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Variation of elastic constants of metal during plastic deformation

O. A. SHISHMAREV (KALININGRAD) and A. G. SHCHERBO (NOVOPOLOTSK)

WE CONSIDER the existing solutions to the problem of dependence of the metal elasticity modulus on plastic deformation to be inadequate since the actual behaviour of elasticity modulus has not been investigated thus far. The paper reports the experimental research on the actual steel elasticity modulus varying due to plastic deformation, all the measurements being as accurate as possible. Methods of elasticity modulus determination were developed. Both Young's modulus and the shear modulus were found to reduce by up to 10%. When plastic deformation of an inverse sign is applied to the material subjected previously to plastic strain, partial restoration of the elasticity modulus values takes place.

Znane rozwiązania dotyczące problemu zależności modułów sprężystości metali od odkształceń plastycznych należy uznać za niewystarczające. W pracy przedstawiono wyniki oryginalnych pomiarów doświadczalnych przeprowadzonych z dużą dokładnością. Opracowano metodę wyznaczania modułów sprężystości. Stwierdzono, że zarówno moduł Younga jak i moduł ścinania ulegają w procesie odkształcenia plastycznego redukcji o wartości do 10%. Poddanie metalu odkształceniom plastycznym o odwrotnym znaku powoduje częściowe przywrócenie początkowych wartości tych modułów.

Известные решения, касающиеся проблемы зависимости модулей упругости металлов от пластических деформаций, следует считать недостаточными. В работе представлены результаты оригинальных экспериментальных исследований, проведенных с большой точностью. Разработан метод определения модулей упругости. Констатировано, что значения как модуля Юнга, так и модуля сдвига в процессе пластического деформирования, редуцируют на величину до 10%. Если подвергнуть металл пластическим деформациям обратного знака, то это вызывает частичный возврат начальных значений этих модулей.

1. Introduction

ABOUT 30 years ago A. M. ZHUKOW published the papers [1, 2] where from them followed that metal elasticity moduli, such as Young's modulus and the shear modulus, which had been considered up to then as independent of plastic deformation, did depend on it to a great extent. But as it can be seen, the interpretation given by A. M. Zhukow was erroneous. Hence his conclusions seem to be doubtful. It is only the quantitative side of the problem that counts (according to the papers [1, 2] the moduli decrease by more than 20%); more important is the determination of the objective qualitative image of the elasticity moduli behaviour at plastic deformation.

So as to avoid different interpretation of terms, it is necessary to give a clear definition of the elasticity modulus. By the elasticity modulus at simple stretching or shearing we understand the proportionality ratio betwen stress and deformation at the elastic deformation of a material. By the elastic deformation of material we mean such a deformation when σ_i— ϵ_i lines (σ_i — stress intensity; ϵ_i — deformation intensity) at unloading and loading coincide (with the accuracy up to the elastic hysteresis loop breadth due to inner friction, which can be neglected at static loading). At elastic deformation stress must be proportional to strain, regardless of how this stressed condition was attained, i.e., by loading or unloading. Besides, the elastic modulus value should not change due to repeated loadings and unloadings.

Proceeding from this definition of the elasticity modulus, it follows that in Zhukow's paper the actual elasticity modulus after elastic deformation was not calculated. In fact the techniques employed by Zhukow amounted to the following procedure: the samples that had been preliminarily stretched or twisted were unloaded and the elasticity modulus was determined in accordance with the $\sigma_i - \epsilon_i$ line. As unloading does not obey the straight line law as it was determined in papers [3, 4], Zhukow approximated the unloading curve by a broken line which he used later to determine the moduli. But the line of repeated loading did not coincide with the unloading curve and formed the hysteresis loop which was broad enough; therefore, according the plasticity postulate, plastic deformations should appear at closed unloading-loading path (1). So, in Zhukow's experiments the plastic deformations were erroneously treated as elastic ones, and the "moduli" that were calculated by taking into consideration those types of deformation cannot be regarded as the elasticity moduli.

2. Experimental analysis

This paper investigates Young's and shear moduli (E and G, respectively) which depend on plastic deformations which appear at simple stretching and shearing. The tests were carried out using special equipment analogous to the one described in [5], where loading was applied by means of suitable weights. The deformations were measured with the aid of the extensometers which made it possible to register absolute longitudinal deformations of 1μ amounting to relative deformation of 0.005%. The minimum relative shear deformations were 0.003%.

