Arch. Mech., 41, 5, pp. 679–695, Warszawa 1989

# Multiaxial secondary creep behaviour of anisotropic materials(*)

J. BETTEN (AACHEN) and M. WANIEWSKI (WARSZAWA)

THE THEORY is based upon the assumption of the existence of a creep potential which can depend only on the irreducible invariants (integrity basis) of Cauchy's stress tensor $\boldsymbol{\sigma}$ or its deviator if the material is isotropic. In this paper a simplified theory is discussed, i.e., the anisotropic behaviour is described by using the linear transformation $\tau_{ij} = \beta_{ijkl}\sigma_{kl}$ instead of the actual stress tensor $\boldsymbol{\sigma}$ in the isotropic concept. The anisotropy of the material is entirely involved in the fourth rank tensor $\boldsymbol{\beta}$, the components of which are related to experimental data. For example, the orthotropic case is considered, furthermore the influence of the anisotropy on the creep behaviour of a thin-walled tube subjected to tension, torsion and internal pressure is investigated. Numerical results are compared with correspondings experimental data in order to discuss the validity of the proposed simplified theory.

Teoria jest oparta na założeniu istnienia potencjału pełzania, który w przypadku materiału izotropowego zależy tylko od nieredukowalnych niezmienników tensora naprężenia Cauchy'ego $\boldsymbol{\sigma}$ lub od jego dewiatora. W pracy dyskutowana jest uproszczona teoria, tzn. opisany jest przypadek anizotropii przy użyciu liniowej transformacji stanu naprężenia $\tau_{ij} = \beta_{ijkl}\delta_{kl}$. Anizotropia materiału w opisie pojawia się w tensorze czwartego rzędu $\boldsymbol{\beta}$, składniki którego odniesione są do danych eksperymentalnych. Rozważono przypadek ortotropii, ponadto analizowano wpływ anizotropii na pełzanie cienkościennych próbek poddanych działaniu rozciągania, skręcania i ciśnienia wewnętrznego. Wyniki teoretyczne porównano z danymi eksperymentalnymi w celu sprawdzenia przydatności proponowanej uproszczonej teorii.

Теория опирается на предположении существования потенциала ползучести, который в случае изотропного материала зависит только от неприводимых инвариантов тензора напряжений Коши $\boldsymbol{\sigma}$ или от его девиатора. В работе обсуждается упрощенная теория, т. зн. описывается случай анизотропии при использовании линейного преобразования напряженного состояния $\tau_{ij} = \beta_{ijkl}\sigma_{kl}$. Анизотропия материала в описании появляется в тензоре четвертого порядка $\boldsymbol{\beta}$, составляющие которого отнесены к экспериментальным данным. Рассмотрен случай ортотропии, кроме этого анализируется влияние анизотропии на ползучесть тонкостенных образцов, подвергнутых действию растяжения, скручивания и внутреннего давления. Теоретические результаты сравнены с экспериментальными данными с целью проверки пригодности предложенной упрощенной теории.

## 1. Introduction

FROM THE PHYSICAL point of view, the assumption of the existence of a creep potential has only limited justification [1], i.e., the creep potential hypothesis can only be used for describing the secondary creep, especially the isotropic behaviour. In this case, the creep potential is a scalar-valued tensor function of the stress tensor $\boldsymbol{\sigma}$. This function is said to be isotropic if the condition

$$F(a_{ip}a_{jq}\sigma_{pq}) \equiv F(\sigma_{ij}) \tag{1.1}$$

(*) This paper was presented at the "Fourth Polish-German Symposium on Mechanics of Inelastic Solids and Structures" held in Mogilany, September 1987.

is fulfilled under any orthogonal transformation ($a_{ik}a_{jk} = \delta_{ij}$). It is evident from the theory of isotropic tensor functions that in an isotropic medium the creep potential $F$ can be expressed as a single-valued function of the irreducible basic invariants

$$S_\nu(\boldsymbol{\sigma}) \equiv \operatorname{tr}(\boldsymbol{\sigma}^\nu) \tag{1.2}$$

or, alternatively, of the irreducible principal invariants

$$J_1(\boldsymbol{\sigma}) \equiv \sigma_{ii}, \quad J_2(\boldsymbol{\sigma}) \equiv -\sigma_{i[i]}\sigma_{j[j]}, \quad J_3(\boldsymbol{\sigma}) \equiv \sigma_{i[i]}\sigma_{j[j]}\sigma_{k[k]} \tag{1.3}$$

of the stress tensor $\boldsymbol{\sigma}$, that is,

$$F = F[S_\nu(\boldsymbol{\sigma})] \quad \text{or} \quad F = [J_\nu(\boldsymbol{\sigma})], \quad \nu = 1, 2, 3, \tag{1.4}$$

respectively.

Assuming incompressibility, it is practical to use the invariants

$$J_2(\boldsymbol{\sigma}') = S_2(\boldsymbol{\sigma}')/2 \equiv \sigma'_{ij}\sigma'_{ji}/2, \quad J_3(\boldsymbol{\sigma}') = S_3(\boldsymbol{\sigma}')/3 \equiv \sigma'_{ij}\sigma'_{jk}\sigma'_{ki}/3 \tag{1.5}$$

of the stress deviator $\sigma'_{ij} = \sigma_{ij} - \sigma_{kk}\delta_{ij}/3$, so that the creep potential (1.4) takes the form

$$F = F[J_2(\boldsymbol{\sigma}'), J_3(\boldsymbol{\sigma}')]. \tag{1.6}$$

The theory of a creep potential is based on the principle of maximum dissipation rate [2, 3, 4, 5], from which, following Lagrange's method in connection with a creep condition $F(\sigma'_{ij}) = \text{const}$ as a subsidiary condition, we obtain the flow rule

$$\dot{\varepsilon}_{ij} = \dot{\lambda}[\partial F(\boldsymbol{\sigma}')/\partial \sigma_{ij}] \tag{1.7}$$

