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Stability of a gas-liquid interface adjacent to a supersonic stream

B. K. SHIVAMOGGI (PRINCETON)

THE FOLLOWING analysis makes an investigation of the inertial effects of the gas motion upon the linear stability characteristics of the wave motion at the interface between a supersonic gas stream and a liquid. The analysis considers a body force directed towards the liquid, and the effects of the surface tension of the liquid. The inertial effects of the gas motion are found to worsen the instanbility of this wave motion. Interestingly enough, the inertial effects of the gas motion disappear in the linear model when the gas stream Mach number equals two .

STUDIES of the stability of the wave motion at the interface between a liquid layer and a gas flowing past it are of interest in the problems of transpiration cooling of the reentry vehicles, particularly in determining the amount of liquid entrained by the gas. CHANG and RUSSELL [1] made a study of the linear stability characteristics of the wave motion at the interface between a liquid layer and a gas stream adjacent to it and found that the nature of waves generated at the interface depends markedly upon the state of flow of the gas. For supersonic gas flow, the gas pressure at the interface is out of phase with the surface tension so that a purely oscillatory constant-amplitude motion of the interface is not possible. For a subsonic gas flow, however, the stabilising effect of the surface tension gives rise to cut-off frequencies.

CHANG and RUSSELL [1] ignored the inertial effects of the gas motion and, probably on this account, their results were in disagreement with the experimental findings of GATER and L'ECUYER [2], where the interfacial wave motion was found to be unstable even for a subsonic gas flow. In any event, the inertial effects of the gas motion become important for waves with speeds of propagation comparable with the gas speed.

The following analysis makes an investigation of the inertial effects of the gas motion upon the linear stability characteristics of the wave motion at the interface between a supersonic gas stream and a liquid. The analysis considers a body force directed towards

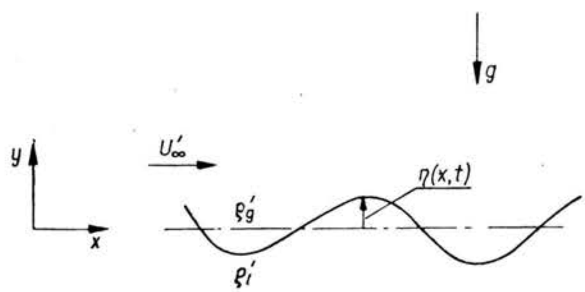

FIG. 1.

the liquid, and the effects of the surface tension of the liquid. The liquid is assumed to be initially quiescent and of infinite depth whose mean level of contact with a gas flowing past it is the horizontal surface $y = 0$, (see Fig. 1). Both the liquid and the gas are assumed to be inviscid and the effects of the viscous boundary layer at the interface are ignored. If the motion of the whole system is supposed to start from rest, it may be assumed to be irrotational. If a typical interfacial disturbance is characterised by a sinusoidal travel ling wave with an amplitude a' and wavelength λ', then all the quantities in the following are nondimensionalised with respect to a reference length $\lambda'/2\pi$, a time $(\lambda'/2\pi g')^{1/2}$, and the inertial effects of the gas motion are characterised by the ratio of the wave speed to the gas speed. The gas density ϱ_q' is small so that the corresponding body force is negligible. The potential function of the motions of the liquid and the gas are taken to be, respectively,

$$(g')^{1/2}\left(\frac{\lambda'}{2\pi}\right)^{3/2}\varphi(x,\eta,t),$$

and

$$U'_\infty[x+\phi(x,\eta,t)]\left(\frac{\lambda'}{2\pi}\right).$$

The latter are governed by the following equations:

$$y<\eta:\quad \varphi_{zz}+\varphi_{yy}=0, \tag{1}$$

$$y>\eta:\quad \phi_{yy}-(M_\infty^2-1)\phi_{xx}-M_\infty^2(2\delta\phi_{xt}+\delta^2\phi_{tt})=0, \tag{2}$$

where

$$\delta=\left(\frac{\lambda' g'}{2\pi}\right)^{1/2}\frac{1}{U'_\infty}\sim\frac{\text{wave speed}}{\text{gas speed}}\ll 1.$$