The samples in the form of tubular specimens made of steel 1X18H10T with the outside diameter of 12 mm and wall thickness of 0.4 mm, and tubular specimens made of steel 10 with the outside diameter and wall thickness 0.6 mm, were preliminarily subjected to recrystallization tempering.

Tests were carried out on all 15 samples. Most of them were preliminarily twisted plastically, since in this case the cross-sections remain stable during plastic deformation. This prevents additional errors in elasticity moduli calculation.

The graphs are drawn in $\sigma_i - \epsilon_i$ coordinates. For stretching, these coordinates were $S_1 = \sigma$ and $\epsilon_1 = \varepsilon$, respectively; for twisting shearing $S_2 = \tau\sqrt{3}$ and $\epsilon_2 = \gamma\sqrt{3}$ (σ and τ actual stresses; ε and γ — conventional deformations).

To fulfill the task set in this paper, it was necessary to apply methods that would allow to define the elasticity moduli with the least possible errors).

Figure 1 presents a typical deformation curve at unloading and repeated loading for the steel sample 1X18H10T, preliminarily subjected to 1.6% of plastic shearing deforma-

(1) Since A. M. Zhukow conducted his experiment in static loading, the elastic hysteresis loop could not appear.

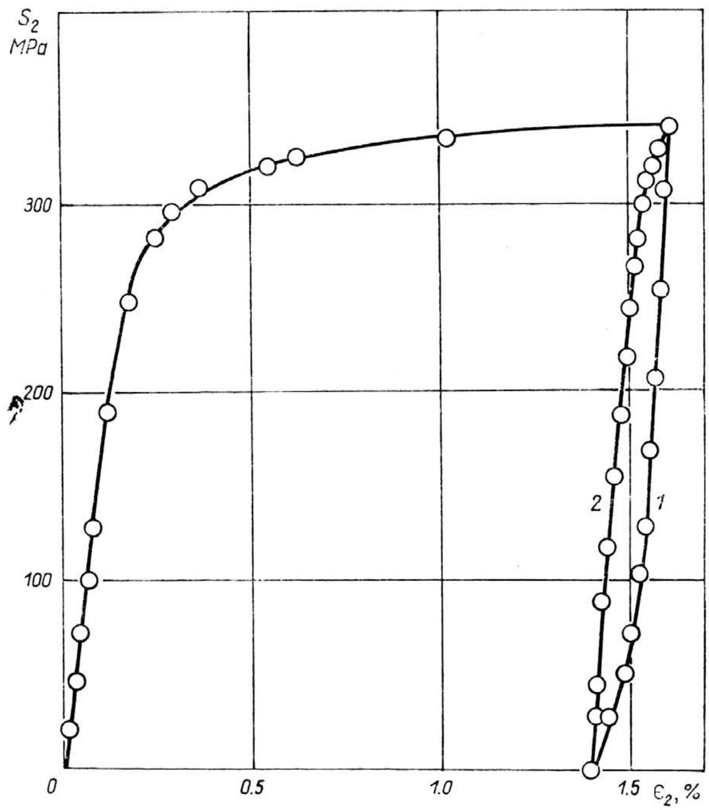

FIG. 1. $\sigma_i - \epsilon_i$ curve at initial unloading (*1*) and repeated loading (*2*).

tion. If all the deformation due to unloading or repeated loading in this case these happened to be equal is to be regarded as the base for calculating the value of the *G* modulus, then it will be 45% lower than the initial modulus value, but, already after the first repeated unloading, the explicit deformation value turned out to be 25–30% lower than that at initial unloading. This circumstance provided the basis for using such an elasticity modulus value calculation technique, according to which the samples underwent several cycles of unloading–loading in the course of which the plastic deformations disappeared "fizzled away" in this range of stress change, and unloading–loading curves practically merged in one line. But, sometimes a rather narrow loop due to inner friction still remained. In such cases the loop breadth was to be subtracted from the whole unloading or loading deformation ([2]).