($\dot{\varepsilon}_{ij}$—tensor of the steady-state creep rate). A creep potential (1.6) and flow rule (1.7) lead to the constitutive equation of isochoric creep behaviour:

$$\dot{\varepsilon}_{ij} = \dot{\lambda}\left[\frac{\partial F}{\partial J_2}\sigma'_{ij} + \frac{\partial F}{\partial J_3}\sigma''_{ij}\right] \tag{1.8}$$

in which

$$\sigma''_{ij} \equiv (\sigma'^{(2)}_{ij})' = \sigma'_{ik}\sigma'_{kj} - 2J_2(\boldsymbol{\sigma}')\delta_{ij}/3 \tag{1.9}$$

is the deviator of the square of the reduced stress $\sigma'_{ij}$.

To determine Lagrange's multiplier $\lambda$ in Eq. (1.8), we first calculate the dissipation rate [6, 7]

$$\sigma'_{ij}\dot{\varepsilon}_{ij} = \left[2\frac{\partial F}{\partial J_2}J_2(\boldsymbol{\sigma}') + 3\frac{\partial F}{\partial J_3}J_3(\boldsymbol{\sigma}')\right]\dot{\lambda}. \tag{1.10}$$

Then, by the hypothesis of equivalent dissipation rate [6, 7], $\sigma\dot{\varepsilon} = \sigma'_{ij}\dot{\varepsilon}_{ij}$ and by assuming Norton's creep law

$$\dot{\varepsilon} = K\sigma^n \equiv \dot{\varepsilon}_c\left(\frac{\sigma}{\sigma_c}\right)^n, \tag{1.11}$$

we find the multiplier

$$\dot{\lambda} = \frac{3K\sigma^{n-1}/2}{\left[\dfrac{\partial F}{\partial J_2}\right]_v + \dfrac{1}{3}\sigma\left[\dfrac{\partial F}{\partial J_3}\right]_v}. \tag{1.12}$$

The index $V$, appended to the brackets in Eq. (1.12), indicates an appropriate equivalent stress state $\sigma$ which can be obtained from the creep condition $F(\sigma'_{ij}) = \text{const}$ corresponding to Eq. (1.6). In this special case the creep potential hypothesis is compatible with the tensor function theory if additional conditions are fulfilled [6].

However, in the anisotropic case the creep potential theory only furnishes restricted forms of constitutive equations. Consequently, the clasical flow rule must be modified for anisotropic solids, as has been pointed out in detail in [7].

## 2. Oriented solids

For anisotropic solids, the flow potential is a function not only of the stress tensor $\sigma_{ij}$ but also of the constitutive tensors $A_{ij}$, $A_{ijkl}$, ..., characterizing the anisotropy of the material [6]

(2.1) $$F = F[\sigma_{ij}, A_{ij}, A_{ijkl}, A_{ijklmn}, ...].$$

Then, by analogy with the condition (1.1), we have the invariance condition

(2.2) $$F(a_{ip}a_{jq}\sigma_{pq}, ..., a_{ip}a_{jq}a_{kr}a_{ls}A_{pqrs}, ...) \equiv F(\sigma_{ij}, ..., A_{ijkl}, ...)$$

fulfilled under any orthogonal transformation.

Now the central problem is: to construct an irreducible integrity basis for the tensors $\sigma_{ij}$, $A_{ij}$, $A_{ijkl}$ etc. Together with the invariants of the single argument tensors $\sigma_{ij}$, $A_{ij}$, $A_{ijkl}$ etc., Eq. (1.2) or (1.3), we have to take into consideration the system of simultaneous or joint invariants [8].

Instead of the generalized representation (2.1), by analogy with the theory of plasticity [9, 10, 11], the anisotropic creep behaviour may be considered by a flow potential

(2.3) $$F = F(\sigma_{ij}, M_{ij})$$

the representation of which is given in the usual manner [6, 11]

(2.4) $$F = F[S_\nu(\sigma), S_\nu(\mathbf{M}), \Omega_1, ..., \Omega_4],$$

where the irreducible invariants

(2.5) $$S_\nu(\boldsymbol{\sigma}) \equiv \mathrm{tr}\boldsymbol{\sigma}^\nu, \quad S_\nu(\mathbf{M}) \equiv \mathrm{tr}\mathbf{M}^\nu, \quad \nu = 1, 2, 3,$$

(2.5') $$\Omega_1 \equiv \mathrm{tr}\boldsymbol{\sigma}\mathbf{M}, \quad \Omega_2 \equiv \mathrm{tr}\boldsymbol{\sigma}\mathbf{M}^2, \quad \Omega_3 \equiv \mathrm{tr}\mathbf{M}\boldsymbol{\sigma}^2, \quad \Omega_4 \equiv \mathrm{tr}\boldsymbol{\sigma}^2\mathbf{M}^2$$

form the integrity basis. The second order tensor generator is defined as the dyadic product $\mathbf{M} = \mathsf{V}\otimes\mathsf{V}$; vector $\mathsf{V}$ specifies a privileged direction of the material (transverse isotropy).

### 2.1. Simplified theory

To describe isochoric creep behaviour of oriented solids, BETTEN [12] proposed a theory adapted previously in the plasticity of anisotropic solids [9, 13, 14] where the following operators are used involving anisotropy effects:

(2.6) $$\tau_{ij} = \alpha_{ij} + \beta_{ijkl}\sigma_{kl} + \gamma_{ijklmn}\sigma_{kl}\sigma_{mn} + \dots .$$

The mechanical anisotropy is specified by the tensors $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$, $\boldsymbol{\gamma}$, ... of rank 2, 4, 6, ..., the components of which are to be determined by experimental data.

