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The origin of dynamic effects during the arrest of a propagating crack

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DEPENDING on both the loading pattern and geometrical factors, the relatively simple K_{Ia} procedure, which is based on static linear elastic fracture mechanics principles, can either overestimate or underestimate the arrest length of a propagating crack. It is sometimes argued that reflected waves returning to the crack tip are responsible for this underestimation of the arrest length. However, this paper shows that the K_{Ia} procedure can underestimate the arrest length, even where it is clearly impossible for reflected waves to have any effect.

1. Introduction

IN SITUATIONS where the prevention of fracture initiation at a defect in a structure cannot be guaranteed, it is important to prevent a crack propagating a large distance and thereby limit the damage to a structure. The development of a crack arrest methodology for engineering structures has received impetus, particularly from considerations concerning the integrity of water-cooled reactor pressure vessels. The ASME Boiler and Pressure Vessel Code [1], which deals with flaw evaluation for emergency and faulted conditions in reactor pressure vessels, is based on the view that a crack arrests when the stress intensity, as determined by static linear elastic fracture mechanics procedures, falls below the crack arrest (K_{Ia}) fracture toughness curve. In the author's opinion, the strict physical justification for this very simple (K_{Ia}) approach stems from the behavior of a semi-infinite crack propagating in an unbounded solid due to the application of time-independent loads. In this case there are no wave reflection effects and the crack tip equation of motion for Mode I propagation is [2].

$$K_I^{\mathrm{DYN}} = f_I(a) K_I^{\mathrm{ST}} = K_{ID}, \tag{1.1}$$

where K_I^{DYN} is the dynamic crack tip stress intensification factor, K_{ID} is the dynamic fracture toughness, which is assumed to be independent of the crack tip velocity $\dot{a}$, K_I^{ST} is the static crack tip stress intensification factor, and $f_I(\dot{a})$ is a known function of crack tip velocity. Since $f_I(\dot{a}) \to 1$ as $\dot{a} \to 0$, the crack arrests when K_I^{ST} equals K_{ID}; this particular result provides some measure of physical justification for the K_{Ia} approach.

However, when the crack has a finite size and/or the structure has finite dimesions, then, assuming that wave reflections do not reach the crack tip, Eq. (1.1) must be replaced by [2–5]

$$K_I^{\mathrm{DYN}} = f_I(\dot{a}) K_I^{*} = K_{ID}, \tag{1.2}$$

where $K_I^{*} \equiv g K_I^{\mathrm{ST}}$, with g being a function of crack length and the configurations' geometry, and may be regarded as a "correction" factor; K_I^{*} is sometimes referred to as the reflectionless stress intensity factor. It is important to appreciate that the derivation of the relation (1.2) is based on an exact dynamic analysis, even though K_I^* is obtained by a purely static stress analysis. It has been argued [6, 7] that wave reflections can be ignored for the LOCA problem, and in this case the relation (1.2) is appropriate. Against this background the propagation of an edge crack due to time-independent loads, which generate a tensile stress ahead of the initial crack, has been studied and it has been shown [4, 8] that the value of K_I^{ST} at arrest exceeds the value of K_I^* at arrest (i.e. K_{ID} from the relation (1.2)); thus in this case the simple K_{Ia} approach provides a conservative estimate of the arrested crack length.

It has also been argued [9, 10, 11] that the K_{Ia} approach can also underestimate the crack length at arrest, due to reflected waves returning to the crack tip, thereby allowing the crack to propagate further than it would if there are no wave reflections. The purpose of this paper is to show that wave reflections need not necessarily be the sole cause of the K_{Ia} approach underpredicting the crack arrest length; underprediction can arise from an exact dynamic analysis in situations where wave reflections clearly play no role. More generally, the paper provides further support for the view that the simple K_{Ia} approach can be either conservative or nonconservative as regards prediction of the crack arrest length. Thus each particular problem should be considered on its own merits, even when wave reflections can be argued as having no effect; this procedure has been adopted for the LOCA problem [6, 7] Furthermore, in situations where sound physical arguments regarding the role of wave reflections cannot be developed, a full numerical dynamic crack propagation analysis using, for example, the Battelle procedures [11] is probably unavoidable.

