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Fibre spinning processes as viscoelastic flows with dominating extensions

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UNDER THE ASSUMPTION of quasi-elongational approximation, i.e., for uniform velocity profiles across the filament, melt-spinning processes as well as film casting and drawing processes are treated as particular cases of flows with dominating extensions (FDE). Certain approximate solutions for slightly non-Newtonian fluids and under the assumption of nearly extensional and weakly nonisothermal flows are discussed in greater detail. Two types of boundary conditions are considered on the free surface of the thread: the first resulting from the force balance, and the second determined by the vanishing radial stress component.

W założeniu quasi-rozciągającego przybliżenia, t.j. dla równomiernych profilów prędkości w poprzek włókna, procesy przędzenia ze stopu jak i procesy kształtowania i rozciągania folii potraktowano jako szczególne przypadki przepływów z dominującymi rozciąganiami (PDR). Przedyskutowano bardziej szczegółowo niektóre przybliżone rozwiązania dla cieczy nieznacznie nienewtonowskich oraz w założeniu przepływów zbliżonych do prostego rozciągania i słabo nieizotermicznych. Rozważono dwa typy warunków brzegowych na swobodnej powierzchni włókna: pierwszy wynikający z równowagi sił i drugi określony przez znikającą promieniową składową naprężenia.

В предположении квазирастягивающего приближения, т.е. для равномерных профилей скорости поперек волокна, процессы прядения из сплава, как и процессы формирования и растяжения фольги трактуются как частные случаи течений с доминирующими растяжениями (ТДР). Обсуждены более подробно некоторые приближенные решения для незначительно неньютоновских жидкостей и в предположении течений сближенных к простому растяжению и слабо неизотермических. Рассмотрены два типа граничных условий на свободной поверхности волокна: первое, вытекающее из равновесия сил и второе, определенное исчезающей радиальной составляющей напряжения.

1. Introduction

IN OUR PAPER [1] it is shown that nonisothermal melt-spinning processes with axial and radial viscosity distributions can be treated as particular cases of the flows with dominating extensions (FDE) defined and discussed elsewhere [2, 3]. It is also proved that the so-called quasi-elongational approximation, i.e., the assumption of uniform velocity profiles across the filament, is fully justified, especially for small radius-to-length ratios and moderate variability of the extensional viscosity function along the spinline (cf. [4, 5]).

In the present paper, we discuss in greater detail certain solutions of the simplified problems based on the assumption of quasi-elongational approximation. In particular, the case of slightly non-Newtonian fluids, i.e., fluids with weak variability of the extensional viscosity with respect to the extension rate, is considered and the corresponding solutions are presented. If the above condition is not satisfied, an alternative approximation for nearly extensional and weakly nonisothermal flows is also proposed.

Moreover, two types of the boundary conditions on the free surface of the thread are discussed. The first one results from the balance of all forces acting on the surface element, while the second one, valid for small radius-to-length ratios, leads to the requirement of vanishing radial stresses on the surface. For non-Newtonian fluids the corresponding approximate solutions are different although some terms are the same.

At the end of the paper, the case of plane flows, relevant to film casting and drawing processes, is treated in a similar way and the essentially distinct results are outlined.

2. Quasi-elongational approximation and axial velocity distributions

Bearing in mind the assumption of quasi-elongational approximation, we consider the following velocity gradient expressed in cylindrical coordinates (r, ϑ, z):

$$[\nabla \mathbf{V}] = \begin{bmatrix} -\frac{1}{2} & 0 & 0 \\ 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} q, \quad q \equiv V'(z), \tag{2.1}$$

where the prime denotes the derivative with respect to z.

The above gradient introduced into the constitutive equation of an incompressible simple fluid in a form valid for purely extensional flows (cf. [6]), viz.

$$\mathbf{T} = -p\mathbf{1} + \beta_1 \mathbf{A}_1 + \beta_2 \mathbf{A}_1^2, \quad \mathrm{tr}\,\mathbf{A}_1 = 0, \tag{2.2}$$

where $\mathbf{T}$ is the stress tensor, p — the hydrostatic pressure, and A_1 — the first Rivlin–Ericksen kinematic tensor, leads to

$$T^{33} - T^{11} = 3\beta V', \tag{2.3}$$

where the extensional viscosity function $\beta = \beta_1 + \beta_2 q$ is simply related to the elongational viscosity: $\eta^* = 3\beta$. The function β as well as the material functions $\beta_i (i = 1, 2)$ are assumed to depend on $q \equiv V'(z)$ and also explicitly on z. The latter dependence is useful for taking into account temperature distributions and crystallization effects along the spinline as discussed by Ziabicki [7, 8].

