{2.21}$$

hence $f(\vartheta) \geqslant 0$, then $K(\vartheta) = \vartheta b^2 f(\vartheta)$ is nonnegative.

3. Coupling acceleration wave — hyperbolicity and symmetry conditions

The basic system of equations for the thermoviscoplastic body comprises: the law of motion,

$$\varrho_0 \dot{\mathbf{v}} - \mathrm{Div}\mathbf{T} = 0, \tag{3.1}$$

where $\mathbf{v} = \dot{\mathbf{u}}$ is the velocity of displacement,

the geometrical compatibility conditions

$$\dot{\mathbf{E}} - \mathrm{Grad}\,\mathbf{v} = 0, \tag{3.2}$$

the energy equation

$$\vartheta \partial_\vartheta \mathcal{N} \dot{\vartheta} + \vartheta \partial_{\mathrm{E}} \mathcal{N} \cdot \dot{\mathbf{E}} + (\vartheta \partial_{\mathbf{a}} \mathcal{N} + \partial_{\mathbf{a}} \Psi) \cdot \dot{\mathbf{a}} + \frac{1}{\varrho_0} \mathrm{Div}\,\mathbf{q} = 0, \tag{3.3}$$

and the evolution equations for $\boldsymbol{\beta}$, $\boldsymbol{\alpha}$, γ and $\varkappa$ (cf. Eqs. (2.1), (2.2), (2.3) and (2.4)),

$$\dot{\boldsymbol{\beta}} - \mathbf{B}_1 \,\mathrm{Grad}\,\vartheta - \mathbf{B}_2 = 0, \tag{3.4}$$

$$\dot{\boldsymbol{\alpha}} - \mathbf{A} = 0, \tag{3.5}$$

$$\dot{\gamma} - \boldsymbol{\Gamma} \cdot \mathbf{A} = 0, \tag{3.6}$$

$$\dot{\varkappa} - \mathcal{K} \cdot \mathbf{A} = 0. \tag{3.7}$$

Instead of the stress $\mathbf{T}$ and the heat flux $\mathbf{q}$, we can take the constitutive functions $\mathcal{T}_\varkappa$ and $\mathbf{Q}$, then we have a system of equations with four independent variables t and $\mathbf{X} = (X_1, X_2, X_3)$ and twenty seven dependent variables.

Hence equivalently to Eqs. (3.1)–(3.5), we have

$$\begin{gathered}
\varrho_0 \dot{\mathbf{v}} - \partial_E \mathcal{T}_\varkappa \cdot \mathrm{Grad}\,\mathbf{E} - \partial_\vartheta \mathcal{T}_\varkappa \mathrm{Grad}\,\vartheta - \partial_{\mathbf{a}} \mathcal{T}_\varkappa \mathrm{Grad}\,\mathbf{a} = \mathbf{0}, \\
\dot{\mathbf{E}} - \mathrm{Grad}\,\mathbf{v} = 0, \\
\varrho_0 \vartheta \partial_\vartheta \mathcal{N} \dot{\vartheta} - \vartheta \partial_\vartheta \mathcal{T}_\varkappa \mathrm{Grad}\,\mathbf{v} + \varrho_0 \mathbf{B}_1 (\vartheta \partial_{\boldsymbol{\beta}} \mathcal{N} + \partial_{\boldsymbol{\beta}} \Psi) \cdot \mathrm{Grad}\,\vartheta + \mathrm{Div}\,\mathbf{q} + H = 0, \\
\dot{\boldsymbol{\beta}} - \mathbf{B}_1 \,\mathrm{Grad}\,\vartheta - \dot{\mathbf{B}}_2 = 0, \\
\dot{\boldsymbol{\alpha}} - \mathbf{A} = 0, \\
\dot{\gamma} - \boldsymbol{\Gamma} \cdot \mathbf{A} = 0, \\
\dot{\varkappa} - \mathcal{K} \cdot \mathbf{A} = 0,
\end{gathered} \tag{3.8}$$

where

$$\begin{aligned}
H = H(\mathbf{E}, \vartheta, \boldsymbol{\alpha}, \boldsymbol{\beta}, \gamma, \varkappa) = \varrho_0 [(\vartheta \partial_{\boldsymbol{\beta}} \mathcal{N} + \partial_{\boldsymbol{\beta}} \Psi) \cdot \mathbf{B}_2 + (\vartheta \partial_{\boldsymbol{\alpha}} \mathcal{N} + \partial_{\boldsymbol{\alpha}} \Psi) \cdot \mathbf{A} \\
+ (\vartheta \partial_\gamma \mathcal{N} + \partial_\gamma \Psi) \boldsymbol{\Gamma} \cdot \mathbf{A} + (\vartheta \partial_\varkappa \mathcal{N} + \partial_\varkappa \Psi) \mathcal{K} \cdot \mathbf{A}].
\end{aligned}$$