Assuming a linear representation

$$\tau_{ij} = \beta_{ijkl}\sigma_{kl}, \tag{2.7}$$

tensor $\boldsymbol{\beta}$ transforms anisotropic stress state $\boldsymbol{\sigma}$ of the actual material on the equivalent isotropic stress state $\boldsymbol{\tau}$ of the fictitious material. The irreducible basic or principal invariants by analogy with the invariants (1.2) and (1.3) are given in this simple way:

$$S_\nu(\tau) \equiv \mathrm{tr}(\boldsymbol{\tau}^\nu), \quad \nu = 1, 2, 3 \tag{2.8}$$

or

$$J_1(\boldsymbol{\tau}) \equiv \tau_{ii}, \quad J_2(\boldsymbol{\tau}) \equiv -\tau_{i[i]}\tau_{j[j]}, \quad J_3(\boldsymbol{\tau}) \equiv \tau_{i[i]}\tau_{j[j]}\tau_{k[k]} \tag{2.8'}$$

Then the anisotropic behaviour can be described by using the invariants (2.8) or (2.8′) in the flow potential

$$F = F[S_\nu(\boldsymbol{\tau})] \quad \text{or} \quad F = F[J_\nu(\boldsymbol{\tau})], \quad \nu = 1, 2, 3 \tag{2.9}$$

instead of the invariants of the actual stress tensor $\boldsymbol{\sigma}$.

Assuming incompressibility (1.5), the flow potential has the form

$$F = F[J_2(\boldsymbol{\tau}'), J_3(\boldsymbol{\tau}')] \tag{2.10}$$

and

$$\tau'_{ij} = \beta'_{\{ij\}kl}\sigma'_{pq}, \tag{2.11}$$

where $\beta'_{\{ij\}kl} = \beta_{ijpq} - \beta_{kkpq}\delta_{ij}/3$ is deviatoric corresponding to the index pair $\{ij\}$.

Starting from a flow potential as a function of the mapped stress tensor $\tau'$, Eq. (2.10), by analogy with Eq. (1.7), (1.8) and (1.9), in anisotropic state, the constitutive equation has been formulated in this simple way:

$$\dot{\varepsilon}_{ij} = \dot{\lambda}[\partial F(\boldsymbol{\tau}')/\partial\tau_{pq}]J_{pqij} \equiv \dot{\gamma}_{pq}J_{pqij}, \tag{2.12}$$

where Jacobi's matrix is defined as $J_{pqij} \equiv \partial\tau'_{pq}/\partial\sigma_{ij} = \beta'_{pq\{ij\}}$, $\dot{\boldsymbol{\gamma}}$ denotes the steady-state creep rate tensor to be specified in the fictitious state.

In a fictitious creep state, defined by Eq. (2.11) and by the equality of the equivalent isochoric creep rates in both states

$$\dot{\gamma} \equiv \dot{\varepsilon}, \tag{2.13}$$

we have, by analogy of Eq. (1.11),

$$\dot{\gamma} \equiv L\tau^m \equiv \dot{\gamma}_c(\tau/\tau_c)^m = \dot{\varepsilon}_c(\tau/\tau_c)^m = \dot{\varepsilon}, \tag{2.14}$$

where $L$, $m$, $\dot{\gamma}_c$, $\tau_c$ and $\dot{\varepsilon}_c$ are material constants.

The equivalent fictitious isotropic creep stress $\tau$ can be determined by the hypothesis of equivalent dissipation rate $D$. Thus, in connection with Eq. (2.13), we require

$$\tau\dot{\gamma} = \tau\dot{\varepsilon} \equiv \sigma'_{ij}\dot{\varepsilon}_{ij} \equiv D. \tag{2.15}$$

From the flow rule (2.12), combined with the relations (2.14) and (2.15), we finally obtain the constitutive equations [12]

$$\dot{\varepsilon}_{ij} = \Phi\beta'_{pq\{ij\}}\left(\frac{\partial F}{\partial J_2}\tau'_{pq} + \frac{\partial F}{\partial J_3}\tau''_{pq}\right) \tag{2.16}$$

in which the function $\Phi$ is defined by

$$(2.17)\qquad \Phi \equiv \frac{1}{2} L\left\{3 \Big/ \left[\left(\frac{\partial F}{\partial J_2}\right)_V + \frac{1}{3}\left(\frac{\partial F}{\partial J_3}\right)_V\right]\right\}^{(m+1)/2} \times \left[\frac{\partial F}{\partial J_2} J_2(\boldsymbol{\tau}') + \frac{3}{2}\frac{\partial F}{\partial J_3} J_3(\boldsymbol{\tau}')\right]^{(m-1)/2}.$$

The index $V$, appended to the round brackets in Eq. (2.17), indicates the equivalent fictitious stress state $(\tau_{ij})_V$. Contrary to Eq. (2.11), the tensor

$$(2.18)\qquad \beta'_{pq\{ij\}} \equiv \beta_{pqij}\ \beta_{pqkk}\delta_{ij}/3$$

in Eq. (2.16) is deviatoric corresponding to the second index pair. By analogy of Eq. (1.9), tensor $\boldsymbol{\tau}''$

$$(2.19)\qquad \tau''_{pq} \equiv (\tau'^{(2)}_{qp})' = \tau'_{qr}\tau'_{rp} - \frac{2}{3} J_2(\boldsymbol{\tau}')\,\delta_{pq} = \frac{\partial J_3(\boldsymbol{\tau}')}{\partial \tau_{ij}}$$

is the deviator of the square of the reduced stress $\tau'_{pq}$.

