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Velocity profiles of suspension flowing through a tube(*)

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THE VELOCITY profiles of solid particle and liquid drop suspensions flowing through tubes of 10 and 14 mm int. diameters have been measured by means of the ultrasound Doppler technique. The relative particle radii varied from $10^{-3}-5\cdot10^{-2}$, particle Reynolds numbers from $10^{-1}-10^{-7}$, and the volume concentration varied from 5% to 50%. It was observed that velocity profiles were blunted. The power N entering the empirical formula for velocity $V = V_0(1-r^N)$ was taken as an indicator of the profile blunting. Results for solid particle suspensions show that N increases with the concentration of suspension and with the relative size of particles. For suspensions of droplets the results depend also on the viscosity ratio of both phases.

Wykorzystując ultradźwiękową technikę doplerowską, zmierzono profile prędkości zawiesiny cząstek sztywnych i kropel w przepływie przez rurki o wymiarach 10 i 14 mm. Względny wymiar cząstek zmieniał się w granicach $10^{-3}-5\cdot10^{-2}$, liczba Reynoldsa dla cząstek od $10^{-1}-10^{-7}$, a koncentracja objętościowa zmieniała się od 5% do 50%. Zaobserwowano spłaszczenia profili prędkości. Jako wskaźnik wielkości spłaszczenia przyjęto wykładnik N w empirycznym wyrażeniu na profil prędkości typu: $V = V_0(1-r^N)$. Zaobserwowano, że wartość N rośnie dla cząstek sztywnych wraz z koncentracją globalną zawiesiny i względnym wymiarem cząstek. Dla zawiesiny kroplowej wielkość spłaszczenia zależy również od stosunku lepkości obu faz zawiesiny.

Используя ультразвуковую доплеровскую технику, измерены профили скорости взвеси жестих частиц и капель в течении через трубки с размерами 10 и 14 мм. Относительный размер частиц изменялся в пределах $10^{-3}-5\cdot10^{-2}$, число Рейнольдса от 10^{-1} до 10^{-7}, а объемная концентрация изменялась от 5% до 50%. Наблюдалась сплюснутость профилей скорости. Как показатель величины сплюснутости принят показатель N в эмпирическом выражении для профиля скорости: $V = V_0(1-r^N)$. Обнаружено, что значение N растет для жестких частиц совместно с глобальной концентрацией взвеси и относительным размером частиц. Для капельной взвеси величина сплюснутости зависит тоже от отношения вязкости обоих фаз взвеси.

Notation

$P(f)$ power spectral density of a Doppler signal,
Q volume flow rate of suspension flowing through a tube,
R tube radius,
Re_T tube Reynolds number $= \dfrac{2\bar{V}R}{\nu_c}$,
Re_p particle Reynolds number $= \dfrac{4}{3}b\left(\dfrac{b}{R}\right)^2\dfrac{\bar{V}}{\nu_c}$,
$V(r)$ velocity of suspension,
$\bar{V}$ mean suspension velocity $= Q/\pi\cdot R^2$,
V_0 maximum velocity in the tube,
$\tilde{V}$ nondimensional velocity of suspension $= V\pi/Q$,
b particle radius,
c volume concentration,

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f frequency of Doppler signal,
f_m mean Doppler frequency,
f_d measured mean Doppler frequency,
r radial distance from the tube axis relatively to the tube radius,
ν_c, ν_d kinematic viscosity of suspending and suspended phase,
λ viscosity ratio $= \nu_d/\nu_c$,
τ tangential shear stress,
μ_0 constant.

OBSERVATIONS of the flow of blood [1] and later of the flow of solid suspensions [2] through narrow tubes reveal that the concentration of particles and the effective viscosity of suspension within a capillary are smaller than in the supplying container. This phenomenon may be associated with the so-called "tubular pinch effect" [3]: "in the low Reynolds number flow of dilute suspensions through a tube solid particles migrate to some "neutral" point between the axis of the tube and its wall".

Unlike the rigid particles, even for a nearly zero Reynolds numbers flow ($Re_p < 10^{-6}$), the suspended deformable liquid drops migrate to the axis of the tube [4] at rates increasing both with the size of droplets, with their deformability and distance to the tube axis.

The problem of lateral migration of single particles in the flow has been theoretically considered [5, 6] but up to now no successful quantitative comparison with the experimental data has been obtained. The situation is even worse for the case of concentrated suspensions. Thus the problem of concentration and velocity profiles of dense suspensions should be treated experimentally.

