Arch. Mech., 32, 6, pp. 831–841, Warszawa 1980

Transient momentum and heat transfer from a cylinder

S. P. LIN (POTSDAM)

By use of a series of integral transformations, the Navier-Stokes equation and the energy equation are solved for the transient momentum and heat transfers from a circular cylinder in an incompressible fluid executing a small amplitude high-frequency fluctuation at finite Reynolds numbers.

Za pomocą szeregu transformacji całkowych rozwiązano równania Naviera-Stokesa i równanie energii dla nieustalonego wypływu pędu i ciepła z cylindra kołowego do cieczy nieściśliwej wykonującej niskoamplitudowe drgania wysokiej częstości przy skończonych liczbach Reynoldsa.

При помощи ряда интегральных преобразований решено уравнение Навье-Стокса и уравнение энергии для неустановившегося истечения импульса и тепла из кругового цилиндра в несжимаемую жидкость, совершающую колебания с малыми амплитудами высокой частоты при конечных числах Рейнольдса.

Notation

- a radius of the cylinder,
- C_p specific heat,
- $\mathbf{D}$ drag force,
- E Eckert number,
- $\mathbf{G}$ Grashof number,
- g gravitational acceleration,
- k thermal conductivity,
- Nu Nusselt number,
- p pressure,
- P_r Prandtl number,
- q heat transfer rate from the cylinder,
- R Reynolds number,
- r radial distance,
- S tension,
- T temperature,
- t time,
- U cylinder velocity,
- u dimensionless cylinder velocity,
- $\mathbf{V}$ fluid velocity,
- $\mathbf{v}$ dimensionless fluid velocity,
- x, y Cartesian coordinates (Fig. 1),
- δ fluctuation amplitude,
- ε dimensionless amplitude δ/a,
- Θ dimensionless temperature,
- θ polar angle,
- μ dynamic viscosity,
- ν kinematic viscosity,
- ∇_1 dimensional gradient operator,
- ∇ dimensionless gradient operator,

ϱ fluid density,
τ dimensionless time,
Φ dissipation function,
ψ stream function,
ω fluctuation frequency.

Subscripts

0 zero-th order solution,
1 first-order solution,
2 second-order solution,
m maximum value.

1. Introduction

THE PROBLEM of transient thermal energy transfer from a thread in an unsteady flow at finite Reynolds numbers are encountered in many industrial and scientific devices. The examples include the cooling of power transmission lines in wind, the heating of electric wires and optical fibers in electronic instruments, the hot wire and hot film anemometers for the measurement of fluctuating flows.

The problem of steady heat transfer without natural convection from a circular cylinder in an incompressible fluid has been studied by COLE and ROSHKO [1], ILLINGWORTH [2] and WOOD [3] with various types of Oseen approximations for low Reynolds number flows. The same problem has been studied by HIEBER and GEBHART [4] with matched asymptotic expansion. The effect of buoyancy was later included by WOOD [5]. The theoretical results are in good agreement with experiments [6, 7, 8, 9] when the Reynolds number is much smaller than one. For the case of large Reynolds numbers, the problem of steady mixed convection from a horizontal circular cylinder has been studied recently by SPARROW and LEE [10] and MERKIN [11] with the boundary layer approximation.

Unsteady heat transfer from hot wires in a low Reynolds number flow has been estimated by DAVIES [12] with an Oseen type approximation. Despite its technological importance, a theoretical study of fluctuating heat transfer from a circular cylinder in a finite Reynolds number flow has not yet appeared. This situation is closely related to the fact that even the isothermal flow field about a cylinder fluctuating at finite Reynolds numbers is not yet theoretically predicted. Numerical solutions and boundary layer solutions for the cases of impulsively started uniform motion and other prescribed simple motions are available as explained in the writer's work [13] and the works of COUTANCEAU *et al.* [14] and BAR-LEV *et al.* [15]. On the other hand, a considerable amount of significant experimental results on fluctuating heat transfer has now been accumulated (see references in [6, 8, 16, 17]). The state of art of this particular subject therefore is simply that the theory lags far behind the experimental work.

