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Effect of acoustic wave incident upon a moving plane in three-dimensional subsonic flow

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An acoustic wave falls upon a plane that moves with subsonic speed in an ideal compressible medium. The problem of gas dynamics is presented as an initial-boundary value problem with a mobile boundary for a three-dimensional wave equation. The solution of the problem is obtained in a closed form, in quadratures, in the general case of an arbitrary orientation of the front of the incident wave with respect to a moving plane. The velocity potential is presented in the form of a double integral that contains an arbitrarily given function which in turn determines the profile of the incident wave. In particular, the obtained solution contains the solution of the problem when an acoustic wave falls upon an immobile plane.

Introduction. Statement of the problem

An acoustic wave falls upon a plane that moves with constant subsonic speed u in an ideal compressible medium. The wave front represents a plane which moves in an immobile medium with the velocity of sound c. The velocity vector $\bar{c}$ forms an angle α with the moving plane, $-\pi/2 < \alpha \leqslant \pi/2$. On the moving plane the trail of the wave front forms an angle $(\pi/2-\beta)$ with the vector $\bar{u}$, $0 \leqslant \beta \leqslant \pi$.

Three-dimensional eddy-free flow of a gas behind the front of an incident wave is considered in a system of coordinate axes $Oxyz$, which moves with the velocity u. The Ox axis is directed along the velocity vector $\bar{u}$. The plane (xy) is combined with the plane on which the wave falls (Fig. 1).

The velocity potential Φ_1 of the disturbed flow of a gas satisfies the three-dimensional wave equation and the condition of flow in the plane (xy): $\Phi_{1z} = 0$.

We shall present the potential Φ_1 in the form $\Phi_1 = \Phi+\varphi$. The function Φ satisfies the wave equation. The function Φ is an arbitrarily given velocity potential in the incident wave in the moving coordinate system. In the general case the potential Φ depends upon four arguments, i.e. three coordinates and time. The parameters of a gas in an incident wave are given through the function $\Phi(x, y, z, t)$. The unknown function φ represents a velocity potential of acoustic field excited by the reflected wave [1].

1. The initial-boundary value problem

The function φ satisfies the three-dimensional wave equation

$$(c^2-u^2)\varphi_{xx}+c^2\varphi_{yy}+c^2\varphi_{zz}-2u\varphi_{xt}-\varphi_{tt} = 0 \tag{1.1}$$

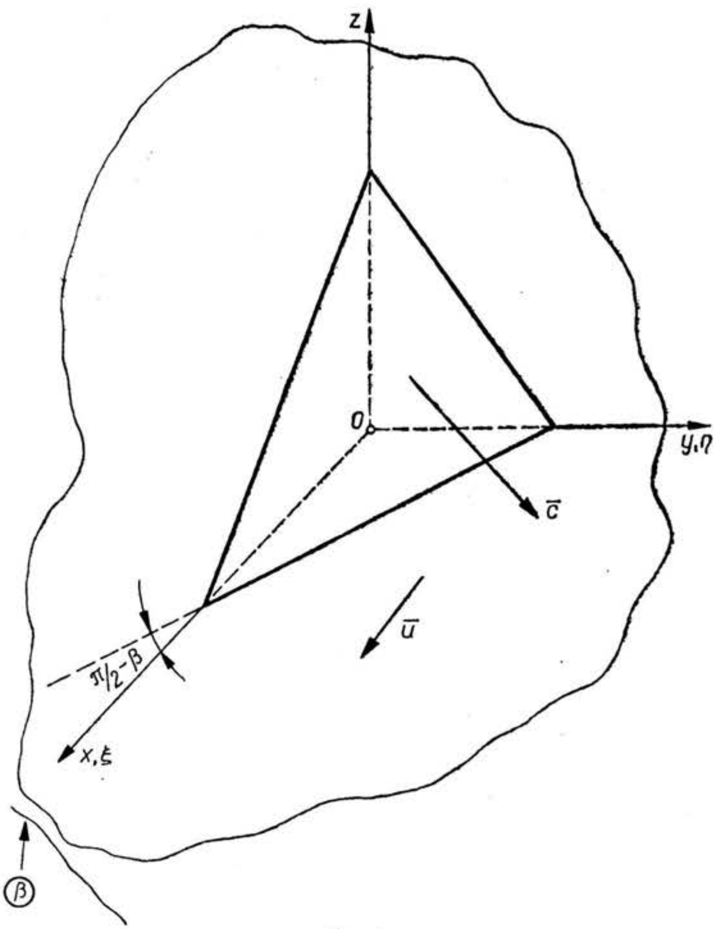

FIG. 1.