Figure 2 presents the tests results of a steel sample 1X18H10T that underwent plastic deformation with the intensity of 1% by means of twisting. Light dots correspond to the unloading curve, and shaded dots — to the first loading curve. The initial shearing modulus

([2]) This method may also be used for defining the elasticity modulus in materials which cyclically lose their strength. In this case it is necessary that the maximum stress corresponding to that cycle of unloading-loading should be much lower than the stress from which the first unloading was initiated.

was $8.07 \cdot 10^4$ MPa. The modulus calculated by the whole deformation on the first unloading was $5.41 \cdot 10^4$ MPa, i.e., it decreased by 32%. The modulus calculated at the first loading (whithout taking into account that loading by means of which the preliminary plastic deformation was produced) within the stress range from 60 to 140 MPa was $6.875 \cdot 10^4$ MPa. This means a decrease by 15% as compared to the initial one. Then, within the intensity range from 60 to 140 MPa, 4 cycles of loading–unloading were carried out. Figure 2 presents the fourth cycle (*AB* line). We see that there is no hysteresis loop, the loading curve (semi-black dots) and the unloading curve (black dots) merge into one straight line and, therefore, the loading and unloading deformations can be considered as elastic. The elasticity modulus has decreased by 5% as compared to the initial one and is equal to $7.65 \cdot 10^4$ MPa.

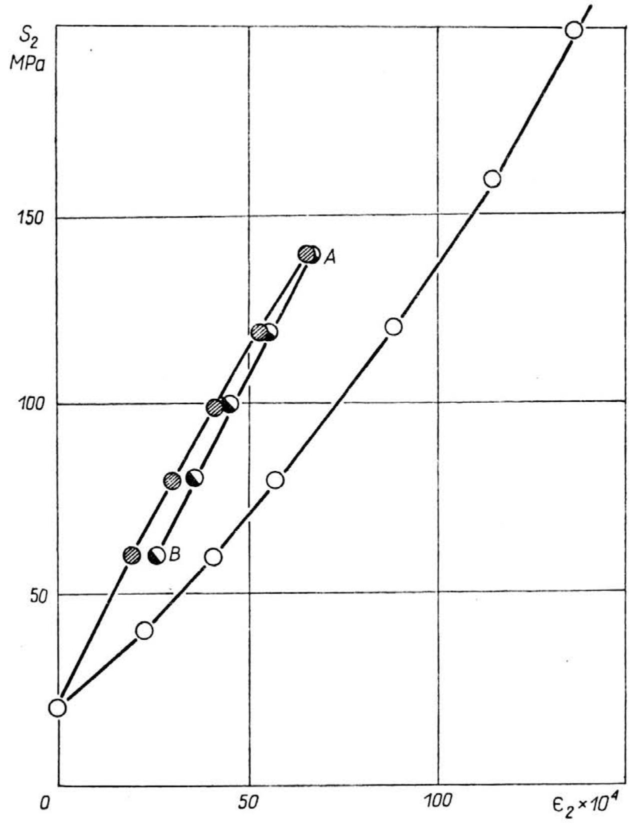

FIG. 2. Results of the experiment on the elasticity modulus definition within tension variation range of 80 MPa. The *AB* line corresponds to the fourth cycle: black dots—unloading, semi-black—loading.

To investigate the elasticity modulus variation depending on the deformation diagram height at unloading and repeated loading (the papers [1, 2] claim that elastic unloading does not obey the linear law and, therefore, the elasticity modulus changes at the process of unloading) an experiment was made, the results of which are presented in Fig. 3. The sample ($G_0 = 8.37 \cdot 10^4$ MPa) in which the plastic deformation of the intensity of 1%