Here the writer develops a method of solution for the problem of forced convective momentum and heat transfer in a fluctuating flow at finite Reynolds numbers for which the boundary layer approximation may not be applicable. The fluctuation of the flow need not be sinusoidal. The usefulness of the general results is demonstrated by applying them to the problems of 1) the transient initial drag on a circular cylinder which starts an arbitrary translation from rest, 2) transient damped vibration of a string, 3) the onset and the migration of the separation point, and 4) the heat transfer from a cylinder in fluctuating finite Reynolds number flows.

2. Theory

2.1. Governing equations

Consider the heat transfer from a circular cylinder of radius a moving with a velocity $-\mathbf{i}U(t)$ in a quiescent fluid as shown in Fig. 1.

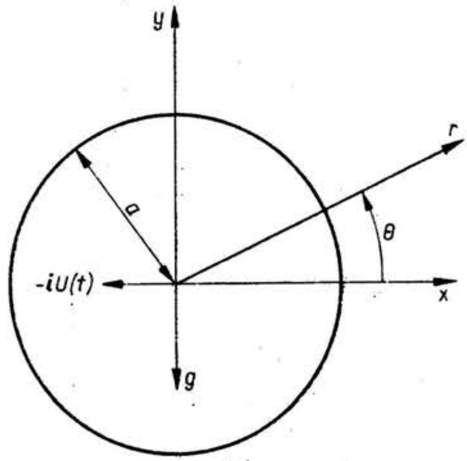

FIG. 1.

With respect to the coordinate system attached to the cylinder, the momentum, mass and energy equations are respectively

$$\frac{\partial \mathbf{V}}{\partial t}+(\mathbf{V}\cdot\nabla_1)\mathbf{V} = -\frac{1}{\varrho}\nabla_1 P+\nu\nabla_1^2\mathbf{V}+\mathbf{i}\dot{U}(t)-\mathbf{j}g,$$

$$(2.1)\qquad \frac{\partial \varrho}{\partial t}+\nabla_1(\varrho\mathbf{V}) = 0,$$

$$k\nabla^2 T = \varrho C_p\left(\frac{\partial}{\partial t}+\mathbf{V}\cdot\nabla_1\right)T-P\nabla_1\mathbf{V}-\mu\Phi,$$

where $\mathbf{V}$ is the fluid velocity, P is the pressure, ∇_1 is the gradient operator, the dot denotes differentiation with respect to time t, ϱ is the density, g is the gravitational acceleration, T is the temperature, C_p is the specific heat, k is the conductivity of fluid, μ is the viscosity, Φ is the dissipation function, and $\mathbf{i}$ and $\mathbf{j}$ are unit vectors in the x- and y-directions, respectively.

Invoking the Boussinesq approximation and introducing the following dimensionless variables

$$V = \mathbf{v}(\omega\delta),\qquad P = p(\varrho_\infty\omega^2\delta^2),\qquad \nabla_1 = \nabla/a,$$
$$T = (T_a-T_\infty),\qquad \Theta = (\Delta T)\Theta,\qquad t = \tau a^2/\nu,$$

where T_a is the surface temperature of the cylinder, T_∞ is the ambient temperature, ω and δ are respectively the characteristic frequency and amplitude of the fluid oscillation, we can write Eq. (2.1) in a dimensionless form:

$$
\begin{aligned}
&\frac{\partial \mathbf{v}}{\partial \tau}+\varepsilon R(\mathbf{v}\cdot\nabla)\mathbf{v} = -\varepsilon R\nabla p+\nabla^2\mathbf{v}-\mathbf{G}R\Theta,\\
(2.2)\qquad &\nabla\cdot\mathbf{v}=0,\\
&\nabla^2\Theta = \left(P_r\frac{\partial}{\partial\tau}+\varepsilon P_r R\mathbf{v}\cdot\nabla\right)\Theta-\varepsilon^2 P_r RE\phi,
\end{aligned}
$$

where P_r, R and $\mathbf{G}$ are respectively the Prandtl number, the Reynolds number and the Grashof number defined by

$$P_r = \nu/(k/\varrho C_p),\quad R=\omega a^2/\nu,\quad \mathbf{G}=\beta(-\mathbf{j}g+\mathbf{i}\dot{U})\Delta T/\omega^2\delta,$$

and

$$\varepsilon=\delta/a,\quad E=\nu\omega/C_p\Delta T,\quad \phi=\Phi/(\omega^2\delta^2/a^2).$$