The coefficients of the system (3.8) are called

$$\begin{gathered}
\varrho_0 \vartheta \partial_\vartheta \mathcal{N} \equiv c; \quad \frac{\partial T^{iL}}{\partial E^n_N} \equiv \mathcal{A}^{iLN}_n; \quad \frac{\partial T^{iK}}{\partial \vartheta} \equiv P^{iK}; \quad \frac{\partial T^{iL}}{\partial \alpha^n_N} \equiv B^{iLN}_n; \quad \frac{\partial T^{iL}}{\partial \beta_N} \equiv C^{iL}_N; \\
\frac{\partial T^{iL}}{\partial \gamma} \equiv D^{iL}; \quad \frac{\partial T^{iL}}{\partial \varkappa} = G^{iL}; \quad \varrho_0 B^{KM}_1 (\vartheta \partial_{\beta_M} \mathcal{N} + \partial_{\beta_M} \Psi) \equiv W^K; \quad \frac{\partial Q^L}{\partial E^n_M} \equiv R^{LM}_n; \\
\frac{\partial Q^L}{\partial \vartheta} \equiv I^L, \quad \frac{\partial Q^L}{\partial \alpha^n_M} \equiv M^{LM}_n, \quad \frac{\partial Q^L}{\partial \beta^M} \equiv N^L_M, \quad \frac{\partial Q^L}{\partial \gamma} \equiv L^K, \quad \frac{\partial Q^L}{\partial \varkappa} \equiv \L^K.
\end{gathered} \tag{3.9}$$

Here $\mathscr{A}_n^{lLM}$ are components of the elastic tensor, c is the specific heat of the material and $\mathbf{P}$ is called the temperature coefficient of stress. The small subscripts and superscripts are associated with the spatial coordinates and the capital with the material coordinates.

We can rewrite Eqs. (3.8) in the form of a system of quasi-linear differential equations

$$\mathbf{U}_{,t}+\mathbf{\Omega}_1(\mathbf{U})\mathbf{U}_{,1}+\mathbf{\Omega}_2(\mathbf{U})\mathbf{U}_{,2}+\mathbf{\Omega}_3(\mathbf{U})\mathbf{U}_{,3}+B(\mathbf{U}) = \mathbf{0}, \tag{3.10}$$

where $\mathbf{\Omega}_i$, $i = 1, 2, 3$ are the square matrices 27×27 containing the denotations (3.9),

$$\mathbf{U} = \begin{bmatrix} \mathbf{v} \\ \mathbf{E} \\ \vartheta \\ \boldsymbol{\beta} \\ \boldsymbol{\alpha} \\ \gamma \\ \varkappa \end{bmatrix}, \quad B \equiv \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ H \\ -\mathbf{B}_2 \\ -\mathbf{A} \\ -\mathbf{\Gamma}\cdot\mathbf{A} \\ -\mathscr{K}\cdot\mathbf{A} \end{bmatrix}. \tag{3.11}$$

The system (3.17) is hyperbolic if the eigenvalues λ of the matrix $\mathbf{\Omega}_K n^K$ are real. It means that

$$\det|\mathbf{\Omega}_1 \mathbf{n}^1+\mathbf{\Omega}_2 n^2+\mathbf{\Omega}_3 n^3-\lambda\mathbf{1}| = 0 \tag{3.12}$$

possesses 27 real roots for each direction $\mathbf{n}$ normal to the characteristic surface $\Sigma: f(\mathbf{X}, t) = 0$ and λ is the normal speed of propagation. λ and $\mathbf{n}$ are defined as follows:

$$\mathbf{n} = \frac{\operatorname{Grad} f}{|\operatorname{Grad} f|}, \quad \lambda = -\frac{\partial f/\partial t}{|\operatorname{Grad} f|}. \tag{3.13}$$