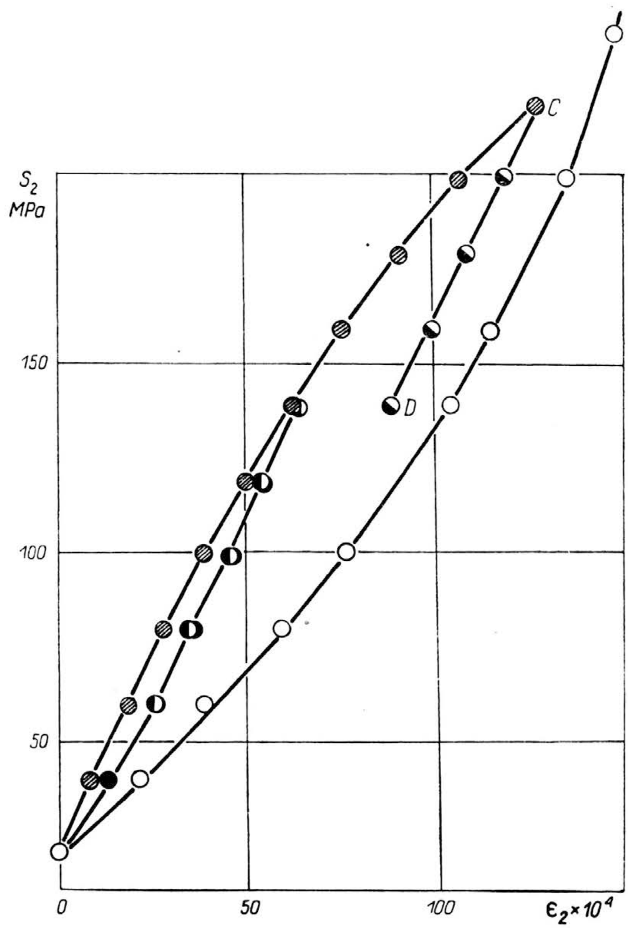

FIG. 3. Graphs illustrating invariation of the elasticity modulus value along the $\sigma_i - \varepsilon_i$ line. *AB* and *CD* — loading–unloading line corresponding to the fourth cycle.

was achieved by twisting was almost completely unloaded: from $S_2 = 240$ MPa to $S_2 = 20$ MPa (light dots correspond to the unloading line) and then loaded again to $S_2 = 140$ MPa (p.A.). The approximate modulus value G decreased by 30% as compared to the initial modulus G_0 calculated from the unloading deformation, and by 11% if the modulus was obtained from the deformation corresponding to the second loading (within the tension range from 20 to 140 MPa). Then, starting from point A, several cycles of unloading–loading were carried out within the stress variation range of 80 MPa (from $S_2 = 60$ MPa to $S_2 = 140$ MPa). In Fig. 3 (the 4-th cycle is presented) black dots stand for unloading, and semi-black — for loading. It can be seen that the unloading-loading lines (AB) almost merged: the hysteresis loop breadth was only 0.001%. The modulus G decreased below the initial value by 4.5% and become equal to $7.98 \cdot 10^4$ MPa.

Then the sample was unloaded to $S_2 = 20$ MPa and afterwards, loaded again to $S = 220$ MPa (shaded dots correspond to the loading line). Then 4 cycles were carried out from point C by means of unloading–loading within the stress variation range of $S =$

= 80 MPa. The unloading–loading line corresponding to the fourth cycle is denoted in the figure by *CD*. It can be seen that the dots practically overlap. The elasticity modulus $G = 7.97 \cdot 10^4$ MPa turned out to be equal (with the accuracy of 0.01) to the elasticity modulus which was found within the stress variation range 60–140 MPa.

Thus, no change of the modulus G depending on the height of the unloading diagram was delivered. This phenomenon was described in the papers [3, 4, 6].

Hence, in order to clarify whether the elasticity modulus value, defined in this way, would not depend on stress variation amplitude within which it has been calculated, a special experiment was made. For the sample subject to initial plastic shear deformation of the intensity of 2.4%, the shear modulus was defined at stress changes amplitude of 80 MPa. It turned out to be equal to $7.6 \cdot 10^4$ MPa. Then the same sample was subjected to four cycles of unloading–loading within the stress variation amplitude of 160 MPa, i.e., 2 times as much as in the previous test. The results of this experiment are shown in Fig. 4.

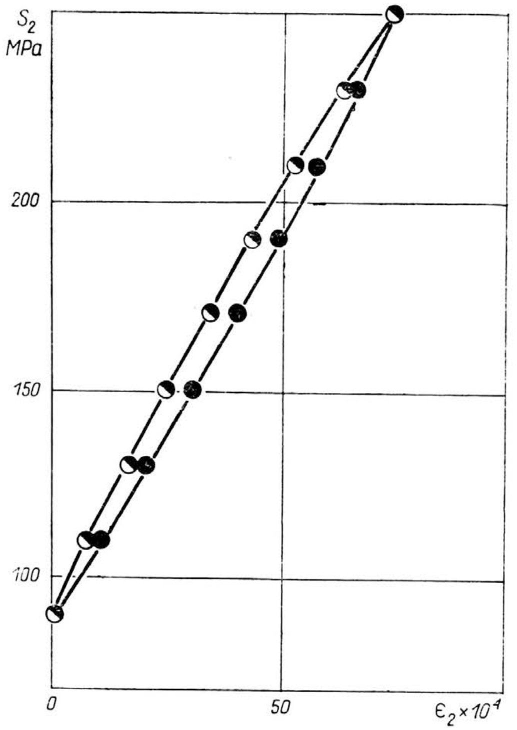

FIG. 4. Results of the experiment on the elasticity modulus definition within tension variation range of 160 MPa. The unloading line does not coincide with the loading line. Loop broadth—0.0051%.