In the present study, we consider only the cases in which

$$(2.3)\qquad \varepsilon\ll 1,\quad \mathbf{G}=0(\varepsilon^3),\quad E=0(\varepsilon)$$

but P_r and R need not be small. The above parameter range is relevant to the problems of forced convective heat transfer in small amplitude high-frequency oscillatory flows including the initial transient momentum transfer at finite R as a special case [13, 18, 19] when the boundary layer theory needs not to be applicable. In this range of parameters, the velocity field and the temperature field described by Eq. (2.2) are decoupled from each other up to $0(\varepsilon^3)$ when the solution is sought in power series of ε.

2.2. Velocity field

In terms of the stream function ψ in cylindrical coordinates (r, θ, z)

$$u_r=-\frac{1}{r}\frac{\partial\psi}{\partial\theta},\quad u_\theta=\frac{\partial\psi}{\partial r},$$

where u_r and u_θ are respectively the r and θ velocity components, the first equation of the set (2.2) can be written as

$$(2.4)\qquad \left(\frac{\partial}{\partial\tau}-\nabla^2\right)\nabla^2\psi=\frac{\varepsilon R}{r}\frac{\partial(\psi,\nabla^2\psi)}{\partial(r,\theta)}.$$

In the above equation

$$
\begin{aligned}
\frac{\partial(\psi,\nabla^2\psi)}{\partial(r,\theta)} &= \frac{\partial\psi}{\partial r}\cdot\frac{\partial\nabla^2\psi}{\partial\theta}-\frac{\partial}{\partial r}\nabla^2\psi\frac{\partial\psi}{\partial\theta},\\
\nabla^2 &= \frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{\partial^2 r^2}{\partial\theta^2}.
\end{aligned}
$$

We expand ψ in the form

$$\psi=\psi_0+\varepsilon\psi_1+\varepsilon^2\psi_2+\dots .$$

Substituting this series into Eq. (2.4), we have from the $0(\varepsilon^0)$ terms

$$\left(\frac{\partial}{\partial\tau}-\nabla^2\right)\nabla^2\psi_0 = 0. \tag{2.5}$$

The boundary conditions are

$$\left(\frac{\partial\psi_0}{\partial r}\right)_{r=1} = 0, \quad \left[-\frac{1}{r}\left(\frac{\partial\psi_0}{\partial\theta}\right)\right]_{r=1} = 0,$$

$$\left(\frac{\partial\psi_0}{\partial r}\right)_{r\to\infty} = [-u(t)+g(r,\tau)]_{r\to\infty}\sin\theta,$$

$$\left[-\frac{1}{r}\frac{\partial\psi_0}{\partial\theta}\right]_{r\to\infty} = [u(t)f(r,\tau)]_{r\to\infty}\cos\theta,$$

where $u(t) = U(t)/[U(t)]_{\max}$, $g(r,\tau)$ and $f(r,\tau)$ are as yet unknown "penalty functions" which must remain smaller than order one if the boundary condition at infinity is to be satisfied to the zero-th order.

The solution of Eq. (2.5) with its boundary conditions is [19]

$$\psi_0 = f_0(r,\tau)\sin\theta,$$

$$f_0 = u(\tau)\left(\frac{1}{r}-r\right)+\frac{1}{r}\int_1^r \chi_0(\omega,\tau)w\,dw,$$

$$\chi_0 = 2\int_0^\tau u(\lambda)\overline{\chi}_0[r,(\tau-\lambda)]d\lambda,$$

$$\overline{\chi}_0(r,\tau) = 1+\frac{2}{\pi}\int_0^\infty e^{-w^2\tau}\frac{J_0(wr)Y_0(w)-Y_0(wr)J_0(w)}{J_0^2(w)+Y_0^2(w)}\frac{dw}{w}$$

$$= 1+\frac{2}{\pi}\int_0^\infty e^{-w^2\tau}C_0(w,r)\frac{dw}{w},$$

$$f = \lim_{r\to\infty} -\frac{1}{r^2}\left[\int_1^r \chi_0(s,\tau)s\,ds+u(\tau)\right], \quad g = f+\lim_{r\to\infty}\chi_0(r,\tau).$$