The main aim of the experiments reported below was to check the applicability of the ultrasound technique to the study of velocity profiles of dense suspensions flowing through the tube. To our best knowledge this technique has not been employed previously in such type of experiments. Almost all known velocity measurements of suspensions have been performed by a Canadian group headed by H. L. Goldsmith and S. G. Mason [7, 8, 9] by the straitghforward method of observation of single tracer particles. The main results are that for sufficiently large particles and for high concentrations the velocity profile of suspensions flowing through the tube becomes flattened in its central part.

Later MCMAHON and PARKER [10] tried to replace the tedious method of Goldsmith and Mason by a more sophisticated one based on microwave Doppler effects. However, as relatively long wavelengths were used, they obtained only a Doppler spectrum from the whole tube and were unable to detect the local velocities of the suspension.

The best experimental method would be the Laser Doppler Anemometry (LDA), the applicability of which, for drop suspensions, has been confirmed in our earlier experiments [11]. As far as we know, no systematic studies of the flow of the dense suspension using this method have been reported. It may be due to the fact that the method has one serious disadvantage—both phases must be carefully matched optically.

In contrast, the Ultrasound Doppler Anemometry (UDA) which we employ in the present work, requires only a rough matching of the acoustic properties of both phases of the suspension. Measurements are based on detection of the Doppler shift of the frequency of the ultrasound beam scattered at a given point in the flow. Due to the relatively

low speed of the wave propagation, the time of echo return can be measured (similar as in the case of radar technique). Thus it enables us to record automatically the velocity as a function of position.

We used the apparatus constructed in the Department of Ultrasound Techniques of the Institute of Fundamental Technological Research [12]. The basic parameters of this apparatus are: frequency—5 MHz, transmitter diameter—3 mm, sampling time—1 μs: for a typical sound speed the spatial resolution which follows is about 0.7 mm along the direction of propagation of the ultrasound beam.

The accuracy of the Doppler frequency measurements is within a 1% error. However, due to the existence of the velocity gradients in the sampled volume and the presence of noise, the measured Doppler frequencies cover the whole frequency spectrum. If $P(f)$ is the power spectral density of detected frequency, the mean Doppler freequency is defined by

$$f_m = \frac{\int_{-\infty}^{\infty} fP(f)df}{\int_{-\infty}^{\infty} P(f)df}. \tag{1}$$

As automatic measurement of f_m is very complicated, usually a simpler, so-called "zero crossing method" is employed; it gives the values f_d:

$$f_d = \left[\frac{\int_{-\infty}^{\infty} f^2P(f)df}{\int_{-\infty}^{\infty} P(f)df}\right]^{1/2}. \tag{2}$$

The difference between f_m and f_d depends on the frequency spectrum $P(f)$ and only for $P(f)$ having the shape of δ function f_m and f_d coincide. For the symmetrical frequency spectrum with a mean width σ an approximate formula is [13]

$$f_d = (f_m^2+\sigma^2)^{1/2}. \tag{3}$$

This method thus gives the value of f_m with an error, the value of which depends on the width of the measured frequency spectrum. In our case an overall error varies then from about 5% at the tube axis to more than 20% at the tube wall. However, as we used the calibration measurements performed for parabolic velocity profiles, the accuracy of processed results is somewhat better—especially for data close to the tube wall.

The number N entering the formula

$$V = V_0(1-r^N) \tag{4}$$

was taken as the indicator of the profile blunting.

As the rate of the flow Q is known, the maximum velocity V_0 can be found as well and Eq. (4) in nondimensional form is

$$\tilde{V} = \frac{N+2}{N}(1-r^N). \tag{5}$$

The adjustment of experimental data to Eq. (5) was done by the least square method, for a set of 3 to 7 simultaneous velocity profiles measurements. The value of N is estab-

lished by this procedure within a 5% error. Examples of such approximating are given in Fig. 1. Our experiments show that the parameter N can be safely used for characterization of measured velocity profiles.