Here, $y=\eta(x, t)$ denotes the disturbed shape of the interface, and U'_∞, M_∞ are the ambient gas velocity and the Mach number. One has the following boundary conditions at the interface:

1) kinematic condition

$$y=0:\quad \varphi_\eta=\eta_t, \tag{3}$$

$$y=0:\quad \phi_\eta=\delta\eta_t+\eta_x, \tag{4}$$

2) dynamic condition

$$y=0:\quad \varphi_t+\eta=k^2\eta_{xx}-\frac{\sigma}{2}kC_p, \tag{5}$$

where

$$k^2=\left(\frac{2\pi}{\lambda'}\right)^2\frac{T'}{p_l' g'},\qquad \sigma=\frac{\varrho_g' U_\infty'^2}{\sqrt{\varrho_l' g' T'^2}},$$

$$C_p=-2\phi_x-2\delta\phi_t$$

and T' denotes the surface tension.

The infinity conditions are

$$y\Rightarrow-\infty:\quad \varphi_y\Rightarrow 0. \tag{6}$$

Since we are looking for travelling waves, introduce

$$\xi = x - ct \tag{7}$$

so that Eqs. (1)–(5) become

$$y < 0: \quad \varphi_{\xi\xi} + \varphi_{yy} = 0, \tag{8}$$

$$y > 0: \quad \phi_{yy} - \Delta^2 \phi_{\xi\xi} = 0, \tag{9}$$

$$y = 0: \quad \varphi_y = -c\eta_\xi, \tag{10}$$

$$y = 0: \quad \phi_y = (1 - \delta c)\eta_\xi, \tag{11}$$

$$y = 0: \quad \eta = c\varphi_\xi + k\sigma(1 - \delta c)\phi_\xi^! + k^2 \eta_{\xi\xi}, \tag{12}$$

$$y \Rightarrow -\infty: \quad \varphi_y \Rightarrow 0, \tag{13}$$

where

$$\Delta^2 = M_\infty^2 - 1 - 2\delta M_\infty^2 c.$$

Let

$$\eta(\xi) = Ae^{i\xi}, \tag{13}$$

then from Eqs. (8)–(11) and (6) one obtains

$$\phi(\xi, y) = -\frac{iA}{\Delta}(1 - \delta c)e^{i(\xi - \Delta y)}, \tag{14}$$

$$\varphi(\xi, y) = -iAce^{y + i\xi}. \tag{15}$$

Using Eqs. (13)–(15) in Eq. (12), there follows

$$1 = c^2 + \frac{ik\sigma(1 - \delta c)^2}{\Delta} - k^2, \tag{16}$$

from which

$$c = \frac{ik\sigma\delta}{2m^3}(M_\infty^2 - 2) \pm \Sigma, \tag{17}$$

where

$$\Sigma = \sqrt{k^2 - \frac{ik\sigma}{m} + 1}, \quad m = \sqrt{M_\infty^2 - 1}.$$

It is seen that the physical condition is always one of instability, regardless of the presence of surface tension. The inertial effects of the gas motion worsen this instability. However, interestingly enough, note that the inertial effects of the gas motion disappear in the linear model at $M_\infty = 2$. A similar intriguing result occurred in the analysis of the flow past a sinusoidal wall with an aligned magnetic field (SEARS and RESLER [3]); the pressure disturbance induced by the wall in the oncoming flow vanished when

$$A^2 = U^2 \bigg/ \frac{H_0^2}{\varrho_\infty} = 2.$$

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Reference

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DEPARTMENT OF AEROSPACE ENGINEERING
PRINCETON UNIVERSITY, NEW JERSEY, USA.

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