If gravity, inertia, surface-tension and air-drag effects can be disregarded [(cf. 4, 7]), and the constant take-up force is fully balanced by the rheological force resulting from Eq. (2.3), we arrive at

$$\beta V' = CV, \tag{2.4}$$

where C denotes a constant. We have made use of the fact that volume discharge is preserved, i.e., $\pi V h^2 = \mathrm{const}$ for any radius h. Integration of Eq. (2.4) with the following boundary conditions (cf. [4, 5, 7, 8]):

$$V(0) = V_0, \quad V(l) = V_1, \tag{2.5}$$

leads to the well-known result (cf. [4])

$$V(z) = V_0 \exp\left[\ln V_1/V_0 \int_0^z dz/\beta \Big/ \int_0^l dz/\beta\right], \tag{2.6}$$

where l denotes the spinline length.

In the paper on "neck-like" deformations in high-speed melt spinning, ZIABICKI [7, 8] also discussed the effects of polymer crystallization. For Newtonian fluids the following simple formula has been proposed:

$$\beta = \beta_0 f(z) \exp(Az), \tag{2.7}$$

$$f(z) = \begin{cases} 1 & \text{for} \quad z \leqslant z_0, \\ 1/(1-K(z-z_0)) & \text{for} \quad z_0 < z < z_0 + \dfrac{1}{K}, \\ \infty & \text{for} \quad z \geqslant z_0 + \dfrac{1}{K}, \end{cases}$$

where A represents temperature dependence of the extensional viscosity function, the quantity K is proportional to the crystallization rate and inversely proportional to the crytical crystallinity, and z_0 denotes the position characteristic of the onset of crystallization.

In particular, for $z \leqslant z_0$ we obtain

$$\ln V = \frac{1}{\exp(-Al)-1}\left[\ln V_1/V_0 \exp(-Az) + \ln V_0 \exp(-Al) - \ln V_1\right], \tag{2.8}$$

and for $z_0 < z \leqslant z_0 + 1/K$

$$\ln V = \frac{1}{(\exp(-Al)-1)(Kz_0+K/A-1)+Kl\exp(-Al)} \times [\ln V_1/V_0 \exp(-Az)(Kz+Kz_0+K/A-1) + \ln V_0 \exp(-Al)(Kz_0+K/A-1) - \ln V_1(Kz_0+K/A-1)]. \tag{2.9}$$

In the case of non-Newtonian fluids, if the viscosity dependence on the extension rate may be described by a power function $\beta = \beta_0 \exp(Az)\cdot q^n$, we arrive at

$$V^a = \frac{1}{\exp(-Al/n+1-1)}\left[(V_1^a - V_0^a)\exp(-Az/n+1) + V_0^a \exp(-Al/n+1) - V_1^a\right], \tag{2.10}$$

where for simplicity we have denoted $a = n/1+n$.

It is noteworthy that inertia effects can easily be included into Eq. (2.4) leading to the result a little different from that presented in Eq. (2.6) (cf. [4]).

3. Flows with dominating extensions and governing equations

Consider the real velocity gradient in the form

$$[\nabla \mathbf{V}^*] = \begin{bmatrix} -\frac{1}{2} & 0 & 0 \\ 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} q + \begin{bmatrix} \frac{\partial u}{\partial r} & 0 & \frac{\partial u}{\partial z} \\ 0 & \frac{u}{r} & 0 \\ \frac{\partial w}{\partial r} & 0 & \frac{\partial w}{\partial z} \end{bmatrix}, \tag{3.1}$$

where the first matrix is exactly the same as that in Eq. (2.1), and the second one describes an arbitrary axi-symmetric gradient resulting from the additional velocity field. Here u and w denote the radial and axial components, respectively. We assume, moreover, that in the process considered, the ratio of the thread radius h (varying from h_0 to h_1) to its length l is a small quantity, i.e., $\varepsilon = h/l \ll 1$. For relatively small vorticity components, we can conclude (cf. [2, 3]) that the second matrix components may be less meaningful as compared to those in the first matrix.