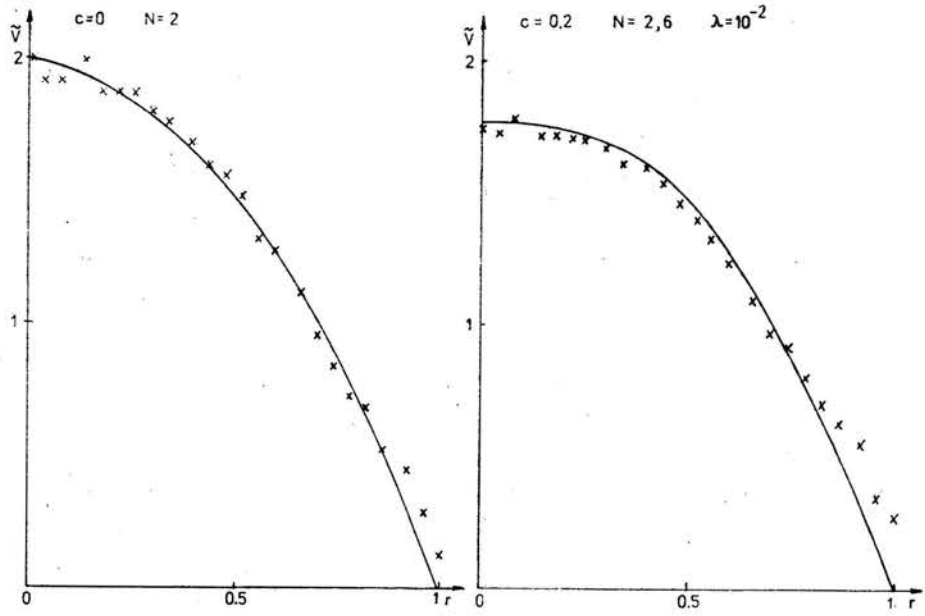

FIG. 1. Dimensionless velocity profiles of suspension of liquid drops for concentrations: $c = 0$ and $c = 0.2$. The solid lines are the best fit through the experimental points of the approximating function (5).

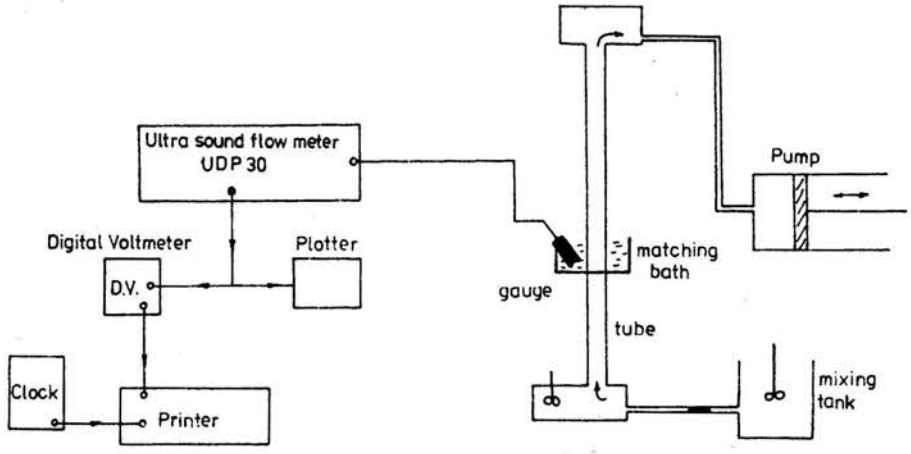

FIG. 2. Schematic diagram of the apparatus.

The scheme of the experimental apparatus is shown in Fig. 2. Plexi tubes, 150 cm long, 10 and 14 mm internal diameter were used and the flow rates were controlled by the constant stroke pump in a 20–50 cm^3/s range. Measurements were carried out for three systems of suspensions:

(i) liquid-liquid suspension with viscosity ratio $\lambda \ll 1$.
(ii) liquid-liquid suspension with viscosity ratio $\lambda \gg 1$.
(iii) solid particle suspension indicated as viscosity ratio $\lambda = \infty$.

The components of liquids used were chosen for the best matching of acoustic properties of both phases with minimum density difference. The chemical and physical data of suspensions used are collected in Table 1.