The solution of the hyperbolic equations have discontinuities of the first derivatives of $\mathbf{U}$ on the characteristic surface. Consequently Σ is called the acceleration wave:

$$[\mathbf{U}] = \mathbf{0}, \quad [\mathbf{U}_{,t}] \neq \mathbf{0}, \quad [\mathbf{U}_{,k}] \neq \mathbf{0}. \tag{3.14}$$

By the definition of an amplitude $\mathbf{r}$: $\mathbf{r} \equiv n^K[\mathbf{U}_{,K}]$ and the standard compatibility relation,

$$[\mathbf{U}_{,t}] = -\lambda n^K[\mathbf{U}_{,K}], \tag{3.15}$$

we obtain from the system (3.10)

$$(\mathbf{\Omega}_K n^K-\lambda\mathbf{1})\mathbf{r} = 0. \tag{3.16}$$

The solution $\mathbf{r} \neq 0$ if and only if Eq. (3.12) holds. The jump form of Eqs. (3.8) can be written as (cf. (3.9))

$$[\dot{v}^l]-\frac{1}{\varrho_0}\mathscr{A}_n^{lKM}[E^n_{M,K}]-\frac{1}{\varrho_0}P^{lK}[\vartheta_{,K}]-\frac{1}{\varrho_0}B_n^{lKM}[\alpha^n_{M,K}]-\frac{1}{\varrho_0}C^{lKM}[\beta_{M,K}] -\frac{1}{\varrho_0}D^{lL}[\gamma_{,L}]-\frac{1}{\varrho_0}G^{lL}[\varkappa_{,L}] = 0,$$

$$[\dot{E}^l_L]-[v^l_{,L}] = 0,$$

$$[\dot{\vartheta}]-c^{-1}\vartheta P^K_l[v^l_{,K}]+c^{-1}(W^K+I^L)[\vartheta_{,K}]+c^{-1}R_n^{LM}[E^n_{M,L}]+ +c^{-1}M_n^{LM}[\alpha^n_{M,L}]+c^{-1}N^{LM}[\beta_{M,L}]+c^{-1}L^K[\gamma_{,K}]+c^{-1}t^K[\varkappa_{,K}] = 0, \tag{3.17}$$

$$[\dot{\beta}_K]-B^L_{1K}[\vartheta_{,L}] = 0,$$
$$[\dot{\alpha}^n_M] = 0, \quad [\dot{\gamma}] = [\dot{\varkappa}] = 0.$$

B(U) are continuous functions of their argumenta. For. Eq. (3.17) we have sixteen nonzero amplitudes b^l, b^l_L, g, c_K; a similar result was obtained by KOSIŃSKI, SZMIT [10] for a thermoelastic material

$$(3.18)\qquad \begin{aligned} [v^l_{,L}] &= b^l n_L,\\ [E^l_{L,K}] &= b^l_L n_K,\\ [\vartheta_{,L}] &= q n_L,\\ [\beta_{K,L}] &= c_K n_L, \end{aligned}$$

hence sixteen equations of the form

$$(3.19)\qquad \begin{aligned} &-\lambda b^l-\frac{1}{\varrho_0}\mathscr{A}^{lKM}_n n_K b^n_M-\frac{1}{\varrho_0}P^{lK}n_K g-\frac{1}{\varrho_0}C^{lKM}n_K c_M = 0,\\ &-\lambda b^l_L-b^l n_L = 0,\\ &-\lambda g-c^{-1}\vartheta P^K_l n_K b^l+c^{-1}(W^K+I^K)n_K g+c^{-1}R^{LM}_n n_L b^n_M+c^{-1}N^{LM}n_L c_M = 0,\\ &-\lambda c_K-B^L_{1K}n_L g = 0. \end{aligned}$$

Equation (3.12) possesses 19 vanishing eigenvalues λ. The remaining nonvanishing solutions can be obtained whenever the following amplitude is defined:

$$(3.20)\qquad s^l \equiv n^K n^L [E^l_{K,L}].$$

Using Eq. (3.20) it is possible to determine the amplitudes (3.18) by

$$(3.21)\qquad \begin{aligned} b^l_L &= s^l n_L,\\ b^l &= -\lambda s^l,\\ c_K &= -\frac{1}{\lambda}B^L_{1K}n_L g \quad \text{for} \quad \lambda \neq 0. \end{aligned}$$