2. Theoretical analysis

Consider the model in which the faces of the semi-infinite crack $-\infty < x < 0$, $y = 0$, at time $t = 0$, are wedged apart a distance h over the interval $-\infty < x < -a_0$ (Fig. 1). The objective of the analysis is to show that the reflectionless stress intensity factor K_I^* exceeds K_I^{ST} as the crack propagates and eventually arrests. This model has already been

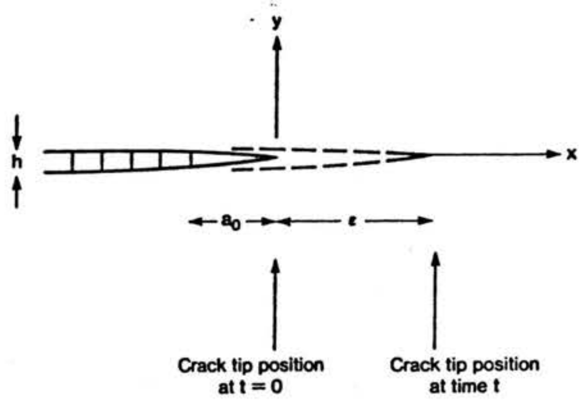

FIG. 1. The propagation of a semi-infinite crack in an unbounded solid due to the application of a constant displacement to the crack faces over the interval $-\infty < x < -a_0$).

analyzed [8] in an earlier paper with the objective of showing that $K_I^* \neq K_I^{ST}$, but this analysis is recast in the present paper with the fresh objectives and implications in mind. For the particular loading system under consideration, the stress $p(\lambda)$ ahead of the crack tip, when it is in its original position, is given by the expression:

$$p(\lambda) = \frac{Eh}{4\pi(1-\nu^2)\sqrt{\left(\frac{a_0}{2}+\lambda^2\right)^2-\left(\frac{a_0}{2}\right)^2}}, \tag{2.1}$$

where E is Young's modulus and v is the Poisson's ratio. The reflectionless stress intensity factor K_I^* at time t, when the crack has extended a distance ε, is given by the expression

$$K_I^* = \sqrt{\frac{2}{\pi}}\int_0^\varepsilon \frac{p(\lambda)d\lambda}{\sqrt{\varepsilon-\lambda}}, \tag{2.2}$$

where $p(\lambda)$ is the tensile stress ahead of the crack tip when it is in its orginal position at time $t = 0$. The relations (2.1) and (2.2) accordingly give

$$K_I^* = \frac{Eh}{4\pi(1-\nu^2)}\sqrt{\frac{2}{\pi}}\int_0^\varepsilon \frac{d\lambda}{\sqrt{\varepsilon-\lambda}\sqrt{\left(\frac{a_0}{2}+\lambda\right)^2-\left(\frac{a_0}{2}\right)^2}} \tag{2.3}$$

$$= \frac{Eh}{2\pi(1-\nu^2)}\sqrt{\frac{2}{\pi a_0}}\int_0^{\pi/2} \frac{d\theta}{\sqrt{1+\frac{\varepsilon}{a_0}\sin^2\theta}}.$$

The static crack tip stress intensification K_I^{ST}, again for a crack extension ε, is given by the expression (2.1), as

$$K_I^{ST} = \frac{Eh}{2(1-\nu^2)\sqrt{2\pi(a_0+\varepsilon)}}. \tag{2.4}$$

Supposing, using the accepted terminology, that K_Q is the static crack tip stress intensification at the onset of crack propagation, i.e.

$$K_Q = \frac{Eh}{2(1-\nu^2)\sqrt{2\pi a_0}}, \tag{2.5}$$

whereupon the relations (2.3) and (2.4) become, respectively,

$$\frac{K_I^*}{K_Q} = \sqrt{\frac{2}{\pi}}\int_0^{\pi/2} \frac{d\theta}{\sqrt{1+\frac{\varepsilon}{a_0}\sin^2\theta}} \tag{2.6}$$

and

$$\frac{K_I^{ST}}{K_Q} = \sqrt{\frac{1}{1+\frac{\varepsilon}{a_0}}}. \tag{2.7}$$

Figure 2 shows plots of both (K_I^*/K_Q) and (K_I^{ST}/K_Q) as functions of the crack jump length (ε/a_0); it is immediately seen that K_I^* exceeds K_I^{ST} for all crack extensions. If the dynamic fracture toughness is independent of crack tip velocity and has the value K_{ID}, the crack