Inserting Eq. (2.16), together with Eq. (2.17), in Eq. (2.15) we obtain the rate of dissipation of creep energy [12]

$$(2.20)\qquad \dot{D} = [2(\partial F/\partial J_2)J_2(\boldsymbol{\tau}') + 3(\partial F/\partial J_3)J_3(\boldsymbol{\tau}')]\Phi.$$

In the isotropic special case, given by $\beta_{pqij} = (\delta_{pi}\delta_{qj} + \delta_{pj}\delta_{qi})/2$, $L \to K$, $m \to n$, $\tau'_{ij} \to \sigma'_{ij}$ and $\tau''_{ij} \to \sigma''_{ij}$, the constitutive equation (2.16), together with Eq. (2.17), immediately lead to the corresponding relations derived in Sect. 1.

## 3. Simplify application for incompressible and orthotropic solids

To verify the mapped stress tensor concept for incompressible solids, we assume the MISES's flow potential [15]

$$(3.1)\qquad F \equiv J_2(\boldsymbol{\tau}') = \frac{1}{3}\tau^2.$$

Inserting the flow potential (3.1) in Eq. (2.16) and (2.17), we obtain the constitutive equations

$$(3.2)\qquad \dot{\varepsilon}_{ij} = \frac{3}{2} L(\tau)^{m-1}\beta'_{pq\{ij\}}\tau'_{pq}.$$

For isotropic state, Eq. (3.2) are reduced to the ODQVIST law [16]

$$(3.3)\qquad \dot{\varepsilon}_{ij} = \frac{3}{2} K(\sigma)^{n-1}\sigma'_{pq}$$

and assuming equivalent secondary creep rate $\dot{\varepsilon}$ in the form

$$(3.4)\qquad \dot{\varepsilon} \equiv [(4/3)J_2(\dot{\boldsymbol{\epsilon}})]^{1/2} = [(2/3)\dot{\varepsilon}_{ij}\dot{\varepsilon}_{ij}]^{1/2},$$

we find the relation (1.11) where

$$(3.5)\qquad \sigma \equiv [(3/2)\sigma'_{ij}\sigma'_{ij}]^{1/2}.$$

Experimental verification of Eq. (1.11) is shown on Fig. 1, for $\dot{\varepsilon}$ and $\sigma$ defined by (3.4) and (3.5), respectively, assuming isotropic creep behaviour presented in double logarithmic scale (straight line fit). Tests were performed on plate specimens of Mg-alloy pulled independetly in two perpendicular directions at 513 K [17].

Assuming the orthotropic case, the creep behaviour can be presented as the second order tensor $\boldsymbol{\omega}$ [12]

$$\beta'_{\{ij\}pq} \equiv \frac{1}{2}(\omega_{ip}\omega_{jq}+\omega_{iq}\omega_{jp})-\frac{1}{3}\omega^{(2)}_{pq}\delta_{ij}. \tag{3.6}$$

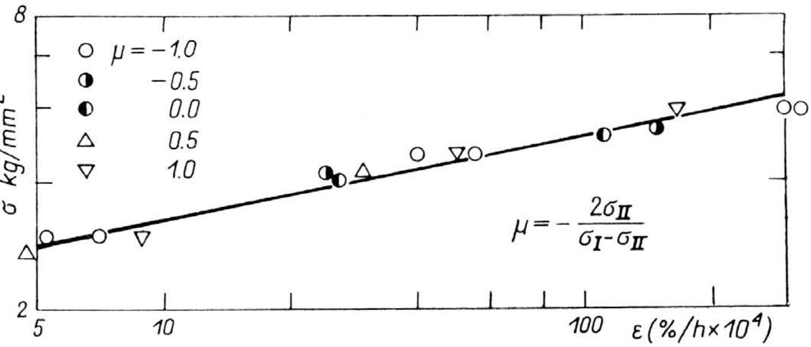



FIG. 1. Isotropic secondary creep behaviour due to [5].

If we assume that the second order tensor of anisotropy $\boldsymbol{\omega}$ is real and symmetric, then its principal values $\omega_{\mathrm{I}}$, $\omega_{\mathrm{II}}$ and $\omega_{\mathrm{III}}$ are all real [18], $\omega_{ij} = \omega_{ji} = \operatorname{diag}\{\omega_{\mathrm{I}}, \omega_{\mathrm{II}}, \omega_{\mathrm{III}}\}$. So the principal invariants of the tensor $\boldsymbol{\omega}$ take the following form, [18]:

$$J_1(\boldsymbol{\omega}) = \omega_{\mathrm{I}}+\omega_{\mathrm{II}}+\omega_{\mathrm{III}}, \tag{3.7}$$

$$J_2(\boldsymbol{\omega}) = -(\omega_{\mathrm{I}}\omega_{\mathrm{II}}+\omega_{\mathrm{II}}\omega_{\mathrm{III}}+\omega_{\mathrm{I}}\omega_{\mathrm{III}}); \quad J_3(\boldsymbol{\omega}) = \omega_{\mathrm{I}}\omega_{\mathrm{II}}\omega_{\mathrm{III}}$$

or, alternatively,

$$J_1(\boldsymbol{\omega}) = \omega_{kk}, \quad J_2(\omega) = \frac{1}{2}(\omega_{ij}\omega_{ji}-\omega_{ii}\omega_{jj}), \quad J_3(\boldsymbol{\omega}) = \det(\omega_{ij}). \tag{3.8}$$