According to our previous definitions, the flows with dominating extensions (FDE) are such flows of the type (3.1) in which the constitutive equations for purely extensional flows of an incompressible simple fluid can be used in a form linearly perturbed with respect to the additional velocity gradients, viz.

$$\mathbf{T}^* = -p\mathbf{1}+\beta_1\mathbf{A}_1+\beta_1\bar{\mathbf{A}}_1+\beta_2\mathbf{A}_1^2+\beta_2(\bar{\mathbf{A}}_1^2)+\frac{\partial\beta_1}{\partial q}\bar{q}\mathbf{A}_1+\frac{\partial\beta_2}{\partial q}\bar{q}\mathbf{A}_1^2+\ \dots, \tag{3.2}$$

where the overbars denote increments of the corresponding quantities, e.g.,

$$\bar{q} = \frac{\partial w}{\partial z}+\frac{1}{3q}\left(\frac{\partial u}{\partial r}\right)^2+\frac{1}{3q}\left(\frac{u}{r}\right)^2+\frac{1}{3q}\left(\frac{\partial w}{\partial z}\right)^2+\frac{1}{6q}\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial r}\right)^2. \tag{3.3}$$

In the next step repeatedly realized in our previous considerations (cf. [2, 3]), the necessary increments of the kinematic tensors are calculated and substituted into Eqs. (3.2). Introducing them into the intertialess equations of equilibrium, we arrive at the system of equations with terms of different orders of magnitude with respect to $\varepsilon = h/l$. After eliminating pressure p and retaining only terms of the highest order of magnitude, the following governing equation is obtained:

$$\frac{\partial}{\partial r}\left\{\beta\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial w}{\partial r}\right)+\frac{\partial}{\partial z}\left[\frac{1}{2}\left(\frac{\partial\beta_1}{\partial q}+\frac{\partial\beta_2}{\partial q}q\right)\left(\frac{\partial w}{\partial r}\right)^2\right]\right\} = 0. \tag{3.4}$$

On defining the modified pressure p^*, viz.

$$p^* = p-T_E^{*11} = -T^{*11}, \tag{3.5}$$

where the subscript E denotes the extra-stress component, we have instead of Eq. (3.4)

$$\begin{aligned}&\frac{\partial p^*}{\partial r} = 0,\\ &\frac{dp^*}{\partial z} = \beta\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial w}{\partial r}\right)+\frac{\partial}{\partial z}\left[\frac{1}{2}\left(\frac{\partial\beta_1}{\partial q}+\frac{\partial\beta_2}{\partial q}q\right)\left(\frac{\partial w}{\partial r}\right)^2\right]+\frac{\partial}{\partial z}(3\beta V') = C(z),\end{aligned} \tag{3.6}$$

where the function $dp^*/dz = C(z)$ must be determined from the appropriate boundary condition on the free surface of the filament.

It is also worth of noting that the following simplified relations

$$T^{*11} = T^{*22} = -p-\beta_1 q+\beta_2 q^2+\beta_2\left(\frac{\partial w}{\partial r}\right)^2-\frac{1}{6}\frac{\partial\beta_1}{\partial q}\left(\frac{\partial w}{\partial r}\right)^2+\frac{1}{6}\frac{\partial\beta_2}{\partial q}q\left(\frac{\partial w}{\partial r}\right)^2,$$

$$(3.7)\qquad T^{*33} = -p+2\beta_1 q+4\beta_2 q^2+\beta_2\left(\frac{\partial w}{\partial r}\right)^2+\frac{1}{3}\frac{\partial\beta_1}{\partial q}\left(\frac{\partial w}{\partial r}\right)^2+\frac{2}{3}\frac{\partial\beta_2}{\partial q}q\left(\frac{\partial w}{\partial r}\right)^2,$$

$$T^{*13} = (\beta_1+\beta_2 q)\frac{\partial w}{\partial r},$$

immediately lead to Eqs. (3.4) or (3.6).

4. Boundary conditions

Together with the nonlinear partial differential equations (3.4) or (3.6), the appropriate boundary conditions should be formulated.

The kinematic boundary conditions must be consistent with Eq. (2.5). Hence we have

$$(4.1)\qquad w(r,0) = 0, \quad w(r,l) = 0;$$

what means that the additional velocity component w should be equal to 0, at least approximately, at both ends of the spinline.