and the following boundary conditions at the plane (xy):

(1.2) $$\varphi_z = 0$$

before the front of the incident wave,

(1.3) $$\varphi_z = -\Phi_z(x, y, 0, t) = A(x, y, t)$$

behind the wave front.

2. The solution of the problem

Starting from the elementary solutions φ^* of Eq. (1.1)

$$\varphi^* = f(\xi, \eta, \tau)r^{-1}, \quad \tau = t-u(c^2-u^2)^{-1}(x-\xi)-c(c^2-u^2)^{-1}r,$$

$$r = \left[(x-\xi)^2+\left(1-\frac{u^2}{c^2}\right)(y-\eta)^2+\left(1-\frac{u^2}{c^2}\right)z^2\right]^{\frac{1}{2}}$$

we shall take the solution of Eq. (1.1) in the form of a double integral [2].

$$\varphi(x, y, z, t) = \iint \frac{f(\xi, \eta, \tau)}{r} d\xi d\eta. \tag{2.1}$$

It may be mentioned that the variables ξ, η, τ are related to each other by the relationship

$$(x-\xi)^2+(y-\eta)^2+z^2+2u(x-\xi)(t-\tau)-(c^2-u^2)(t-\tau)^2 = 0. \tag{2.2}$$

Let us now consider the space defined by the variables x, y, t [2, 3]. We shall consider as the initial moment of time, $t = 0$, that moment of time when any point of the trail of the incident wave on the plane (xy) coincides with the origin of the coordinates 0. The plane W, which is defined by the equation

$$(\cos\alpha\cos\beta)\xi+(\cos\alpha\sin\beta)\eta+(u\cos\alpha\cos\beta-c)\tau' = 0, \tag{2.3}$$

divides the space $(x\ y\ t)$ into two parts, i.e. V_0 and V with different values of the derivative φ_z (Fig. 2). According to the condition (1.2) in the semi-space V_0 which conforms to small values of time, the derivative $\varphi_z = 0$. And according to the condition (1.3) in the semi-space V that conforms to large values of time, the derivative $\varphi_z = A(x\ y\ t)$.

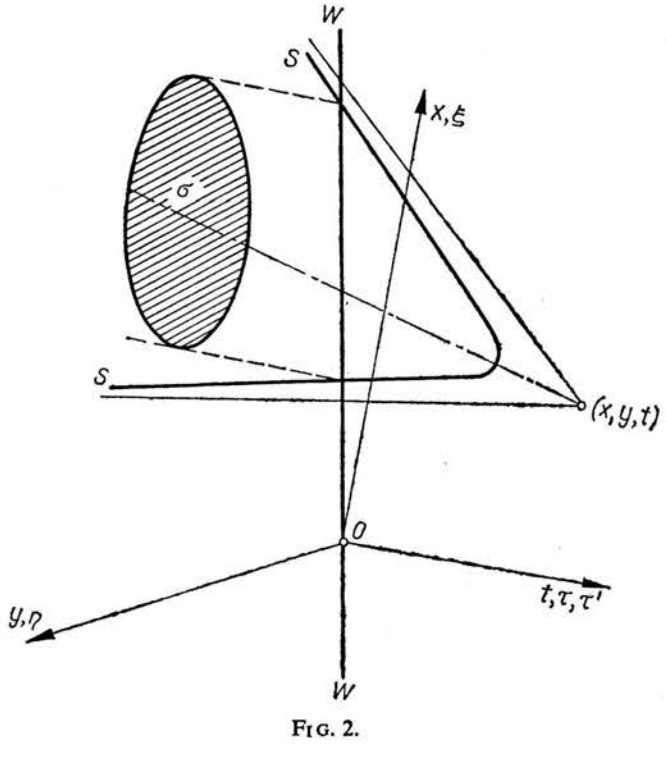

FIG. 2.