## 4. Identification of the tensor of anisotropy

Components of the tensor $\boldsymbol{\omega}$ and essential creep parameters $L$, $m$ involved in the constitutive equations (2.16) are related to experimental data. For instance, there are some experiments carried out under combined stress states which identify anisotropic behaviour of material during the creep process [19, 20, 21, 22, 23, 24, 25, 26, 27]. A double logarithmic plots of $\sigma$ vs $\varepsilon$, where $\sigma$ and $\varepsilon$ are defined as Eqs. (3.5) and (3.4), respectively, present results by ODING *et al.* [20], Fig. 2a and by KOWALEWSKI [27], Fig. 2b. The tests were performed on thin-walled tubes of austenitic, chromium–nickel steel at 873 K and of pure copper at 573 K. The scatter of the points is too large to exlude the general straight line fit, Eq. (1.11). However, good agreement with experiments gives the straight line fit for the

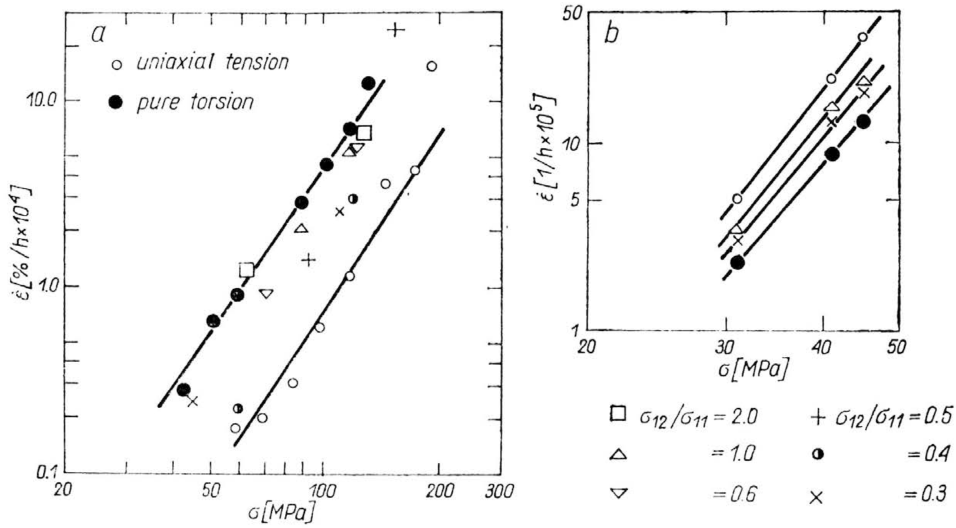



FIG. 2. Anisotropic secondary creep behaviour (experimental values due to [20, 27]).

appropriate stress tensor directions, see the experimental data of pure shear and uniaxial tension creep tests, Fig. 2a and Fig. 2b.

In this section, according to the test performed on the thin-walled tubes under combined stress states [20, 27], Fig. 3, we try to verify the constitutive equation (3.2) based on the relation (3.1) and on the following assumption:

$$\dot{\gamma} \equiv (2\dot{\gamma}_{ij}\dot{\gamma}_{ji}/3)^{1/2} \tag{4.1}$$

for incompressible and orthotropic solids, Eq. (3.6).

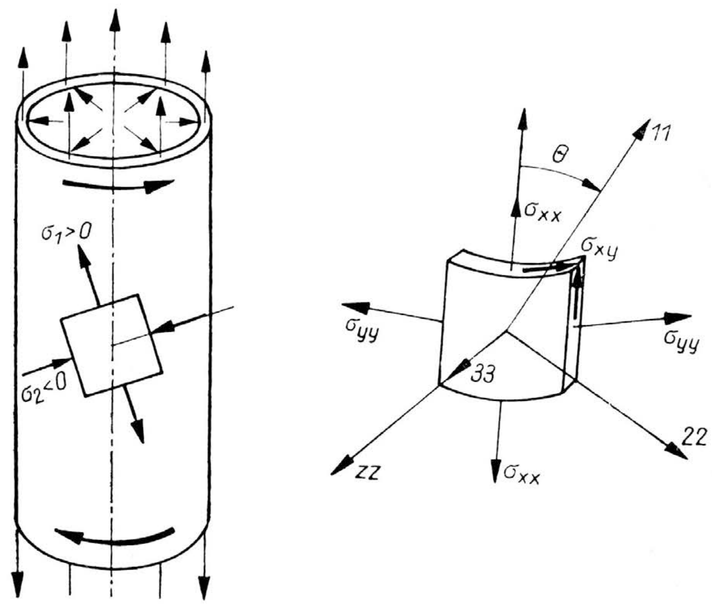



FIG. 3. The plane stress state of anisotropic material.

Let us consider a thin-walled circular cylindrical tube (thickness $s$, mean radius $r$) subjected to combined tension, torsion and internal pressure $p$. Its stress state is given by the components $\sigma_{xx}$, $\sigma_{yy}$, $\sigma_{xy}$ with circumferential $(yy)$, radial $(zz)$ and tube axis $(xx)$ principal directions. We consider an orthotropic material with privileged directions (11), (22) and (33) = $(zz)$, $\theta$ is the angle between the directions $(xx)$ and (11), Fig. 3.