The remaining boundary conditions are determined in terms of stresses. It is possible to distinguish two types of the boundary condition prescribed on the free surface.

The first type of condition results from the balance of forces acting on a surface element based on the coordinates $r\Delta\vartheta$, Δz and $\xi\Delta z$ (cf. [5]). For small values of $\xi = dh/dz$, i.e., for $\xi \ll 1$, and this is the case for melt-spinning processes, we finally arrive at (cf. [1])

$$(4.2)\qquad \xi(T^{*33}-T^{*11}) = T^{*13} \quad \text{for} \quad r = h.$$

Since from the assumption of constant output it results that

$$(4.3)\qquad \xi = -\frac{1}{2}\frac{V'}{V}h,$$

the above boundary condition can be expressed as

$$(4.4)\qquad -\frac{1}{2}\frac{V'}{V}h\left[3\beta V'+\frac{1}{2}\left(\frac{\partial\beta_1}{\partial q}+\frac{\partial\beta_2}{\partial q}q\right)\left(\frac{\partial w}{\partial r}\right)^2\right] = \beta\frac{\partial w}{\partial r} \quad \text{for} \quad r = h.$$

The second type of condition is based on the assumption that for very small values of $\xi = dh/dz$, i.e., for almost a cylindrical shape of the thread, what is equivalent to rather moderate draw ratios, the radial stress component T^{*11} vanishes on the free surface. Taking into account Eq. (3.5), we therefore have

$$(4.5)\qquad p^* = 0 \quad \text{for} \quad r = h,$$

and since p^* does not depend on the radius r, the above result implies that $p^* \equiv 0$ everywhere. Thus, bearing in mind Eq. (3.6), we obtain

$$(4.6)\qquad \frac{dp^*}{dz} = C(z) \equiv 0.$$

The condition (4.4) seems to be more exact than the simplified condition (4.6), but it leads to more complex solutions (cf. Sect. 5). A proper choice of the condition should be made for any particular problem with known geometrical parameters of the spinning process.

5. Solutions for slightly non-Newtonian fluids

The nonlinear partial differential equations (3.4) or (3.6) together with the boundary conditions discussed in Sec. 4 cannot be solved in a straightforward manner. Therefore we propose some approximate methods.

For slightly non-Newtonian fluids, for which the parameter

$$k = \frac{1}{\beta}\left(\frac{\partial \beta_1}{\partial q} + \frac{\partial \beta_2}{\partial q} q\right) q = \frac{1}{\beta}\left(\frac{\partial \beta}{\partial q} - \beta_2\right) q \tag{5.1}$$

is small enough, i.e., for $k \leqslant \varepsilon = h/l$, we can seek a solution in the following perturbed form (cf. [1]):

$$w(r, z) = w_0(r, z) + k w_1(r, z), \tag{5.2}$$

where w_0 denotes the corresponding Newtonian solution.

Thus the governing equation $(3.6)_2$ for purely viscous fluids ($\beta \equiv \beta_0$) takes the following form:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r \frac{\partial w_0}{\partial r}\right) = -\frac{1}{\beta_0}\frac{\partial}{\partial z}(3\beta V') + \frac{C_0}{\beta_0}. \tag{5.3}$$

Integration of the above equation leads to

$$w_0 = -\frac{1}{4\beta_0}\frac{\partial}{\partial z}(3\beta_0 V') r^2 + \frac{C_0 r^2}{4\beta_0} + D_0, \tag{5.4}$$

and from the condition of constant total volume output, we have

$$\int_0^h w_0 \cdot 2\pi r\, dr = 0; \tag{5.5}$$

hence it appears

$$D_0 = \frac{3}{8\beta_0}\frac{\partial}{\partial z}(\beta_0 V') h^2 - \frac{C_0 h^2}{8\beta_0}. \tag{5.6}$$

After calculating the constant C_0 either from the condition (4.4) or (4.6), and substituting it into Eq. (5.4), in both cases we arrive at

$$w_0 = -\frac{3}{4}\frac{V'^2}{V}\left(r^2 - \frac{h^2}{2}\right), \tag{5.7}$$

where we have used the relation

$$\frac{1}{\beta}\frac{\partial}{\partial z}(\beta V') = V'^2/V \tag{5.8}$$

resulting from Eq. (2.4) not only for Newtonian fluids.