Table 1

components of dispersed phase	$//_d$	glycerine+ water	castor oil+ d. phtalate+ m. phtalate	methacrylate spheres
components of cont. phase	$//_c$	castor oil+ dib. phtalate	glycerine+ water	glycerine+ water
viscosity ratio	λ	10^{-2}	$2 \cdot 10^2$	"∞"
density [g/cm^3]	ϱ_d ϱ_c	1.01 1.01	1.02 1.02	1.19 1.25
viscosity [cP]	ν_d ν_c	1.2 100	200 1.3	— 800
particle relative radius	b/R	$3.10^{-3}-2.10^{-2}$	$2.10^{-2}-5.10^{-2}$	$10^{-2}-2.5\cdot10^{-2}$
tube Re number	Re_T	~ 10	~ 800	~ 1
particle Re number	Re_p	$10^{-6}-10^{-7}$	$10^{-2}-10^{-1}$	$10^{-6}-10^{-5}$
sound velocity [m/s]	C_d C_c	1520 1580	1580 1525	2350 1900
acoustic susceptibility [g/m^{2}s]	$C_d \cdot \varrho_d$ $C_c \cdot \varrho_c$	$1.6 \cdot 10^{-3}$	$1.6 \cdot 10^{-3}$	$2.7 \cdot 10^{-3}$ $2.4 \cdot 10^{-3}$

Suspensions of drops were produced by injecting one liquid into another through a fine bore needle and by subsequent mixing in a special tank. Special care was taken to avoid the coalescence of droplets and to obtain suspensions whose structures wouldn't change during the experiments. This was checked by measuring the distribution of droplet diameters before and after each experiment. For suspensions of solid particles screen-fractionated samples of polimethacrylate spheres were used.

The dependence of the profile blunting characterized by the parameter N as a function of concentration c is shown in Figs. 3–5.

The results shown in Fig. 3 for a solid particle suspension indicate that the profile blunting observed increases with the concentration. Within the accuracy of an experimental error, the value of blunting is independent, in the tested range, on the rate of flow. As it can be seen, for larger particles profile blunting at the same concentration is higher. For particles of a similar relative dimension the experimental results of KARNIS *et al.* [7] (triangles in Fig. 3) are close to our measurements. The data obtained by MCMAHON

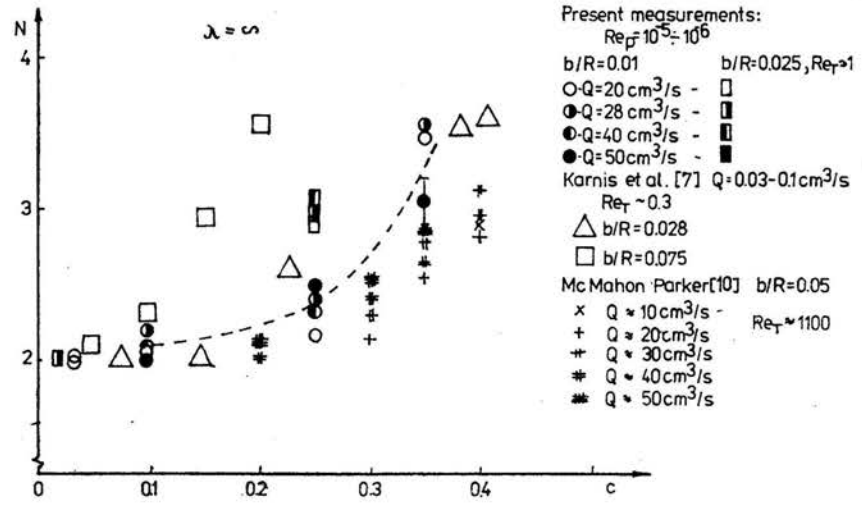

FIG. 3. Profile blunting for solid particle suspension, vs. concentration.