For the special case of harmonic oscillation, it can be shown that $f = g = 0$ for all τ [19]. It is most likely that f and g remain zero for the case of nonharmonic fluctuations, although a rigorous verification must be given *a posteriori*. For the present expansion $\psi = \varepsilon^n\psi_n$ to be valid up to $0(\varepsilon^n)$, $u(\tau)$ must be such that both f and g remain $0(\varepsilon^{n+1})$. Otherwise, we will end up with a parabolic equation with conditions specified both at $r = a$ and $r\to\infty$ in the $0(\varepsilon^{n+1})$ solution. This is an over-specification of the boundary condition and the present method of successive approximation cannot be continued.

The differential system for the $0(\varepsilon)$ solution is

$$\left(\frac{\partial}{\partial\tau}-\nabla^2\right)\nabla^2\psi_1 = \frac{R}{2}\sin 2\theta\cdot F_1(r,\tau), \tag{2.6}$$

(2.7) $$\left(\frac{\partial\psi_1}{\partial r}\right)_{r=1} = 0, \quad \left[-\frac{1}{r}\frac{\partial\psi}{\partial\theta}\right]_{r=1} = 0, \quad \left(\frac{\partial\psi_1}{\partial r}\right)_{r\to\infty} = 0, \quad \left[-\frac{1}{r}\frac{\partial\psi_1}{\partial\theta_r}\right]_{r\to\infty} = 0,$$

where

$$F_1(r,\tau) = -r^{-1}\left[\chi_{0r}\left(\chi_0 - u - ur^{-2} - r^{-2}\int_1^r w\chi_0\,dw\right) + \chi_{0rr}\left(-ur^{-1} + ur - r^{-1}\int_1^r w\chi_0\,dw\right)\right].$$

The solution of Eq. (2.6) can be written as

$$\psi_1 = f_1 R\sin 2\theta,$$

where f_1 must satisfy

(2.8) $$\left(\frac{\partial}{\partial\tau} - D^2\right)D^2 f_1 = \frac{1}{2}F_1(r,\tau),$$
$$D^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} - \frac{4}{r^2}.$$

By use of the method of variation of parameters, we can integrate Eq. (2.8) to give

(2.9) $$\left(\frac{\partial}{\partial\tau} - D^2\right)f_1 = \frac{1}{2}G_1(r,\tau),$$

where

$$G_1(r,\tau) = \frac{1}{4}\left[r^2\int_1^r \frac{F_1}{r}\,dr - r^{-2}\int_1^r r^3 F_1, dr\right].$$

By use of the transformation

$$f_1 = \chi_1 - 2r^{-2}\int_1^r \chi_1 s\,ds,$$

Eq. (2.9) can be reduced to

$$\frac{\partial\chi_1}{\partial\tau} - \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\chi_1}{\partial r}\right) = \frac{G_1}{2} + \int_1^r \frac{G_1}{s}\,ds = G(r,\tau).$$

The solution of this equation with the boundary condition corresponding to Eq. (2.7) is

$$\chi_1 = \int_0^\tau\int_1^\infty G(r',\lambda)\,e^{-w^2(\tau-\lambda)}C_1(w,r)C_1(w,r')\,wr'\,dw\,dr'\,d\lambda,$$

where

$$C_1(w,r) = \frac{J_0(wr)Y_1(w) - Y_0(wr)J_1(w)}{[J_1^2(w) + Y_1^2(w)]^{1/2}}.$$

Similarly, the higher order solutions can be obtained.