For slightly non-Newtonian fluids satisfying the condition (5.1), the governing equation $(3.6)_2$ can be presented in the form

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial w}{\partial r}\right) = -\frac{3}{\beta}\frac{\partial}{\partial z}(\beta V') + \frac{C}{\beta} - \frac{9}{8\beta}\frac{\partial}{\partial z}\left[\left(\frac{\partial \beta_1}{\partial q} + \frac{\partial \beta_2}{\partial q} q\right)\left(\frac{V'^2}{V}\right)^2\right] r^2 \tag{5.9}$$

from which $w = w_0 + kw_1$ can directly be calculated. Proceeding in a way similar to that employed for purely viscous fluids, we finally arrive at

$$(5.10) \qquad w = -\frac{3}{4\beta}\frac{V'^2}{V}\left(r^2-\frac{h^2}{2}\right)-\frac{9}{32}\frac{V'}{V}h^2\frac{1}{\beta}\left(\frac{\partial\beta_1}{\partial q}+\frac{\partial\beta_2}{\partial q}q\right)\left(\frac{V'^2}{V}\right)^2\left(r^2-\frac{h^2}{2}\right)$$
$$-\frac{9}{128\beta}\frac{\partial}{\partial z}\left[\left(\frac{\partial\beta_1}{\partial q}+\frac{\partial\beta_2}{\partial q}q\right)\left(\frac{V'^2}{V}\right)^2\right]\left[\left(r^4-\frac{h^4}{3}\right)-2h^2\left(r^2-\frac{h^2}{2}\right)\right],$$

if the boundary condition (4.4) is used. For the simplified condition (4.6), we obtain instead of Eq. (5.10) the following simpler expression:

$$(5.11) \qquad w = -\frac{3}{4\beta}\frac{V'^2}{V}\left(r^2-\frac{h^2}{2}\right)-\frac{9}{128\beta}\frac{\partial}{\partial z}\left[\left(\frac{\partial\beta_1}{\partial q}+\frac{\partial\beta_2}{\partial q}q\right)\left(\frac{V'^2}{V}\right)^2\right]\left(r^4-\frac{h^4}{3}\right).$$

It is seen that for purely Newtonian fluids ($\beta \equiv \beta_0$) Eq. (5.11) leads to the previous result (5.7). On the other hand, the additional terms appearing in Eqs. (5.10) and (5.11) for non-Newtonian fluids may differ in magnitudes and signs.

The fact that the kinematic boundary conditions (4.1) cannot be satisfied exactly either by Eq. (5.7) or by Eqs. (5.10) and (5.11) deserves some comments. Because of the approximate method involved, these conditions can be satisfied only approximately. Such a possibility results from the observation that, for typical S-shaped velocity distributions (cf. [4, 7, 8]), the quantities V'^2/V are very small at both ends of the spinline. It also turns out from Eq. (2.6) that at the ends

$$(5.12) \qquad \begin{aligned} \frac{V'^2(0)}{V(0)}h_0^2 &= C_1^2 V_0 h_0^2 \frac{1}{\beta^2(0)}, \\ \frac{V'^2(l)}{V(l)}h_1^2 &= C_1^2 V_1 h_1^2 \frac{1}{\beta^2(l)}, \end{aligned}$$

where C_1 is a constant. So the additional velocity component w at both ends of the spinline is inversely proportional to squares of the corresponding extensional viscosities. These quantities are small, especially for $z = l$, where solidification occurs.

6. Solutions for nearly extensional and weakly nonisothermal flows

For nonlinear fluids with strong variability of the extensional viscosity function on the extension rate, i.e. in the case in which the parameter k defined by Eq. (5.1) is not small, another approximate solution can be presented under the assumption that the flow considered differs little from the purely extensional flow of a cylinder, being simultaneously weakly nonisothermal, i.e., with the coefficients $\beta_i (i = 1, 2)$ weakly depending on the distance z.

Therefore, seeking solution of Eq. $(3.6)_2$ in the form

$$(6.1) \qquad w(r, z) = V'^2(z)\overline{w}(r),$$

where $\overline{w}$ is a function of r only, we conclude that for $\partial\beta_i/\partial z = 0$ and $V' \equiv q = \text{const}$, all the coefficients in that equation are either constants or zero. Thus we arrive at the equation

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \overline{w}}{\partial r}\right) = \frac{C}{\beta}, \tag{6.2}$$

the solution of which takes again the form (5.7).