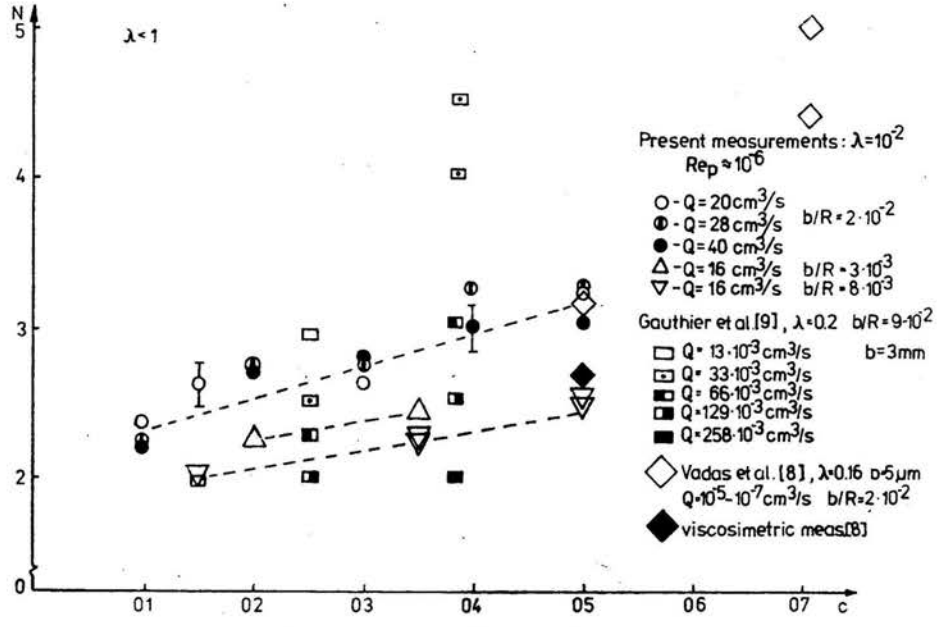

FIG. 4. Profile blunting for liquid—liquid suspension, $\lambda \ll 1$, vs. concentration.

and PARKER [10] by the microwave method are much below these results; these data contradict other measurements, because for larger particles and higher Reynolds numbers as those which were used, higher values of N should be expected.

In Fig. 4 the results for a drop suspension with the viscosity ratio $\lambda \ll 1$ are presented. The profile blunting is smaller than for a comparable suspension of solid particles, but also increases with the size of particles and with their concentration. The results of GAUT-

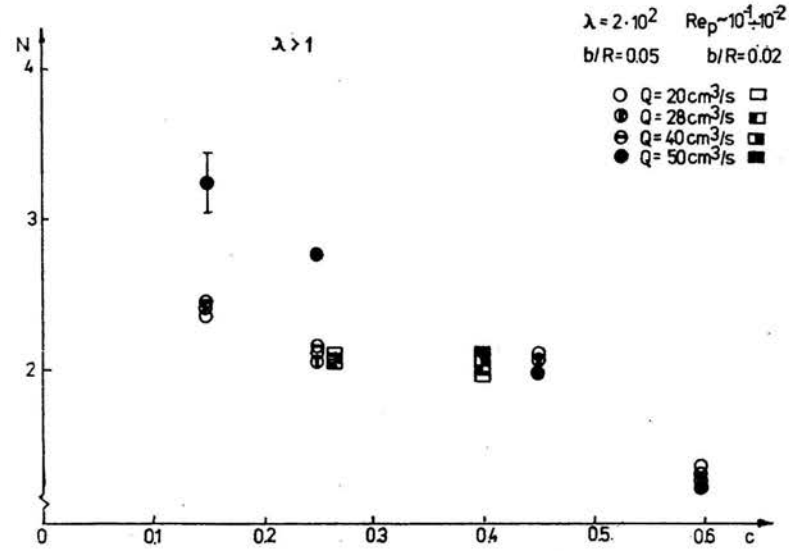

FIG. 5. Profile blunting for liquid—liquid suspension, $\lambda \gg 1$, vs. concentration.

HIER *et al.* [9] for a suspension of 3 mm drops, and of VADAS *et al.* [8] for a 5 μm emulsion are also shown in this figure. For a suspension with highly deformable large drops [9], the profile blunting depends strongly on the rate of flow. Our data are closer to that of the emulsion because of the comparable relative size of particles. In both cases no significant influence of the rate of flow on the velocity profile was observed.

The third system considered (Fig. 5) was that of a suspension of drops with the viscosity ratio $\lambda \gg 1$. The observations were made for the flow at much higher Reynolds numbers (~ 800) but we were unable to perform measurements at lower flow rates. The observed profile blunting for the medium concentration range decreases with concentration. This fact to our best knowledge has not been reported before.

Our results can be summarized as follows: the presence of solid particles and droplets with the viscosity ratio $\lambda \ll 1$ in the flow at low Reynolds numbers leads to blunting of the velocity profile. The degree of blunting depends mainly on the relative dimension of particles and their concentration. For suspensions of drops, blunting is relatively smaller and depends also on the viscosity ratio.