Using the notations from Fig. 3 and considering Eq. (3.6), the diagonal form of the "orthotropic tensor" $\omega_{ij}$ is given by

$$\omega_{ij} = \operatorname{diag}\left\{\sqrt{\tau/(\sigma)_{11}}, \quad \sqrt{\tau/(\sigma)_{22}}, \quad \sqrt{\tau/(\sigma)_{33}}\right\}, \tag{4.2}$$

where the equivalent stresses $(\sigma)_{11} = \operatorname{diag}\{\sigma_{11}, 0, 0\}$; $(\sigma)_{22} = \operatorname{diag}\{0, \sigma_{22}, 0\}$ and $(\sigma)_{33} = \operatorname{diag}\{0, 0, \sigma_{33}\}$ are obtained by the using creep law (1.11) in tests on specimens cut along the mutually perpendicular directions (11), (22) or (33). Then, with the notation from Fig. 3, we have

$$\dot{\varepsilon} = K_{11}\sigma_{11}^{n_{11}} = K_{22}\sigma_{22}^{n_{22}} = K_{33}\sigma_{33}^{n_{33}}. \tag{4.3}$$

In any privileged orthonormal frame [28], the components of the stress tensor, Fig. 3, are given by

$$\sigma_{ij} = \begin{pmatrix} \sigma_{xx}\cos^2\theta + \sigma_{xy}\sin 2\theta + \sigma_{yy}\sin^2\theta; & \frac{1}{2}(\sigma_{yy}-\sigma_{xx})\sin 2\theta + \sigma_{xy}\cos 2\theta; & 0 \\ \frac{1}{2}(\sigma_{yy}-\sigma_{xx})\sin 2\theta + \sigma_{xy}\cos 2\theta; & \sigma_{xx}\sin^2\theta - \sigma_{xy}\sin 2\theta + \sigma_{yy}\cos^2\theta; & 0 \\ 0; & 0; & 0 \end{pmatrix}. \tag{4.4}$$

Let us assume that the stress state is coincident with the privileged orthonormal frame of material, $\theta = 0$, Fig. 3. Then the representation (4.4) has a form

$$\sigma_{ij} = \begin{pmatrix} \sigma_{xx}; & \sigma_{xy}; & 0 \\ \sigma_{xy}; & \sigma_{yy}; & 0 \\ 0; & 0; & 0 \end{pmatrix} = \begin{pmatrix} \sigma_{11}; & \sigma_{12}; & 0 \\ \sigma_{12}; & \sigma_{22}; & 0 \\ 0; & 0; & 0 \end{pmatrix}. \tag{4.5}$$

Inserting Eqs. (2.11) and (3.6) in Eq. (3.1) and assuming the form (4.5), we obtain the quadratic equation

$$\begin{aligned} &\frac{2}{9}\left(2\omega_{\rm I}^4 + \frac{1}{2}\omega_{\rm II}^4 + \frac{1}{2}\omega_{\rm III}^4 + \omega_{\rm I}^2\omega_{\rm II}^2 + \omega_{\rm I}^2\omega_{\rm III}^2 - \frac{1}{2}\omega_{\rm II}^2\omega_{\rm III}^2\right)\sigma_{11}^2 \\ &\qquad - \frac{2}{9}\left(2\omega_{\rm I}^4 + 2\omega_{\rm II}^4 - \omega_{\rm III}^4 + \frac{5}{2}\omega_{\rm I}^2\omega_{\rm II}^2 - \frac{1}{2}\omega_{\rm I}^2\omega_{\rm III}^2 - \frac{1}{2}\omega_{\rm II}^2\omega_{\rm III}^2\right)\sigma_{11}\sigma_{22} \\ &+ \frac{2}{9}\left(\frac{1}{2}\omega_{\rm I}^4 + 2\omega_{\rm II}^4 + \frac{1}{2}\omega_{\rm III}^4 + \omega_{\rm I}^2\omega_{\rm II}^2 - \frac{1}{2}\omega_{\rm I}^2\omega_{\rm III}^2 + \omega_{\rm II}^2\omega_{\rm III}^2\right)\sigma_{22}^2 + 3\omega_{\rm I}^2\omega_{\rm II}^2\sigma_{12}^2 = \tau^2. \end{aligned} \tag{4.6}$$

Equation (4.6) in the stress space represents an ellipsoid, i.e., the surface of constant steady--state creep rate.

To determine the principal components $\omega_{\rm I}$, $\omega_{\rm II}$ and $\omega_{\rm III}$ of $\boldsymbol{\omega}$, the following three creep tests are required, i.e., the thin-walled tubular specimen is subjected to:

uniaxial tension

$$\sigma_{ij} = \operatorname{diag}\{\sigma_{11}, 0, 0\}, \tag{4.7}$$

pure torsion

$$(4.8)\qquad \sigma_{ij} = \begin{pmatrix} 0 & ; \sigma_{12}; & 0 \\ \sigma_{12}; & 0 & ; 0 \\ 0 & ; 0 & ; 0 \end{pmatrix}$$

or internal pressure

$$(4.9)\qquad \sigma_{ij} = \begin{pmatrix} \sigma_{11}; & 0 & ; 0 \\ 0 & ; \sigma_{22}; & 0 \\ 0 & ; 0 & ; 0 \end{pmatrix}, \quad \sigma_{11} = \frac{rp}{2s}, \quad \sigma_{22} = \frac{rp}{s}$$

Then Eq. (4.6) is reduced to the following three forms:

$$(4.10)\qquad 2\omega_{I}^4 + \frac{1}{2}\omega_{II}^4 + \frac{1}{2}\omega_{III}^4 + \omega_I^2\omega_{II}^2 + \omega_I^2\omega_{III}^2 - \frac{1}{2}\omega_{II}^2\omega_{III}^2 = \frac{9\tau^2}{2\sigma_{11}^2} \equiv \frac{9\tau^2}{2(\sigma)_{11}^2},$$

$$(4.11)\qquad \omega_{II}^2\omega_{III}^2 = \frac{\tau^2}{3\sigma_{12}^2} \equiv \frac{\tau^2}{(\sigma)_{12}^2},$$

$$(4.12)\qquad \omega_{II}^4 + \omega_{II}^2\omega_{III}^2 + \omega_{III}^4 = \frac{\tau^2}{\left(\frac{rp}{2s}\right)^2} \equiv \frac{3\tau^2}{(\sigma)_p^2}.$$

The scheme of determination of the equivalent stresses, $(\sigma)_{11}$, $(\sigma)_{12}$ and $(\sigma)_p$, to be defined in Eqs. (4.10)–(4.12), is presented on Fig. 4. in double logarithmic scale.