2.3. Temperature field

The solution to the energy equation is sought in the form

$$\Theta = \tau_0+\varepsilon\tau_1+\varepsilon^2\tau_2+ \dots .$$

Substituting this series expansion into the third equation of the set (2.2) and collecting the $0(\varepsilon^0)$ terms, we have

$$\left(P_r\frac{\partial}{\partial\tau}-\nabla^2\right)\tau_0 = 0. \tag{2.10}$$

Only the case of constant wall temperature will be illustrated here. The method of solution for other types of boundary conditions remains essentially the same. Thus the constant wall temperature boundary condition is

$$\tau_0(1, \tau) = 1, \quad \tau > 0,$$
$$\tau_0(1, \tau) = 0, \quad \tau \leqslant 0.$$

The solution of Eq. (2.10) with the above condition is

$$\tau_0 = 1+\frac{2}{\pi}\int\limits_0^\infty \exp[-w^2\tau/P_r]C_0(w, r)dw/w.$$

Similarly, the governing equation for the $0(\varepsilon)$ solution is

$$\left(\nabla^2-P_r\frac{\partial}{\partial\tau}\right)\tau_1 = P_rR\left(\frac{\partial\psi_0}{\partial r}\frac{\partial\tau_0}{r\partial\theta}-\frac{1}{r}\frac{\partial\psi_0}{\partial\theta}\frac{\partial\tau_0}{\partial r}\right). \tag{2.11}$$

The solution of this equation can be written as

$$\tau_1 = P_rRJ(r, \tau)\cos\theta.$$

Substitutions of the above into Eq. (2.11) gives

$$\left(P_r\frac{\partial}{\partial\tau}-\frac{\partial^2}{\partial r^2}-\frac{1}{r}\frac{\partial}{\partial r}+\frac{1}{r^2}\right)J = H(r, \tau), \tag{2.12}$$

where

$$H(r, \tau) = \frac{2}{\pi r}f_0(r, \tau)\int\limits_0^\infty \exp[-\omega^2\tau/P_r]\frac{J_0'(wr)Y_0(w)-Y_0'(wr)J_0(w)}{J_0^2(w)+Y_0^2(w)}dw,$$

and the prime denotes differentiation of the Bessel function with respect to its argument. By use of the integral transform

$$J = \frac{1}{r}\int\limits_1^r Ysds \tag{2.13}$$

Eq. (2.12) can be reduced to

$$P_r\frac{\partial Y}{\partial\tau}-\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial Y}{\partial r}\right) = \frac{1}{r}\frac{\partial}{\partial r}[H(r, \tau)r] = \frac{1}{r}H_1(r, \tau), \tag{2.14}$$

if $Y_r(1, \tau) = 0$. Note that the condition $\tau_1(1, \tau) = 0$ is satisfied since $J(1, \tau) = 0$ according to Eq. (2.13). Thus the side conditions for Eq. (2.14) are

$$Y_r(1, \tau) = 0,$$

and

$$J(\tau, r) = 0, \quad \tau \leqslant 0 \text{ i.e.,} \quad Y(\tau, r) = 0, \quad \tau \leqslant 0.$$

The solution of Eq. (2.14) with the above conditions is

$$Y(r, \tau/P_r) = \int\limits_1^\infty \int\limits_0^{\tau/Pr} \int\limits_0^\infty H_1(r', \lambda) C_1(w, r') C_1(w, r) \exp\left[-w^2\left(\frac{\tau}{P_r} - \lambda\right)\right] w d w d\lambda \, dr'.$$

Similarly, for the $0(\varepsilon^2)$ solution, we have

$$\left(\nabla_2 - P_r \frac{\partial}{\partial \tau}\right) \tau_2 = \frac{(P_r R)^2}{r} [G_M \cos^2\theta + G_N \sin^2\theta + G_Q \cos 2\theta], \tag{2.15}$$

where

$$G_M = f_0 \frac{\partial J}{\partial r}, \quad G_N = J \frac{\partial f_0}{\partial r}, \quad G_Q = 2f_1 \frac{\partial \tau_0}{\partial r}.$$

The solution of Eq. (2.15) can be written as

$$\tau_2 = (P_r R)^2 [M(r, \tau) \cos^2\theta + N(r, \tau) \sin^2\theta + Q(r, \tau) \cos 2\theta].$$