The fourth cycle of unloading–loading is shown. The black dots stand for unloading, the semi-black dots for loading, the hysteresis loop breadth being 0.0058%. In order to evaluate the elasticity modulus from deformations which correspond to loading and unloading (they were equal), the loop breadth was subtracted. The elasticity modulus turned out to be equal to $7.69 \cdot 10^4$ MPa, i.e., practically equal to that obtained from the unloading cycle with stress amplitude 2 times less (the difference is 1%).

Testing of all other samples gave similar results: elasticity moduli values (both G and E) decreased at plastic deformation. The results of these experiments for ten samples are presented in Table 1. The test data of the other five samples are not included in the table, since the elasticity moduli behaviour of these samples was discussed in the text and partially given in Table 2.

Table 1.

Sample numbers		1	2	3	4	5	6	7	8	9	10
Material		st. 10	st. 10	st. 10	steel 1X18H10T						
Plastic deformation value being imparted ϵ_i, %	Stretching	3	1.5	9.8	—	—	—	—	—	—	—
	Shear	1.4	3	—	1.6	1.4	1.2	1.5	1.1	0.9	1.5
Modulus value decrease, %	ΔE	10	6	5	4	6.5	—	—	—	—	—
	ΔG	—	7.7	—	8	4.5	6	7.5	5	6.7	4.7

Table 2.

Sample numbers	1-st deformation increment $\Delta\epsilon_2'$, %	modulus decrease value ΔG, %	2-nd deformation increment $\Delta \epsilon_2''$, %	modulus decrease increment ΔG, %	3-d deformation increment $\Delta \epsilon_2'''$, %	modulus decrease value ΔG, %	whole deformation %
11	0.97	4.5	1.43	8.7	—	—	2.4
12	1.0	4.0	1.28	10.0	—	—	2.28
13	0.97	3.0	0.7	4.0	0.81	10	2.5

The data given in Table 1 do not allow for any conclusions concerning the dependence of the decrease of elasticity moduli on the value of plastic deformation imparted to the samples. But the tests performed on three samples gave the convincing evidence that if plastic deformation is imparted in consecutive stages with time intervals between these stages of 5–6 hours, then every new "portion" of plastic deformation is followed by a decrease of the elasticity modulus. Using an example to illustrate this result, let us present one of these 3 tests.

The sample was initially plastically twisted to a deformation of intensity of 1%. The elasticity modulus G was then decreased by 4.5% as compared to the initial one. Then, after keeping it under loading for 5 hours, the sample was given the additional plastic shear deformation of 1.43% after which the modulus decreased by another 4.2%. The whole shear modulus reduction amounted to about 9%.

The results of two other tests were quite analogous what can be seen from Table 2.

Several tests were carried out to investigate the behaviour of the G value which decreases following initial plastic deformation, i.e., plastic deformation of the opposite sign. Figure 5