Equations (3.18) reduce to five equations for s and g

$$(3.22)\qquad \begin{aligned} &(\mathbf{Q}-\varrho_0\lambda^2\mathbf{1})\mathbf{s}+\left(\mathbf{p}-\frac{1}{\lambda}\mathbf{\Lambda}\right)g = 0,\\ &(\lambda^2\vartheta\mathbf{p}+\lambda\boldsymbol{\omega})\cdot\mathbf{s}+(\lambda\Pi-Z-c\lambda^2)g = 0, \end{aligned}$$

where

$$(3.23)\qquad \begin{aligned} &Q^l_n \equiv A^{lKM}_n n_K n_M\,(^2), \quad p_l \equiv P^K_l n_K, \quad \Lambda^l \equiv C^{lKM}n_K B^L_{1M}n_L,\\ &\Pi \equiv (W^K+I^K)n_K, \quad \omega_l \equiv R^{LM}_n n_L n_M \delta^n_l, \quad Z = n_L N^{LM}B^K_{1M}n_K. \end{aligned}$$

([2]) **Q** is the symmetric acoustic tensor.

The nonvanishing values of s and g exist when

$$\det\begin{bmatrix} Q_1^1-\varrho_0\lambda^2 & Q_2^1 & Q_3^1 & p_1-\frac{1}{\lambda}\Lambda_1 \\ Q_1^2 & Q_2^2-\varrho_0\lambda^2 & Q_3^2 & p_2-\frac{1}{\lambda}\Lambda_2 \\ Q_1^3 & Q_2^3 & Q_3^3-\varrho_0\lambda^2 & p_3-\frac{1}{\lambda}\Lambda_3 \\ \lambda^2\vartheta p_1+\lambda\omega_1 & \lambda^2\vartheta p_2+\lambda\omega_2 & \lambda^2\vartheta p_3+\lambda\omega_3 & \lambda\Pi-Z-c\lambda^2 \end{bmatrix}=0. \tag{3.24}$$

From this polynomial we have eight λ speeds of acceleration waves and as roots of Eq. (3.24) do not admit symmetric waves (λ and $-\lambda$ generally are not the roots of Eq. (3.24)). Similarly, nonsymmetric waves were obtained in the papers of GURTIN and PIPKIN [8], CHEN [3] and [4], BOGY and NAGHDI [1], MÜLLER [14]. On the contrary, SULICIU [18] in his paper assumed symmetry of the waves and then obtained the restriction on constitutive functions. For example, in our case we can obtain symmetry if we neglect certain coupling between mechanical and thermal properties such that $\mathbf{\Lambda}$, $\boldsymbol{\omega}$ and Π vanish (KOSIŃSKI, SZMIT [10]). We conclude that the nonsymmetry is caused mainly by thermal effects. To show this let us take a rigid conductor for which the arguments of the constitutive equations contain ϑ and $\boldsymbol{\beta}$ only. The same is for $\mathbf{B}_1$ and $\mathbf{B}_2$. Here there is one amplitude g and Eqs. (3.22) reduce to

$$c\lambda^2-\lambda(\mathbf{W}+\mathbf{I})\cdot\mathbf{n}+Z=0. \tag{3.25}$$

There are two acceleration waves; we assume that waves propagate alike in the direction $\mathbf{n}$ and $-\mathbf{n}$, such that

$$\partial_{\beta}\mathbf{Q}(\mathbf{B}_1\mathbf{n})\cdot\mathbf{n}\equiv Z<0.$$

The roots of Eq. (3.25) (cf. (3.9) where $c>0$) can be written in the form after GURTIN and PIPKIN [8]:

$$\begin{aligned} \vec{\lambda}_1 &= u_0\left\{\sqrt{1+\left[\frac{(\mathbf{W}+\mathbf{I})\cdot\mathbf{n}}{2u_0c}\right]^2}+\frac{(\mathbf{W}+\mathbf{I})\cdot\mathbf{n}}{2u_0c}\right\}>0, \\ \vec{\lambda}_2 &= u_0\left\{-\sqrt{1+\left[\frac{(\mathbf{W}+\mathbf{I})\cdot\mathbf{n}}{2u_0c}\right]^2}+\frac{(\mathbf{W}+\mathbf{I})\cdot\mathbf{n}}{2u_0c}\right\}<0, \qquad u_0\equiv\sqrt{-Z/c}. \end{aligned} \tag{3.26}$$