Equation (2.15) then demands that M, N and Q satisfy

$$\left(P_r \frac{\partial}{\partial \tau} - \frac{1}{r} \frac{\partial}{\partial r} - \frac{\partial^2}{\partial r^2}\right) \begin{bmatrix} M \\ N \end{bmatrix} = \frac{1}{r} \begin{bmatrix} G_M \\ G_N \end{bmatrix}, \tag{2.16}$$

$$\left(P_r \frac{\partial}{\partial \tau} - D^2\right) Q = \frac{1}{r}\left[G_Q - \frac{2}{r}(M+N)\right] = \frac{1}{r} L(r, \tau). \tag{2.17}$$

The solution of Eq. (2.16) with its homogeneous boundary conditions yields [20]

$$\begin{bmatrix} M \\ N \end{bmatrix} = \int\limits_1^\infty \int\limits_1^{\tau/R_r} \int\limits_0^\infty \exp\left[-w^2\left(\frac{\tau}{P_r} - \lambda\right)\right] \begin{bmatrix} G_M(r', \lambda) \\ G_N(r', \lambda) \end{bmatrix} C_1(w, r) C_1(w, r') w d w d\lambda d r'.$$

The solution of Eq. (2.17) with homogeneous boundary conditions is

$$Q = Z - \frac{2}{r^2} \int\limits_1^r Z s ds,$$

where

$$Z = \int\limits_0^{\tau/P_r} \int\limits_1^\infty L(r', \lambda) \int\limits_0^\infty \exp\left[-w^2\left(\frac{\tau}{P_r} - \lambda\right)\right] C_1(w, r) C_1(w, r') w d w d r' d\lambda.$$

The heat transfer q from the cylinder to the surrounding fluid is then given by

$$(2.18)\qquad q = -k\Delta T\int\limits_0^{2\pi}\left[\frac{\partial}{\partial r}\left(\tau_0+\varepsilon\tau_1+\varepsilon^2\tau_2\right)+0(\varepsilon^3)\right]_{r=1} d\theta$$

$$= -k\Delta T[2\pi\tau_{0r}(1,\tau)+\varepsilon^2\pi P_r R\{M_r(1,\tau)+N_r(1,\tau)\}]+0(\varepsilon^3),$$

where the subscript r denotes differentiation.

3. Specific problems

3.1. Initial drag

Consider the drag force $\mathbf{D}$ per unit length of a cylinder which starts from rest a motion described by $u(\tau)$. During the initial stage when the distance traveled by the cylinder is much smaller than the cylinder diameter, the fluid velocity field is approximated by ψ_0. The corresponding stress field can be readily obtained and integrated over the cylinder surface to give [13]

$$(3.1)\qquad \mathbf{D} = \pi\varrho\nu U_m[\dot{u}(\tau)-2\chi_{0,}(1,\tau)],$$

where $U_m = [U(\tau)]_{\max}$ and the subscript r denotes differentiation with respect to r. For $\tau \ll 1$, we have

$$\overline{\chi}_{0r}(1,\tau) = -(\pi\tau)^{-1/2}-\frac{1}{2}+\frac{1}{4}(\tau/\pi)^{1/2}-\frac{1}{8}\tau-325\tau^{3/2}\Big/\left\{1024\Gamma\left(\frac{5}{2}\right)\right\}+0(\tau^2),$$

and

$$(3.2)\qquad \mathbf{D} = \pi\varrho\nu U_m\Bigg[\dot{u}+4\int\limits_0^{\tau}\dot{u}(\lambda)\{\pi(\tau-\lambda)\}^{-1/2}d\lambda+2\{v(\tau)-v(0)\}$$

$$-\pi^{-1/2}\int\limits_0^{\tau}\dot{u}(\lambda)(\tau-\lambda)^{1/2}d\lambda+\frac{1}{2}\int\limits_0^{\tau}\dot{u}(\lambda)(\tau-\lambda)d\lambda$$

$$+\frac{325}{256\Gamma\left(\frac{5}{2}\right)}\int\limits_0^{\tau}\dot{u}(\lambda)(\tau-\lambda)^{3/2}d\lambda+0(\tau^2)\Bigg].$$