The observed blunting of the velocity profiles may perhaps be related to the non-Newtonian behaviour of suspension. It was theoretically shown [14] that such behaviour should be expected even for the dilute suspension of liquid drops. There are also several experimental observations of rheological behaviour suspensions of solid particles [15]. In viscosimetric measurements [2, 8, 16] suspensions usually behave as pseudoplastic liquids, the viscosity of which decrease at higher shear rates. For a power-law model of such a liquid the shear rate — shear stress relation is

$$\tau = \mu_0 \left(\frac{dV}{dr}\right)^n \tag{6}$$

and the exact solution for the velocity profile in the flow through a tube becomes

$$V = V_0(1-r^N), \quad N = \frac{n+1}{n}. \tag{7}$$

Viscosimetric measurements of an emulsion presented by VADAS *et al.* [8] indicate a nonlinear relation between shear-stress and shear rate. The value of the exponent N, as determined from this measurement (see diamonds in Fig. 3), is too low to explain the observed profile blunting. It seems therefore that also a nonuniform distribution of particles (nonuniform concentration) has an appreciable influence on the velocity profile. For a suspension of drops at $\lambda < 1$, as we observed [11], the concentration at the tube axis may be 30% higher than the average concentration, so the apparent viscosity increases in that region, too.

Theoretical analysis of the considered problem is extremely difficult due to the screening effects as well: such an effect surely occurs due to the presence of neighbouring particles which stops the particle rotation and hence the velocity gradients across the particle vanishes.

The decrease of the value of the parameter N with concentration of droplets observed for the third system ($\lambda \gg 1$) seems to contradict the results for solid particles ($\lambda = \infty$). However, it should be mentioned that droplets suspended in the shear flow as follows from Taylor's formula [17]

$$D = \frac{G \cdot b \cdot \nu_c}{k} \frac{19\lambda+16}{16\lambda+16}, \tag{8}$$

where D—parameter of deformation, G—velocity gradient, k—interfacial tension, even in the limit of the viscosity ratio $\lambda \to \infty$ remain deformable. Due to the deformability of droplets, their behaviour qualitatively differs from that of solid particles; a single solid particle migrates to some intermediate position between the tube wall and the tube axis [3], while liquid particles for $\lambda < 1$ migrate to the tube axis [4].

There are no experimental reports about the behaviour of droplets for $\lambda > 1$. But the theoretical results of CHAN, LEAL [18] suggest that such drops can migrate even towards the wall of the tube. This and also the effect of inertia (Re $\sim$ 800) can be responsible for the observed increase of concentration of highly viscous drops near the tube wall. This change of concentration is probably responsible for the rise of the velocity near the axis of the tube and, therefore, for a decrease of the value of the parameter N. Further experiments are required to elucidate these effects.

References

1. R. FAHRAEUS, T. LINDQUIST, Am. J. Physiol., **96**, 562, 1931.
2. S. SEGRÉ, A. SILBERBERG, J. Colloid Sci., **18**, 312, 1963.
3. S. SEGRÉ, A. SILBERBERG, J. Fluid Mech., **14**, 115, 1962.
4. H. L. GOLDSMITH, S. G. MASON, J. Colloid Sci., **17**, 448, 1962.
5. R. G. COX, H. BRENNER, Chem. Eng. Sci., **23**, 147, 1968.
6. B. P. HO, L. G. LEAL, Fluid Mech., **65**, 365, 1974.
7. A. KARNIS, H. L. GOLDSMITH, S. G. MASON, J. Colloid Sci., **22**, 531, 1966.

8. E. B. VADAS, H. L. GOLDSMITH, S. G. MASON, Trans. Soc. Rheol., **20**, 373, 1976.
9. F. J. GAUTHIER, H. L. GOLDSMITH, S. G. MASON, Biorheology, **9**, 205, 1972.
10. T. A. MCMAHON, R. R. PARKER, Trans. Soc. Rheol., **19**, 445, 1975.
11. T. KOWALEWSKI, IFTR Reports, 49, 1976.
12. A. NOWICKI, Ph. D. dissertation, IFTR PAN, Warsaw 1975.
13. P. PERONEAU, Ph. D. dissertation, University Paris-Sud, 1977.
14. W. R. SCHOWALTER, C. E. CHAFFEY, H. BRENNER, J. Colloid Sci., **26**, 152, 1968.
15. D. J. JEFFREYS, A. ACRIVOS, AIChEJ, **22**, 417, 1976.
16. R. L. HOFFMAN, Trans. Soc. Rheol., **16**, 155, 1972.
17. G. I. TAYLOR, Proc. Roy Soc., A**146**, 501, 1934.
18. P. CHAN, G. LEAL, J. Fluid Mech., **92**, 131, 1979.

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