Here the arrow means that these speeds are in the direction of $\mathbf{n}$. The speeds (3.26) are invariant under a change of the propagation vector from $\mathbf{n}$ to $-\mathbf{n}$:

$$\begin{aligned} \overleftarrow{\lambda}_1 &= -\vec{\lambda}_2>0, \\ \overleftarrow{\lambda}_2 &= -\vec{\lambda}_1<0. \end{aligned} \tag{3.27}$$

Thus

$$|\vec{\lambda}_1|-|\vec{\lambda}_2| = \vec{\lambda}_1-\overleftarrow{\lambda}_1 = \frac{1}{c}(\mathbf{W}+\mathbf{I})\cdot\mathbf{n} = \frac{\Pi}{c}. \tag{3.28}$$

It follows that if the inner product $(\mathbf{W}+\mathbf{I})\cdot\mathbf{n}=0$, then acceleration waves in the direction $\mathbf{n}$ and $-\mathbf{n}$ are symmetrical to each other. For $(\mathbf{W}+\mathbf{I})\cdot\mathbf{n}\neq 0$ we can see that the angle between the vector $(\mathbf{W}+\mathbf{I})$ and $\mathbf{n}$ involves the relations $|\vec{\lambda}_1|<|\overleftarrow{\lambda}_2|$ or $|\vec{\lambda}_1|>|\overleftarrow{\lambda}_2|$. If, in particular, $\partial_{\beta}\mathcal{N} = 0$ and $\mathbf{B}_1$ is a scalar function, then the vector $(\mathbf{W}+\mathbf{I})$ is proportional to the heat flux $\mathbf{q}$:

$$\mathbf{W}+\mathbf{I} = \frac{\partial_{\vartheta} B_1}{B_1}\mathbf{q}. \tag{3.29}$$

Let $(\partial_{\vartheta} B_1/B_1) > 0$, then we conclude that (cf. GURTIN, PIPKIN [8] and MÜLLER [14]) the speed of a wave propagating in the direction of $\mathbf{q}$ is greater than that of a wave propagating in the direction of $-\mathbf{q}$:

$$\vec{\lambda}_1 - \overleftarrow{\lambda}_2 = \frac{1}{c}\frac{\partial_{\vartheta} B_1}{B_1}(\mathbf{q}\cdot\mathbf{n}). \tag{3.30}$$

Next, let us assume that 1) the material ahead of the acceleration wave is at the homogeneous equilibrium (undisturbed) state, 2) the material has a centre of symmetry. This means that $\mathbf{1} \in G$ where G is the symmetry group of a body $\mathcal{B}$:

$$\begin{aligned} \mathcal{N}(\vartheta, \boldsymbol{\beta}) &= \mathcal{N}(\vartheta, -\boldsymbol{\beta}),\\ \Psi(\vartheta, \boldsymbol{\beta}) &= \Psi(\vartheta, -\boldsymbol{\beta}),\\ \mathbf{Q}(\vartheta, \boldsymbol{\beta}) &= -\mathbf{Q}(\vartheta, -\boldsymbol{\beta}). \end{aligned} \tag{3.31}$$

Thus

$$\begin{aligned} \partial_{\boldsymbol{\beta}}\mathcal{N}(\vartheta, \mathbf{0}) &= \mathbf{0},\\ \partial_{\boldsymbol{\beta}}\Psi(\vartheta, \mathbf{0}) &= \mathbf{0},\\ \mathbf{Q}(\vartheta, \mathbf{0}) &= \mathbf{0}. \end{aligned} \tag{3.32}$$

By definition of the acceleration wave, the continuity of the equation coefficients (3.25) follows and, additionally, from the assumption 1 they are constants equal to their values at the point $(\vartheta^{\#}, \boldsymbol{\beta}^{\#}) = (\vartheta^{\#}, \mathbf{0})$. Now we may rewrite Eq. (3.25) by using the properties of ψ and $\mathcal{N}$ (2.18), $(3.32)_{1,2}$.