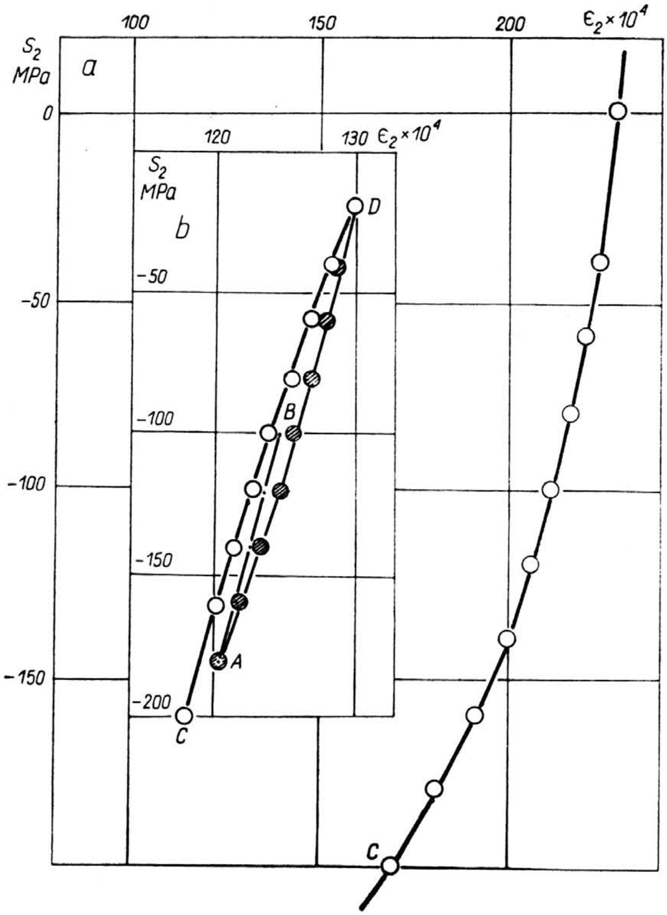

FIG. 5. Results of the experiment on how the elasticity modulus behaves when it is reduced by initial plastic deformation, secondary plastic deformation of the opposite sign. a — $S_2 - \epsilon_2$ curve at initial loading after reversal, b — unloading curves (light dots) and secondary loading (shaded dots) after reversal. The AB line corresponds to the fourth unloading–loading cycle.

presents the results of tests made on the sample ($G_0 = 8.37 \cdot 10^4$ MPa) which was subject to initial plastic shear deformation of intensity of 2.4% (after which the shear modulus became equal to $7.6 \cdot 10^4$ MPa i.e. decreased by 9%), and then, after the period of time sufficient to stop the creep, twisted in the opposite direction by a value equal to 1.2% of the plastic deformation. It can be seen from the figure that, contrary to the results described in [1, 2], there is no straight portion on the deformation diagram corresponding to twisting the sample in the opposite direction (Fig. 5a). It was also not found on the diagrams of three other samples tested. Therefore, speaking about the definition of initial shear modulus does not make sense at this point. Then the sample was unloaded from a stressed state corresponding to point C (Fig. 5b) to $S_2 = 20$ MPa (light dots on Fig. 5b) and loaded again to point A (shaded dots), corresponding to $S_2 = -180$ MPa. Then four cycles of unloading–loading were carried out from point A, the tension variation being 80 MPa. The shear modulus was defined by the fourth cycle (straight line AB at Fig. 5b) where unloading and loading signs coincided, and the modulus turned out to be $7.9 \times \times 10^4$ MPa. It increased by 4% a compared to the modulus evaluated when the sample was subjected to plastic deformation in the initial direction.

Another sample with the initial shear modulus of $G_0 = 8.15 \cdot 10^4$ MPa underwent the plastic shear deformation of $S = 2.5\%$. The modulus, calculated according to the method mentioned above, decreased to $7.53 \cdot 10^4$ MPa (i.e. by 7.6%); then the sample underwent the opposite sign plastic deformation of 2.3%. Further on we defined the modulus in the way described above to be equal to $7.75 \cdot 10^4$ MPa. It increased by 3%.

The analogous effect of partial restoration of elasticity moduli after the sample undergoes plastic deformation of opposite sign was supported by tests made on other samples.

3. Conclusions

The results obtained allow to draw the following conclusions:

1. The elasticity moduli E and G of metals after being subject to plastic deformation of intensity 1–10% decrease. The decrease in all cases did not exceed 10% as compared to the initial moduli values. We couldn't establish any rule of moduli variation dependence on plastic deformation imparted to the samples.

2. The elasticity modulus value does not vary along the $\sigma_i - \epsilon_i$ line, both at unloading and loading.

3. When the initially deformed sample undergoes plastic deformation, opposite sign partial restoration of the elasticity modulus occurs.

It should be noted that this effect of variation of actual elasticity modulus at plastic deformation agrees with the effect known from physics according to which a monocrystal is always elastically anisotropic, and an elastic modulus of polycrystalline metals has an integral character. Actually, the crystals of tempered metals are in a chaotic random state and the elasticity modulus has a corresponding definite value. At plastic deformation, however, the crystals assume a predominated orientation texture and, as a result, the elasticity modulus inevitably changes.

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KALININGRAD, USSR.

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