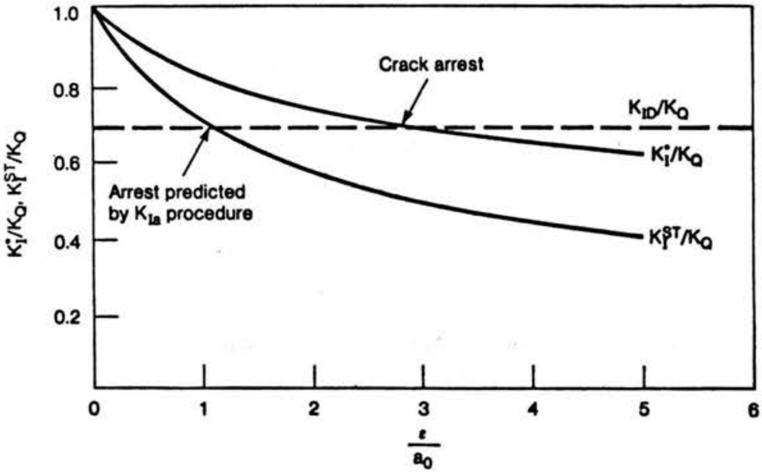

FIG. 2. The reflectionless stress intensity factor K_I^* and the static stress intensity factor K_I^{ST} plotted as functions of the crack extension ε; the results are normalized with respect to the initial static stress intensity factor K_Q. Crack arrest occurs where $K_I^* = K_{ID}$.

arrests when $K_I^* = K_{ID}$ (see the relation (1.2)). Thus the simple K_{Ia} procedure which predicts arrest when $K_I^{ST} = K_{ID}$ underestimates the arrest crack length for a particular situation where wave reflection effects play no role.

It should be emphasized that the result of this section is for a particular model in which a time-independent displacement is applied to the faces of the semi-infinite crack. If time-independent pressures are applied to the faces, $K_I^* = K_I^{ST}$ during crack propagation, and the K_{Ia} approach is exact as regards arrest crack length predictions [2, 8].

3. Discussion

The main point arising from the analysis in the preceding section is the clear demonstration that the simple K_{Ia} approach can underestimate the arrest crack length, even when wave reflection effects are clearly playing no role. The demonstration that a crack is able to propagate further than is predicted by the K_{Ia} procedure, without reflected waves necessarily reaching the crack tip, implies that in other situations, wave reflections may not be the sole cause of the K_{Ia} procedure underpredicting the crack arrest length. However, this should not be seen as an implication that the effects of wave reflections on crack propagation and arrest cannot be discounted. For example, wave reflections clearly play an important role in the rectangular double cantilever beam test since crack reinitiation has been observed after arrest, and this phenomenon is difficult to explain without involving the role of reflected waves returning to the crack tip [11]. Reflected waves apparently also play an important role in the continuous propagation stage, as will now be demonstrated.

Assuming there are no wave reflections, then, because $K_I^* \equiv K_I^{ST}$ at the onset of crack propagation, and $f_I(\dot{a}) \sim (1-\dot{a}/c_R)$ [3], where c_R is the Rayleigh wave velocity, Eq. (1.2) shows that the initial crack velocity for any situation should be v_0 given by the relation

$$\frac{v_0}{c_R} = \frac{(1-K_{ID})}{K_Q}, \tag{3.1}$$

where K_Q is the initial value of K_I^{ST}. Now, if the rectangular double cantilever beam test is simulated by the model in Fig. 3. where a semi-infinite crack in an infinite strip is wedged open, in the limit as the strip width tends to zero there will be no stress acting normal

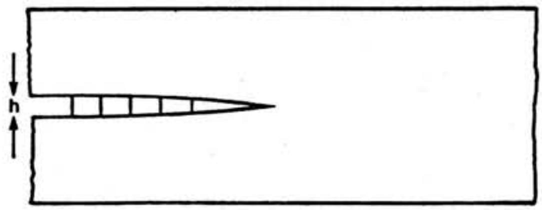

FIG. 3. A simple representation of the rectangular double cantilever beam test.

to the strip ahead of the crack, apart from the localized stress at the crack tip. Thus, assuming no wave reflection effects, K_I^* should equal K_Q throughout crack propagation and the crack ought to maintain a constant speed during the propagation process. This is indeed observed experimentally, but the experimentally observed crack velocities [12] are substantially lower than predicted by the relation (3.1). It must be concluded, therefore, that wave reflections from surfaces parallel to the crack are responsible for this decrease of the crack speed.

Finally, although this paper has clearly shows that the static fracture mechanics K_{Ia} approach underestimates the crack arrest length for the particular displacement-controlled situation examined here, it should certainly not be seen as an implication that such an underestimation always occurs. Each situation must be considered on its merits and in this context as indicated in the Introduction: when an edge crack propagates in a semi-infinite solid as a result of loadings which generate a tensile stress field ahead of the initial crack, the static approach is conservative in that it overestimates the crack length at arrest [8].

4. Conclusions

This paper clearly shows that the simple static fracture mechanics (K_{Ia}) approach can underestimate the arrest crack length, even though wave reflections play no role in the propagation event.

The demonstration is for a specific model in which a fixed displacement is applied to the crack's surface, and it must be emphasized that the question of conservatism, or otherwise, of the K_{Ia} approach should be considered for each particular problem.

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