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Degeneration of a shock wave into an acoustic one in a strong electromagnetic field

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A SHOCK wave, ionizing gas, propagates in a channel. A strong electric field is applied to the walls of the channel. The case of the finite magnetic Reynolds number $Re_m > 1$, when the induced magnetic field cannot be neglected, is considered. The solutions of the appropriate equations indicate that the generated magnetic field attenuates the shock wave until it degenerates into an acoustic one. The damping of the shock wave is stronger than in the case of small Re_m, when the constant outer field was applied and the induced magnetic field was neglected; this situation was considered in a previous paper.

VARIOUS patterns of flow can be observed when a shock wave, ionizing gas, propagates in a channel where a strong electric field is applied to the walls. The variety of patterns depends on the values of the parameters describing the flow and on the construction singularities of the electrical net scheme (see e.g. [1–6]). For small magnetic Reynolds numbers $Re_m < 1$, when the induced electromagnetic field can be neglected, the outer electric field accelerates the shock wave. If, additionally, an outer magnetic field of a constant intensity is applied, then an attenuation of the shock wave can be observed. Values of the parameters, for which the attenuation of the shock wave and its degeneration into an acoustic wave takes place, as well as the asymptotic laws of such a degeneration are given in [6]. The present note appears to be the continuation of the paper mentioned for the case of the finite magnetic Reynolds numbers $Re_m \geqslant 1$, when the induced magnetic field cannot be neglected. For the flow patterns given in Fig. 1, the generated magnetic field

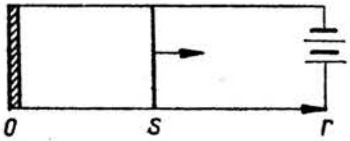

FIG. 1.

strongly attenuates the shock wave until it degenerates into an acoustic one, causing the formation of a new shock wave propagating in the opposite direction [1].

Let us consider the process of deceleration of the shock wave in this case. The flow behind the shock wave in the nondimensional variables

$$v = a_0 V, \quad \varrho = \varrho_0 R, \quad p = \varrho_0 a_0^2 P, \quad E = E_0 \mathscr{E}, \quad H = \frac{cE_0}{a_0}\mathscr{H},$$
$$t = t_0 \tau, \quad r = a_0 t_0 x, \quad t_0 = \varrho_0 a^2/\sigma_0 E^2, \quad \sigma = \sigma_0 \varphi(P, R),$$

is described by the following system of equations:

$$\frac{\partial R}{\partial \tau}+\frac{\partial RV}{\partial x}=0,$$

$$R\left(\frac{\partial V}{\partial \tau}+V\frac{\partial V}{\partial x}\right)+\frac{\partial P}{\partial x}=-\varphi\mathscr{H}(\mathscr{E}+V\mathscr{H}),\qquad \mathscr{E}=\mathscr{E}(\tau),$$

(1)

$$\frac{\partial P}{\partial \tau}+V\frac{\partial P}{\partial x}+\gamma P\frac{\partial V}{\partial x}=(\gamma-1)\varphi(\mathscr{E}+V\mathscr{H})^2,$$

$$\frac{\partial \mathscr{H}}{\partial x}=\mathrm{Re}_m\varphi(\mathscr{E}+V\mathscr{H}),\qquad \mathrm{Re}_m=\frac{4\pi\sigma_0 t_0 a_0^2}{c^2},$$

where t is the time, r is the distance from the wall of the channel, p, ϱ, v are the pressure, the density and the velocity of the gas, σ is the conductivity, E, H are the intensities of electric and magnetic field respectively, c is the velocity of light, a_0 is the sound velocity in a nondisturbed medium, t_0, σ_0, E_0 are some reference values of the respective parameters.

Following [6] and [7], the equation for the intensity of the weak shock wave $\varepsilon = 1-a_0^2/D$ can be obtained in the form

(2) $$\frac{4}{\gamma+1}\dot{\varepsilon}+\frac{2}{\gamma+1}\frac{\varepsilon}{\tau}+\frac{2\varepsilon}{\gamma+1}\varphi\mathscr{H}[\mathscr{H}-2(\gamma-1)\mathscr{E}]+\varphi\mathscr{E}[\mathscr{H}-(\gamma-1)\mathscr{E}]=0.$$

To analyse this equation, it is nesessary to know the change in time of the intensity of the magnetic field across the shock. From the equation for the magnetic induction it follows that the value of the intensity of the magnetic field in the shock wave is defined by the value of the total current in the net, that is $\mathscr{H}=J(\tau)=\int_0^{x_s} j dx$. As results from the experiments, for a constant electric field $\mathscr{E}=1$, the total current increases proportionally with time $J=\alpha\tau$. In this case Eq. (2) for sufficiently large values of time takes the form

(3) $$\dot{\varepsilon}+\frac{\varepsilon}{2}\left(\frac{1}{\tau}+\varphi\alpha^2\tau^2\right)+\frac{\gamma+1}{4}\varphi\alpha\tau=0.$$

For the constant conductivity of the medium $\varphi=1$, the solution of the equation takes the form

(4) $$\varepsilon=\frac{1}{\sqrt{\tau}}e^{-\alpha^2\tau^3/6}\left[\mathrm{const}-\frac{\gamma+1}{4}\alpha\int_{\tau_0}^{\tau}\xi^{3/2}e^{\alpha^2\xi^3/6}d\xi\right].$$

Hence the shock wave is damped very fast and it is degenerated into the acoustic wave in a finite interval of time, what can be observed in experiments. It could happen that the conductivity of the medium $\varphi(P, R)$ tends to zero behind the shock wave as the wave is attenuated, that is $\varphi\left(\frac{1}{\gamma}, 1\right)=0$. Then, for the case when the analytical dependence on the thermodynamical parameters of the medium occurs, we obtain from Eq. (2) the following equation for the intensity of the shock

(5) $$\dot{\varepsilon}+\frac{\varepsilon}{2\tau}+\frac{\varepsilon}{2}(m+n)[J(\tau)-(\gamma-1)]=0,\qquad m=\left(\frac{\partial\varphi}{\partial P}\right)_0,\qquad n=\left(\frac{\partial\varphi}{\partial R}\right)_0.$$

Hence.

$$\varepsilon = \frac{\text{const}}{\sqrt{\tau}} \exp\left\{-\frac{m+n}{2} \int_{\tau_0}^{\tau} [J(\tau)-(\gamma-1)]d\tau\right\}$$

and for $J(\tau) = \alpha\tau$ and for sufficiently large values of time we obtain

$$\varepsilon = \frac{\text{const}}{\sqrt{\tau}} e^{-\frac{m+n}{4}\alpha\tau^2}. \tag{6}$$

This means that in this case also the shock wave is damped very fast and it is degenerated into an acoustic wave.

It is interesting to take note of the stronger character of the damping of the shock wave in the induced magnetic field the formulae ((4) and (6)), as compared with the case of the applied constant outer field and small magnetic Reynolds number [6]. In the last case mentioned damping may not occur.

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