$$\varrho_0\vartheta^{\#}\partial_{\vartheta}\mathcal{N}^{\#}\lambda^2 - \varrho_0\vartheta^{\#}\mathbf{B}_1^{\#}\partial^2_{\boldsymbol{\beta\beta}}\Psi^{\#}(\mathbf{B}_1^{\#}\mathbf{n})\cdot\mathbf{n} = 0. \tag{3.33}$$

The wave speed

$$\lambda^2 = \frac{\mathbf{B}_1^{\#}\partial^2_{\boldsymbol{\beta\beta}}\Psi^{\#}(\mathbf{B}_1^{\#}\mathbf{n})\cdot\mathbf{n}}{\partial_{\vartheta}\mathcal{N}^{\#}} = u_0^2|_{\#}. \tag{3.34}$$

It results from the above discussion that for any adiabatic process in a thermoviscoplastic material acceleration waves are symmetric. Indeed in such a process $\mathbf{q} \equiv \mathbf{0}$, hence $\mathbf{B}_1\partial_{\boldsymbol{\beta}}\Psi \equiv \mathbf{0}$ (cf. $(2.6)_4$, and for det $[\mathbf{B}_1] \neq \mathbf{0}$ we have

$$\partial_{\boldsymbol{\beta}}\Psi = \mathbf{0}. \tag{3.35}$$

Moreover, by Eqs. (3.9) and (3.22)

$$\boldsymbol{\omega} = \mathbf{0}, \quad Z = \mathbf{0}, \quad \boldsymbol{\Lambda} = 0, \quad \Pi = 0 \quad (\mathbf{W}+\mathbf{I} = \mathbf{0}) \tag{3.36}$$

and Eq. [3.24] takes the form

$$\begin{aligned} \varrho_0^3 cz^3 - \varrho_0 z^2\{\vartheta\mathbf{p}\cdot\mathbf{p} + \varrho_0 c\,\mathrm{tr}\,\mathbf{Q}\} + \varrho_0 z\{\vartheta(\mathbf{p}\cdot\mathbf{p}\,\mathrm{tr}\,\mathbf{Q} - \mathbf{p}\cdot\mathbf{Q}\mathbf{p}) - c\mathrm{II}_{\mathbf{Q}}\}\\ - \{\vartheta(\mathbf{Q}\mathbf{p}\cdot\mathbf{Q}\mathbf{p} - \mathbf{p}\cdot\mathbf{p}\mathrm{II}_{\mathbf{Q}} - \mathbf{p}\otimes\mathbf{p}\cdot\mathbf{Q}\,\mathrm{tr}\,\mathbf{Q}) + c\det\mathbf{Q}\} = 0, \end{aligned} \tag{3.37}$$

where

$$\mathrm{II}_{\mathbf{Q}} = \frac{1}{2}\left(\mathrm{tr}\,\mathbf{Q}^2 - (\mathrm{tr}\,\mathbf{Q})^2\right), \quad z \equiv \lambda^2.$$

LEMMA 1. For the positive definite acoustic tensor $\mathbf{Q}$ and the positive specific heat c the necessary and sufficient conditions for the existence of real roots of the polynomial (3.37) are

$$S \equiv m^2+n^3 < 0,$$

where

$$2m = -\frac{2C_2^3}{27C_3^3}+\frac{C_2C_1}{2C_3^2}-\frac{C_0}{C_3}, \quad 3n = \frac{3C_1C_3-C_2^2}{3C_3^2},$$

C_1, C_2, C_3 and C_0 correspond to the coefficient of (3.37) written in the form

$$C_3z^3-C_2z^2+C_1z-C_0 = 0,$$

$$\begin{aligned} &C_3 \equiv \varrho_0^3 c,\\ &C_2 \equiv \varrho_0\{\vartheta\mathbf{p}\cdot\mathbf{p}+\varrho_0 c\,\mathrm{tr}\,\mathbf{Q}\},\\ &C_1 \equiv \varrho_0\{\vartheta(\mathbf{p}\cdot\mathbf{p}\,\mathrm{tr}\,\mathbf{Q}-\mathbf{p}\cdot\mathbf{Q}\mathbf{p})-c\,\mathrm{II}_{\mathbf{Q}}\},\\ &C_0 \equiv \{\vartheta(\mathbf{Q}\mathbf{p}\cdot\mathbf{Q}\mathbf{p}-\mathbf{p}\cdot\mathbf{p}\,\mathrm{II}_{\mathbf{Q}}-\mathbf{p}\otimes\mathbf{p}\cdot\mathbf{Q}\,\mathrm{tr}\,\mathbf{Q})+c\det\mathbf{Q}\}. \end{aligned} \tag{3.38}$$

The proof is based on the theory of algebraic equations. From the assumption of the Lemma we have:

$$C_3 > 0, \quad C_2 > 0, \quad C_1 > 0, \quad C_0 > 0. \tag{3.39}$$

The inequality $S < 0$ is the necessary and sufficient condition for the existence of real roots of Eqs. (3.38). Next from the inequalities (3.39) we find that they are positive.