In the solution (2.1) we shall specify the integration range. With this aim we shall change over from the double integral (2.1) to the surface integral spread over the surface of the hyperboloid S which is defined, respecively, by the equation

(2.4) $$(x-\xi)^2+(y-\eta)^2+z^2+2u(x-\xi)(t-\tau')-(c^2-u^2)(t-\tau')^2 = 0$$

and the inequality

(2.5) $$\tau' < t,$$

where ξ, η, τ' — current coordinates (Fig. 2).

The element dS of the surface S is bounded to the element of the plane (xy) by $dS = [EG-F^2]^{\frac{1}{2}} d\xi\, d\eta$, where the values E, G, F are the coefficient at the differential elements in the first main quadratic form.

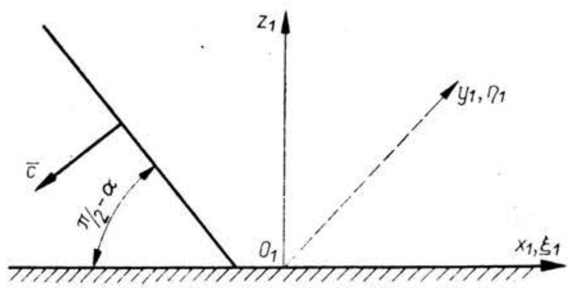

FIG. 3.

Considering the variables ξ, η and τ in Eq. (2.2) as the function ξ and η we get the coefficients E, G, F in the form

$$E = (c^2-u^2)^{-2}r^{-2}[(c^2-u^2)^2r^2+u^2c^2r^2+c^4(x-\xi)^2+2uc^3r(x-\xi)],$$

$$G = 1+r^{-2}(y-\eta)^2,$$

$$F = (c^2-u^2)^{-2}r^{-2}[u^2c^2r(y-\eta)+c^4(x-\xi)(y-\eta)].$$

Using the boundary conditions (1.2) and (1.3) of the problem and the known relationship $2\pi f(x, y, t) = -\varphi_z(x, y, 0, t)$, we shall present the solution (2.1) in the form of the surface integral

(2.6) $$\varphi(x, y, z, t) = -\frac{1}{2\pi}\iint_{\Sigma} \frac{A(\xi, \eta, \tau)}{r}\frac{dS}{\sqrt{EG-F^2}},$$

where the integration range Σ is a part of the surface S intercepted by the plane W, $(\Sigma \subset V)$.

Now we shall reverse the process, i.e. we shall change over from the surface integral (2.6) to a double integral with the integration range located in the plane $(x\, y)$. The solution of the problem will be obtained in the form of a double integral:

(2.7) $$\varphi(x, y, z, t) = -\frac{1}{2\pi}\iint_{\sigma} \frac{A(\xi, \eta, \tau)}{r}\, d\xi d\eta,$$

where the integration range σ is bounded by the ellipse l given by Eqs. (2.3) and (2.4). Equations (2.3) and (2.4) are the equations of the ellipse which is given in the parametric form. In these equations the value τ' is considered as a parameter. The ellipse l represents the projection of the line of intersection of the surface S with the plane W on the plane $(x\, y)$.

3. The case when an acoustic field is instantaneously superimposed upon the plane

In particular, assuming that the parameter α in the formula (2.7) is equal to $\pi/2$, we get the solution of the problem when an acoustic field, which is given by the potential $\Phi(x, y, z, t)$, is instantaneously superimposed upon the plane or an acoustic wave falls vertically upon the plane (for example, acoustic loads on the wing in subsonic flow). For $\alpha = \pi/2$ the integration range σ is the circle with its center located at a point which is defined by the coordinates $x_c = x+ut$, $y_c = y$ and with the radius equal to $\sqrt{c^2t^2-z^2}$.