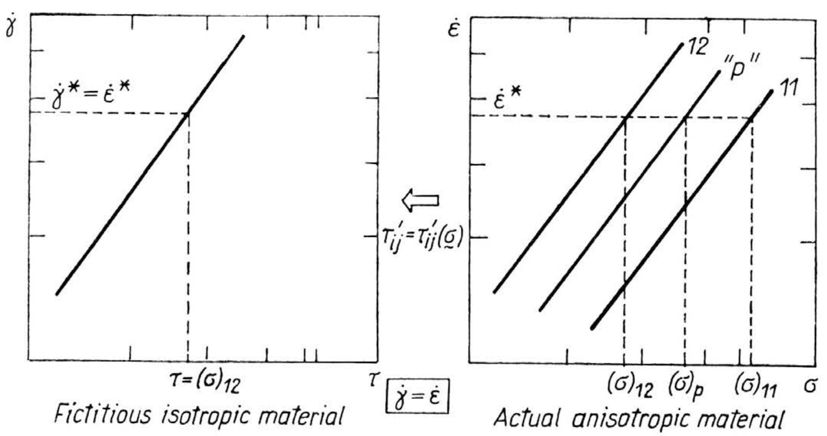



FIG. 4. "$\tau$-concept" — determination of the equivalent stress.

Resolving Eqs. (4.10)–(4.12), we obtain the principal values of tensor $\boldsymbol{\omega}$:

$$(4.13)\qquad \omega_I^2 = \frac{\tau}{(\sigma)_{11}},$$

$$(4.14)\qquad \omega_{II}^2 = \frac{(\sigma)_{11}}{(\sigma)_{12}^2}\tau,$$

$$(4.15)\qquad \omega_{III}^2 = \frac{\omega_{II}^2}{2}\left(\sqrt{12\frac{(\sigma)_{12}^4}{(\sigma)_{11}^2(\sigma)_p^2} - 3} - 1\right).$$

Finally, the equation of ellipsoid (4.6) in the stress space $\sigma_{xx}$, $\sigma_{yy}$ and $\sigma_{xy}$ has the form

$$(4.16)\qquad \frac{(\sigma)^2_{12}}{(\sigma)^2_{11}}\sigma^2_{xx}-\frac{1}{3}\left[3\frac{(\sigma)^2_{12}}{(\sigma)^2_{11}}+\frac{(\sigma)^2_{11}}{(\sigma)^2_{12}}-\frac{(\sigma)^2_{11}}{4(\sigma)^2_{12}}(\sqrt{a}-1)^2-\frac{1}{2}(\sqrt{a}-1)\right]\sigma_{xx}\sigma_{yy}$$
$$+\frac{1}{3}\left[2\frac{(\sigma)^2_{12}}{(\sigma)^2_{11}}+\frac{(\sigma)^2_{11}}{(\sigma)^2_{12}}+\frac{1}{2}\left(\frac{(\sigma)^2_{11}}{(\sigma)^2_{12}}-1\right)(\sqrt{a}-1)\right]\sigma^2_{yy}+3\sigma^2_{xy}=(\sigma)^2_{12},$$

where

$$a = 12\frac{(\sigma)^4_{12}}{(\sigma)^2_{11}(\sigma)^2_{p}}-3.$$

The stress state defined in Eq. (4.5) causes the steady-state creep rate, the representation of which is given by the way

$$(4.17)\qquad \dot{\varepsilon}_{ij}=\begin{pmatrix}\dot{\varepsilon}_{11}; & \dot{\varepsilon}_{12}; & 0\\ \dot{\varepsilon}_{12}; & \dot{\varepsilon}_{22}; & 0\\ 0; & 0; & -(\dot{\varepsilon}_{11}+\dot{\varepsilon}_{22})\end{pmatrix}.$$

Inserting Eqs. (2.11), (3.6), (4.13)–(4.15) in Eq. (3.2) and assuming the form (4.5), we obtain the following representation of isochoric creep rate tensor:

$$(4.18)\qquad \dot{\varepsilon}_{11}=\frac{2}{9}L(\tau)^{m-1}\left[\left(2\omega^4_{\rm I}+\frac{1}{2}\omega^4_{\rm II}+\frac{1}{2}\omega^4_{\rm III}+\omega^2_{\rm I}\omega^2_{\rm II}+\omega^2_{\rm I}\omega^2_{\rm III}-\frac{1}{2}\omega^2_{\rm II}\omega^2_{\rm III}\right)\sigma_{11}\right.$$
$$\left.-\left(\omega^4_{\rm I}+\omega^4_{\rm II}-\frac{1}{2}\omega^4_{\rm III}+5\omega^2_{\rm I}\omega^2_{\rm II}-\frac{1}{4}\omega^2_{\rm I}\omega^2_{\rm III}-\frac{1}{4}\omega^2_{\rm II}\omega^2_{\rm III}\right)\right]\sigma_{22},$$