Thus, for nearly extensional and weakly nonisothermal flows, when the filament can be considered as composed of purely extended cylindrical parts with constant viscosity distributions along each part (cf. [4, 5]), we arrive at the equation

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \overline{w}}{\partial r}\right) + A(z)\left(\frac{\partial \overline{w}}{\partial r}\right)^2 = B(z), \tag{6.3}$$

where

$$\begin{aligned} A(z) &= \frac{1}{\beta}\left[\frac{1}{2}\frac{\partial}{\partial z}\left(\frac{\partial \beta_1}{\partial q} + \frac{\partial \beta_2}{\partial q} q\right)V'^2 + 2\left(\frac{\partial \beta_1}{\partial q} + \frac{\partial \beta_2}{\partial q} q\right)V'V'',\right. \\ B(z) &= -\frac{3}{\beta V'^2}\frac{\partial}{\partial z}(\beta V') + \frac{C(z)}{V'^2\beta}. \end{aligned} \tag{6.4}$$

The above expressions weakly differing from constants or zero, may be considered as quantities depending on the parameter z. Equation (6.3) is the special Riccati equation (for $\overline{w}'$) and its solutions amount to

$$\begin{aligned} \overline{w}(r) &= -\frac{1}{A}\ln I_0\left(\sqrt{-AB}\, r\right) + D \qquad \text{for} \quad AB < 0, \\ \overline{w}(r) &= \frac{1}{A}\ln J_0\left(\sqrt{AB}\, r\right) + D \qquad \text{for} \quad AB > 0, \end{aligned} \tag{6.5}$$

where J_0 and I_0 denote the Bessel and the modified Bessel functions of zero order, respectively. From the condition (5.5) we also have

$$D = \frac{2}{Ah^2}\int_0^h \ln I_0\left(\sqrt{-AB}\, r\right) r\, dr \qquad \text{for} \quad AB < 0, \tag{6.6}$$

and a similar expression for $AB > 0$.

The quantities $C(z)$, or rather $B(z)$, can be determined from the boundary conditions (4.4) or (4.6). In the first case we obtain the following complex equation:

$$-\frac{1}{2}\frac{V'}{V}h\left[3\beta V' + \frac{1}{2}\left(\frac{\partial \beta_1}{\partial q} + \frac{\partial \beta_2}{\partial q} q\right)\frac{B}{A}\frac{I_1^2\left(\sqrt{-AB}\, r\right)}{I_0\left(\sqrt{-AB}\, r\right)}\right] = \beta\left[-\sqrt{\frac{B}{A}}\frac{I_1\left(\sqrt{-AB}\, r\right)}{I_0\left(\sqrt{-AB}\, r\right)}\right], \tag{6.7}$$

if $AB < 0$. In the case of the simplified boundary condition we have

$$C(z) \equiv 0, \qquad B(z) = -\frac{3}{\beta V'^2}\frac{\partial}{\partial z}(\beta V'). \tag{6.8}$$

7. Solutions for plane flows

The case of plane flows, relevant for film casting and drawing of molten polymers (cf. [9]) can be treated in a way similar to that presented in the previous sections devoted to axi-symmetric flows. Therefore, in this section we quote only such formulae which essentially differ from the previous ones.

The fundamental flow characterized by the following gradient

$$[\nabla \mathbf{V}] = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} q, \quad q \equiv V'(x), \tag{7.1}$$

expressed in Cartesian coordinates (x, y) leads again to Eq. (2.4), the solution of which takes the form (2.6) with this distinction from the axi-symmetric case that $\beta \equiv \beta_1$, where β_1 is the only coefficient in the constitutive equation (cf. [6]):

$$\mathbf{T} = -p\mathbf{1} + \beta_1 \mathbf{A}_1, \quad \operatorname{tr} \mathbf{A}_1 = 0, \tag{7.2}$$

valid for plane extensional flow. The planar extensional viscosity function amounts to $\eta_p^* = 4\beta_1$.