4. The case when an acoustic wave falls upon an immobile plane

In particular, assuming that the parameter u in the formula (2.7) is equal to 0, we get the solution of the problem if the plane is immobile (for example, sound stroke of supersonic aeroplane and its effect on ground).

The solution of the problem will be obtained in the form

(4.1)
$$\varphi(x_1, y_1, z_1, t) = -\frac{1}{2\pi}\iint\limits_{\sigma}\frac{A^*(\xi_1, \eta_1, \tau_0)}{r_0}\,d\xi_1 d\eta_1,$$
$$\{\tau_0 = t-c^{-1}r_0,\ r_0 = [(x_1-\xi_1)^2+(y_1-\eta_1)^2+z_1^2]^{\frac{1}{2}}\},$$

where the integration range σ is the ellipse given by the following equation:

$$(\sin^2\alpha)\xi_1^2+\eta_1^2-2(x_1+ct\cos\alpha)\xi_1-2y_1\eta_1+x_1^2+y_1^2+z_1^2-c^2t^2 = 0.$$

The solution (4.1) is given in the fixed axes of coordinates $O_1 x_1 y_1 z_1$ when the plane $(x_1 y_1)$ is combined with the plane upon which the acoustic wave falls; direct the axis $O_1 x_1$ opposite to the direction of motion of the wave trail on the plane. We consider as the initial moment of time, $t = 0$, that moment of time when the trail of the incident wave on the plane $(x_1 y_1)$ coincides with the axis $O_1 y_1$.

In the formula (4.1) the function $A^* = -\Phi^*_{z_1}(x_1, y_1, 0, t)$. The function $\Phi^*(x_1, y_1, z_1, t)$ is an arbitrarily given velocity potential in the incident wave in the fixed axes of the coordinates $O_1 x_1 y_1 z_1$. Using the Lagrange's integral for unsteady eddy-less flows of a gas and the solution (4.1), we shall find the pressure of an acoustic wave on the plane $(x_1 y_1)$ in the form of the formula when $z_1 = 0$

(4.2)
$$p(x_1, y_1, 0, t) = -\varrho\left\{\frac{\partial\Phi^*}{\partial t}+\frac{\partial\varphi}{\partial t}\right\},$$

$$\frac{\partial\varphi}{\partial t} = -\frac{1}{2\pi}\int\limits_{\xi_1'}^{\xi_2}\int\limits_{\eta_1'}^{\eta_2}\frac{\partial}{\partial\tau_0}\{A^*(\xi, \eta, \tau_0)\}\frac{d\eta d\xi}{\sqrt{(x_1-\xi)^2+(y_1-\eta)^2}}$$
$$-\frac{c}{2\pi}\int\limits_{\xi_1'}^{\xi_2}\frac{A^*(\xi, \eta_1', -c^{-1}\xi\cos\alpha)+A^*(\xi, \eta_2, -c^{-1}\xi\cos\alpha)}{\sqrt{(\xi\cos\alpha+ct)^2-(x_1-\xi)^2}}\,d\xi,$$

(4.2) [cont.]

$$\xi_1' = \frac{1-\cos\alpha}{\sin^2\alpha}(x_1-ct), \qquad \xi_2 = \frac{1+\cos\alpha}{\sin^2\alpha}(x_1+ct),$$

$$\eta_1' = y_1 - [(\xi\cos\alpha+ct)^2-(x_1-\xi)^2]^{\frac{1}{2}},$$

$$\eta_2 = y_1 + [(\xi\cos\alpha+ct)^2-(x_1-\xi)^2]^{\frac{1}{2}},$$

where ϱ density of the unperturbed gas.

Assuming that the angle α in the formula (4.2) is equal to $\pi/2$ we get the pressure of an acoustic field on the immobile plane $(x_1\, y_1)$ then the acoustic field is instantaneously superimposed upon the plane, in particular, when the wave falls vertically upon the plane.

Considering the shock wave of a supersonic aeroplane as an acoustic wave, we can compute from the formula (4.2) the pressure on a ground and, therefore, we can obtain the intensity of sound stroke.

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