For the special example of impulsively started uniform translation, $u(\tau) = H(\tau)$ where $H(\tau)$ is the Heaviside function, we have

$$\mathbf{D} = \varrho\nu U_m[\pi\dot{H}(\tau)+4\pi^{1/2}\tau^{-1/2}+2\pi-(\pi\tau)^{1/2}+\frac{1}{2}\pi\tau = \frac{325\pi}{256\Gamma\left(\frac{5}{2}\right)}\tau^{3/2}+0(\tau^2).$$

The first term is the added mass drag $\pi\varrho\nu U_m\dot{H}(\tau) = \pi a^2\varrho U_m\dot{H}(t)$. The second and the third terms were obtained by WANG [21] who omitted the added mass term and used the method of matched asymptotic expansions. This drag formula also agrees up to $0(\tau^{1/2})$

with that of COLLINS and DENNIS [22, 23] who also omitted the added mass term and used double series expansion in a diffusion time and convection time.

For other $u(\tau)$, **D** can be obtained from Eq. (3.2), which is a new contribution by the writer.

3.2. Damped vibration of a string

Consider a string of radius a stretched along the x axis under a tension of magnitude S. Small amplitude vibration of such a string is governed by

$$\pi a^2 \varrho_S y_{tt}(x,t) = S y_{xx}(x,t) + D(x,t),$$

where ϱ_S is the density of the string, $y(x, t)$ the transverse displacement of the string at the station x at time t, D the drag per unit length of the string. With D given locally by Eq. (3.1) the writer solves numerically the above integral differential equation [19]. It was shown that the usual theory based on the quasi-steady drag formula overestimates considerably the period and the decay rate of damped vibration of a string in a viscous fluid.

3.3. Separation point

The separation occurs at the point where the shear stress $\sigma_{r\theta}$ vanishes, i.e.

$$\sigma_{r\theta} = \mu(U/a)\left(\frac{\partial u_r}{r\partial\theta} - \frac{u_\theta}{r} + \frac{\partial u_\theta}{\partial r}\right)_{r=1} = 0.$$

In terms of the stream function, the above equation can be written as

$$\frac{\partial^2}{\partial r^2}(\psi_0 + \varepsilon\psi_1) + 0(\varepsilon^2) = 0.$$

Substitution of the expressions for ψ_0 and ψ_1 into the above equation leads to

$$\cos\theta(\tau) = \frac{-\chi_{0r}(1,\tau)}{2\chi_{1rr}(1,\tau)\varepsilon R}, \tag{3.3}$$

which gives the location of the separation point, as a function of time, on a circular cylinder which starts its motion from rest. Of course this result remains valid as long as $\frac{1}{a}\int\limits_0^t \mathrm{U}(t)\,dt \ll 1$. Thus, for small amplitude high-frequency oscillation the above results may be valid for indefinite periods of time. Note that $\chi_{0r}(1,\tau) \to 0$ as $\varepsilon^n \to 0$, $n = 1$.

3.4. Heat transfer

The heat transfer from a circular cylinder in a fluctuating flow is given by Eq. (2.18). In terms of the Nusselt number Nu Eq. (2.18) can be written as

$$\mathrm{Nu} = \frac{q}{-k\Delta T} = \tau_{0r}(1,\tau) + \frac{\varepsilon^2}{2}(P_r R)^2[M_r(1,\tau) + N_r(1,\tau)] + 0(\varepsilon^3). \tag{3.4}$$

It should be pointed out that this equation does not state that $\mathrm{Nu} \sim P_r^2$, since P_r also appears in the integrands of M_r and N_r.

A very important application of the present results will be found in the area of hot-film anemometry. The numerical computation based on Eq. (3.4) may be used to give a theoretical estimate of the difference between static and dynamic calibration [6, 17, 24–26] of hot-film in the range of high frequency spectrum.