Similarly, for a thermo-viscoelastic material in the state of equilibrium possessing a center of symmetry, the waves at the initial time $t = t_0$ are symmetric. For $t > t_0$ only the fastest wave of constant speed is symmetric; slower waves are propagated into the disturbed region and cease to be symmetric.

Thus from Eqs. (3.31) and (3.32)

$$\begin{aligned} \Psi(\mathbf{E},\vartheta,\boldsymbol{\alpha},\boldsymbol{\beta},\gamma,\varkappa) &= \Psi(\mathbf{E},\vartheta,\boldsymbol{\alpha},-\boldsymbol{\beta},\gamma,\varkappa),\\ \mathcal{T}_\varkappa(\mathbf{E},\vartheta,\boldsymbol{\alpha},\boldsymbol{\beta},\gamma,\varkappa) &= \mathcal{T}_\varkappa(\mathbf{E},\vartheta,\boldsymbol{\alpha},-\boldsymbol{\beta},\gamma,\varkappa),\\ \mathbf{Q}(\mathbf{E},\vartheta,\boldsymbol{\alpha},\boldsymbol{\beta},\gamma,\varkappa) &= -\mathbf{Q}(\mathbf{E},\vartheta,\boldsymbol{\alpha},-\boldsymbol{\beta},\gamma,\varkappa), \end{aligned} \tag{3.40}$$

then

$$\begin{aligned} \partial_{\boldsymbol{\beta}}\Psi(\mathbf{E},\vartheta,\boldsymbol{\alpha},\mathbf{0},\gamma,\varkappa) &= \mathbf{0},\\ \mathbf{Q}(\mathbf{E},\vartheta,\boldsymbol{\alpha},\mathbf{0},\gamma,\varkappa) &= \mathbf{0}. \end{aligned} \tag{3.41}$$

Therefore if we take the coefficient of the polynomial (3.24) at $(\mathbf{E}^{\#},\vartheta^{\#},\boldsymbol{\alpha}^{\#},\mathbf{0},\gamma^{\#},\varkappa^{\#})$, then

$$\boldsymbol{\omega} = \mathbf{0}, \quad \boldsymbol{\Lambda} = 0, \quad \Pi = 0, \tag{3.42}$$

and the next

$$\begin{aligned} &\varrho_0^3 c^{\#} z^4-\varrho_0 z^3\{\vartheta^{\#}\mathbf{p}^{\#}\cdot\mathbf{p}^{\#}-Z^{\#}\varrho_0^2+\varrho_0 c^{\#}\mathrm{tr}\,\mathbf{Q}^{\#}\}+\varrho_0 z^2\{\vartheta^{\#}(\mathbf{p}^{\#}\cdot\mathbf{p}^{\#}\mathrm{tr}\,\mathbf{Q}^{\#}\\ &\quad-\mathbf{p}^{\#}\cdot\mathbf{Q}^{\#}\mathbf{p}^{\#})-Z^{\#}\varrho_0\,\mathrm{tr}\,\mathbf{Q}^{\#}-c^{\#}\mathrm{II}_{\mathbf{Q}^{\#}}\}-z\{\vartheta^{\#}(\mathbf{Q}^{\#}\mathbf{p}^{\#}\cdot\mathbf{Q}^{\#}\mathbf{p}^{\#}-\mathbf{p}^{\#}\cdot\mathbf{p}^{\#}\mathrm{II}_{\mathbf{Q}^{\#}}\\ &\quad-\mathbf{p}^{\#}\otimes\mathbf{p}^{\#}\cdot\mathbf{Q}^{\#}\mathrm{tr}\,\mathbf{Q}^{\#})+Z^{\#}\varrho_0\mathrm{II}_{\mathbf{Q}^{\#}}+c^{\#}\det\mathbf{Q}^{\#}\}-Z^{\#}\det\mathbf{Q}^{\#} = 0. \end{aligned} \tag{3.43}$$