Now we consider the real velocity gradient in the form

$$[\nabla \mathbf{V}^*] = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} q + \begin{bmatrix} \dfrac{\partial u}{\partial x} & \dfrac{\partial u}{\partial y} \\ \dfrac{\partial v}{\partial x} & \dfrac{\partial v}{\partial y} \end{bmatrix}, \tag{7.3}$$

where u and v denote the additional velocity components in the x and y directions, respectively. Introducing the notation

$$p^* = p - T_E^{*22} = -T^{*22}, \tag{7.4}$$

we obtain, instead of Eqs. (3.6),

$$\begin{aligned} &\frac{\partial p^*}{\partial y} = 0, \\ &\frac{dp^*}{dx} = \beta \frac{\partial^2 u}{\partial y^2} + \frac{1}{2} \frac{\partial}{\partial x} \left[\left(\frac{\partial \beta}{\partial q} \right) \left(\frac{\partial u}{\partial y} \right)^2 \right] + \frac{\partial}{\partial x} (4\beta V') = C(x). \end{aligned} \tag{7.5}$$

The above governing equations result directly from the following relations for stresses:

$$\begin{aligned} T^{*11} &= -p + 2\beta q + \frac{1}{4} \frac{\partial \beta}{\partial q} \left(\frac{\partial u}{\partial y} \right)^2, \\ T^{*22} &= -p - 2\beta q - \frac{1}{4} \frac{\partial \beta}{\partial q} \left(\frac{\partial u}{\partial y} \right)^2, \\ T^{*12} &= \beta \frac{\partial u}{\partial y}. \end{aligned} \tag{7.6}$$

The boundary condition (4.2) determined on the free surface takes the form

$$\xi(T^{*11} - T^{*22}) = T^{*12} \quad \text{for} \quad y = \pm h, \tag{7.7}$$

where

$$\xi = -\frac{V'}{V}h \tag{7.8}$$

is assumed small as compared with unity. The simplified boundary condition (4.6) remains unchanged.

For purely Newtonian fluids, Eq. $(7.5)_2$ leads to the simplified equation

$$\frac{\partial^2 u_0}{\partial y^2} = -\frac{1}{\beta_0}\frac{\partial}{\partial x}(4\beta_0 V') + \frac{C_0}{\beta_0}, \tag{7.9}$$

the solution of which amounts to

$$u_0 = -2\frac{V'^2}{V}\left(y^2 - \frac{h^2}{3}\right), \tag{7.10}$$

for both types of the boundary conditions.

For slightly non-Newtonian fluids for which

$$k = \frac{1}{\beta}\frac{\partial \beta}{\partial q}q \tag{7.11}$$

is small enough, i.e., for $k \leqslant \varepsilon = h/l$, we arrive at

$$u = -2\frac{V'^2}{V}\left(y^2 - \frac{h^2}{3}\right) - \frac{4}{\beta}\frac{V'}{V}h^2\frac{\partial \beta}{\partial q}\left(\frac{V'^2}{V}\right)^2\left(y^2 - \frac{h^2}{3}\right) - \frac{2}{3\beta}\frac{\partial}{\partial x}\left[\frac{\partial \beta}{\partial q}\left(\frac{V'^2}{V}\right)^2\right]\left[\left(y^4 - \frac{h^4}{5}\right) - 2h^2\left(y^2 - \frac{h^2}{3}\right)\right], \tag{7.12}$$

if the condition expressed by Eq. (7.7) is satisfied or

$$u = -2\frac{V'^2}{V}\left(y^2 - \frac{h^2}{3}\right) - \frac{2}{3\beta}\frac{\partial}{\partial x}\left[\frac{\partial \beta}{\partial q}\left(\frac{V'^2}{V}\right)^2\right]\left(y^4 - \frac{h^4}{5}\right), \tag{7.13}$$

if the simplified condition ($C(x) \equiv 0$) can be accepted.

8. Conclusions

It has been shown again that melt-spinning processes and plane film-casting and drawing processes can be considered, under the quasi-elongational approximation, as particular cases of flows with dominating extensions (FDE). An arbitrary viscosity distribution along the filament axis makes it possible to take into account both temperature and crystallization effects (cf. [7, 8]). The proposed approximate solutions depend only on two material functions defining the extensional viscosity as well as on their variability with respect to the extension rate. In the case of plane flows, only one function: the planar extensional viscosity and its derivative with respect to the extension gradient is involved.

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