References

1. J. COLE, A. ROSHKO, *Heat transfer from wires at Reynolds numbers in the Oseen range*, Proceeding of Heat Transfer and Fluid Mechanics Institute, 13, 1954.
2. C. R. ILLINGWORTH, *Flow at small Reynolds number*, in: *Laminar boundary layers* [ed. L. ROSENHEAD], Clarendon Press., Oxford 1963.
3. W. W. WOOD, *Calculation for anemometry with fine hot wires*, J. Fluid Mech., **32**, 9, 1968.
4. C. A. HIEBER, B. GEBHART, *Low Reynolds number heat transfer from a circular cylinder*, J. Fluid Mech., **32**, 21, 1968.
5. W. W. WOOD, *Free and forced convection from fine hot wires*, J. Fluid Mech., **55**, 419, 1972.
6. S. CORRSIN, *Turbulence. Experimental methods*, Handb. Phys., 812, 523, Springer, Berlin 1963.
7. D. C. COLLIS, M. J. WILLIAMS, *Two-dimensional convection from heated wires at low Reynolds numbers*, J. Fluid Mech., **6**, 357, 1959.
8. B. GEBHART, L. PERA, *Mixed convection from long horizontal cylinders*, J. Fluid Mech., **45**, 49, 1970.
9. N. HATTORI, *Forced convection heat transfer from a uniformly heated cylinder at relatively low Reynolds numbers*, Trans. Japan Soc. Mech. Eng., **41** (348), 2429, 1975.
10. E. M. SPARROW, L. LEE, *Analysis of mixed convection about a horizontal cylinder*, Int. J. Heat Mass Transfer, **19**, 229, 1976.
11. J. H. MERKIN, *Mixed convection from a horizontal circular cylinder*, Int. J. Heat Mass Transfer, **20**, 73, 1977.
12. H. G. DAVIES, *Fluctuating heat transfer from hot wires in low Reynolds number fow*, J. Fluid Mech., **73**, 39, 1976.
13. S. P. LIN, *Initial drag on a cylinder*, Q. Jl. Mech. Appl. Math., **29**, 61, 1976.
14. M. COUTANCEAU, R. BOUARD, *Experimental determination of the main features of the viscous flow in the wake of a circular cylinder in uniform translation, Part 2, Unsteady flow*, J. Fluid Mech., **79**, 257, 1977.
15. M. BAR-LEV, H. T. YANG, *Initial flow field over an impulsively started circular cylinder*, J. Fluid Mech., **72**, 625, 1975.
16. G. ANTONINI, G. GUIFFANT, P. PEROT, *The effect of transverse oscillation on heat transfer from a horizontal hot-wire to a liquid, Holographic visualization*, Int. J. Heat Mass Transfer, **20**, 88, 1977.
17. G. COMTE-BELLOT, *Hot wire anemometry*, Annual Review of Fluid Mech., **8**, 209, 1976.
18. L. LANDAU, E. M. LIFSHITZ, *Fluid mechanics*, p. 91, Pergamon 1959,
19. S. P. LIN, *Damped vibration of a string*, J. Fluid Mech., **72**, 787, 1975.
20. H. S. CARSLAW, J. C. JAEGER, *Conduction of solids*, p. 370. Oxford 1959.
21. C. Y. WANG, *The flow past a circular cylinder which is started impulsively from rest*, J. Math. Phys., **46**, 195, 1967.
22. W. M. COLLINS, S. C. R. DENNIS, *The initial flow past an impulsively started circular cylinder*, Q. Jl. Mech. Appl. Math., **26**, 53, 1973.
23. W. M. COLLINS, S. C. R. DENNIS, *Flow past an impulsively started circular cylinder*, J. Fluid Mech., **60**, 105. 1973.
24. H. H. BRUUN, *A note on static and dynamic calibration of constant-temperature hot-wire probes*, J. Fluid Mech., **76**, 145, 1976.
25. A. E. PERRY, G. L. MORRISON, *Static and dynamic calibration of constant-temperature hot-wire systems*, J. Flud. Mech., **47**, 765, 1971.
26. Y. K. CHU, *A hot wire calibration facility*, Report No. MIE-024, Clarkson College of Technology, 1977.

CLARKSON COLLEGE OF TECHNOLOGY, POTSDAM, N.Y., USA.

Received October 15, 1977.