The polynomial in Eq. (3.43) can be written in the short form

$$w(z) = d_0 z^4 - d_3 z^3 + d_2 z^2 - d_1 z + d_0, \tag{3.44}$$

where

$$d_0 = \varrho_0^3 c^{\#},$$
$$d_3 = \varrho_0 \{\vartheta^{\#} \mathbf{p}^{\#} \cdot \mathbf{p}^{\#} - Z^{\#} \varrho_0^2 + \varrho_0 c^{\#} \operatorname{tr} \mathbf{Q}^{\#}\},$$
$$d_2 = \varrho_0 \{\vartheta^{\#} (\mathbf{p}^{\#} \cdot \mathbf{p}^{\#} \operatorname{tr} \mathbf{Q}^{\#} - \mathbf{p}^{\#} \cdot \mathbf{Q}^{\#} \mathbf{p}^{\#}) - Z^{\#} \varrho_0 \operatorname{tr} \mathbf{Q}^{\#} - c^{\#} \mathrm{II}_{\mathbf{Q}^{\#}}\},$$
$$d_1 = \vartheta^{\#} (\mathbf{Q}^{\#} \mathbf{p} \cdot \mathbf{Q}^{\#} \mathbf{p} - \mathbf{p}^{\#} \cdot \mathbf{p}^{\#} \mathrm{II}_{\mathbf{Q}^{\#}} - \mathbf{p}^{\#} \otimes \mathbf{p}^{\#} \cdot \mathbf{Q}^{\#} \operatorname{tr} \mathbf{Q}^{\#}) + Z^{\#} \varrho_0 \mathrm{II}_{\mathbf{Q}^{\#}} + c^{\#} \det \mathbf{Q}^{\#},$$
$$d_0 = Z^{\#} \det \mathbf{Q}^{\#}.$$

LEMMA 2. Let the tensor $\mathbf{Q}^{\#}$ be positive definite, $c^{\#} > 0$ and $Z^{\#} \equiv \mathbf{n} \cdot \partial_\beta \mathbf{Q}^{\#} > 0$, then the necessary and sufficient condition of existence of real roots of the polynomial (3.44) are

a) $$\bar{S} \equiv \bar{m}^2 + \bar{n}^3 < 0,$$

b) $$w(z_1) < 0, \quad w(z_2) > 0, \quad w(z_3) < 0,$$

where z_1, z_2, z_3 are roots of the polynomial $w'(z) = 4d_4 z^3 - 3d_2 z^2 + 2d_2 z - d_1$ such that $z_1 < z_2 < z_3$ and

$$2\bar{m} = -\frac{d_3^2}{32 d_4^2} + \frac{d_3 d_2}{8 d_4^2} - \frac{d_1}{4 d_4},$$

$$3\bar{n} = \frac{8 d_4 d_2 - 3 d_3^2}{16 d_4^2}.$$

Proof. From the assumption we have: $d_4 > 0$, $d_3 > 0$, $d_2 > 0$, $d_1 > 0$, $d_0 > 0$. From the theory of the third-order algebraic equation involves it follows that the roots z_1, z_2, z_3 are positive (cf. Lemma 1). The necessary condition is evident: if $z_1 > 0$, $z_2 > 0$, $z_3 > 0$, then a) and b) are fulfilled (see Fig. 1).

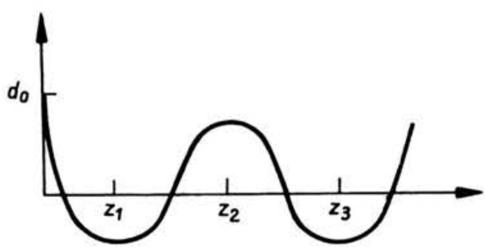

FIG. 1.

Sufficient condition: from a), b) and the assumptions it results that the roots z_1, z_2, z_3 are positive and the graph of $w(z)$ looks like Fig. 1.

Generally it is not so simple to obtain such conditions for Eq. (3.24) of the existence of real roots. It is opposed to thermoelasticity where the positive definition of $\mathbf{Q}^{\#}$ is the necessary and sufficient condition of acceleration wave propagation.

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