$$(4.19)\qquad \dot{\varepsilon}_{22}=\frac{2}{9}L(\tau)^{m-1}\left[-\left(\omega^4_{\rm I}+\omega^4_{\rm II}-\frac{1}{2}\omega^4_{\rm III}+\frac{5}{4}\omega^2_{\rm I}\omega^2_{\rm II}-\frac{1}{4}\omega^2_{\rm I}\omega^2_{\rm III}-\frac{1}{4}\omega^2_{\rm II}\omega^2_{\rm III}\right)\sigma_{11}\right.$$
$$\left.+\left(\frac{1}{2}\omega^4_{\rm I}+2\omega^4_{\rm II}+\frac{1}{2}\omega^4_{\rm III}+\omega^2_{\rm I}\omega^2_{\rm II}-\frac{1}{2}\omega^2_{\rm I}\omega^2_{\rm III}+\omega^2_{\rm II}\omega^2_{\rm III}\right)\right]\sigma_{22},$$

$$(4.20)\qquad \dot{\varepsilon}_{33}=\frac{2}{9}L(\tau)^{m-1}\left[\left(-\omega^4_{\rm I}+\frac{1}{2}\omega^4_{\rm II}-\omega^4_{\rm III}+\frac{1}{4}\omega^2_{\rm I}\omega^2_{\rm II}-\frac{5}{4}\omega^2_{\rm I}\omega^2_{\rm III}+\frac{1}{4}\omega^2_{\rm II}\omega^2_{\rm III}\right)\sigma_{11}\right.$$
$$\left.+\left(\frac{1}{2}\omega^4_{\rm I}-\omega^4_{\rm II}-\omega^4_{\rm III}+\frac{1}{4}\omega^2_{\rm I}\omega^2_{\rm II}+\frac{1}{4}\omega^2_{\rm I}\omega^2_{\rm III}-\frac{5}{4}\omega^2_{\rm II}\omega^2_{\rm III}\right)\right]\sigma_{22}$$

and

$$(4.21)\qquad \dot{\varepsilon}_{12}=\frac{3}{2}L(\tau)^{m-1}\omega^2_{\rm I}\omega^2_{\rm II}\sigma_{12}.$$

## 5. Comparisons with experiments

Basing on the experimental data obtained from the creep tests under combined tension and torsion stress states carried out by ODING *et al.* [20] and by KOWALEWSKI [27] on the thin-walled tubular specimens, Fig. 2a and Fig. 2b, we discuss the validity of the proposed simplified theory. To find components of the tensor of anisotropy, we started off with the experimental data received from uniaxial tension and pure torsion creep tests. The data allow to specify the values of $(\sigma)_{11}$ and $(\sigma)_{12}$ for various equivalent creep rates $\varepsilon$, Fig. 4. Due to the lack of creep tests performed on the tubular specimens deformed under internal pressure, the value of $(\sigma)_p$ is used as the parameter and the stress state described by Eq. (4.5) is reduced by $\sigma_{22} = 0$. Then Eq. (4.16) has a simplified form

$$\alpha\sigma_{xx}^2 + 3\sigma_{xy}^2 = \tau^2 \tag{5.1}$$

where

$$\alpha = (\sigma)_{12}^2/(\sigma)_{11}^2 \quad \text{and} \quad \tau = (\sigma)_{12} \tag{5.2}$$

or

$$\sigma_{xx}^2 + 3(1/\alpha)\sigma_{xy}^2 = \tau^2, \tag{5.3}$$

where

$$\tau = (\sigma)_{11} \tag{5.4}$$

**Table 1.**

mat.: austenitic steel  $T = 873$ K
$m = 2.80$  $L = 1.05e-11$  $\alpha = 0.29$  $(\sigma)_p = (\sigma)_{12}/1.10$
$\omega_I = 1.11$  $\omega_{II} = 0.90$  $\omega_{III} = 1.17$
experimental values due to [20]

| $\dot\varepsilon = \dot\gamma$ $*10^{-5}$/h | $\sigma_{12}/\sigma_{11}$ | $(\sigma)_{xy} = \tau$ MPa | $\sigma_{11}$ MPa exp | $\sigma_{11}$ MPa Eq. (5.1) | $\sigma_{12}$ MPa exp | $\sigma_{12}$ MPa Eq. (5.1) | $\dot\varepsilon_{11}$ $*10^{-5}$/h exp | $\dot\varepsilon_{11}$ $*10^{-5}$/h Eq. (5.5) | $\dot\varepsilon_{12}$ $*10^{-5}$/h exp | $\dot\varepsilon_{12}$ $*10^{-5}$/h Eq. (4.21) |
|---|---|---|---|---|---|---|---|---|---|---|
| 1.3 | 0 | 29.1 | 58.8 | 53.8 | 0 | 0 | 1.3 | 1.3 | 0 | 0 |
| 2.0 | 0 | 33.9 | 68.6 | 62.8 | 0 | 0 | 2.0 | 2.0 | 0 | 0 |
| 2.2 | 0.4 | 34.9 | 49.0 | 47.0 | 19.6 | 13.9 | 1.0 | 2.2 | 1.7 | 1.3 |
| 2.4 | 0.3 | 36.0 | 39.2 | 40.2 | 11.8 | 16.6 | 2.0 | 2.0 | 1.2 | 1.7 |
| 2.8 | ~ | 38.1 | 0 | 0 | 24.5 | 22.0 | 0 | 0 | 2.4 | 2.4 |
| 3.0 | 0 | 39.2 | 83.4 | 72.6 | 0 | 0 | 3.0 | 3.0 | 0 | 0 |
| 6.1 | 0 | 50.5 | 98.1 | 93.5 | 0 | 0 | 6.1 | 6.1 | 0 | 0 |
| 6.5 | ~ | 51.4 | 0 | 0 | 29.4 | 29.7 | 0 | 0 | 5.6 | 5.6 |
| 9.0 | ~ | 57.8 | 0 | 0 | 34.3 | 33.4 | 0 | 0 | 7.7 | 7.7 |
| 9.2 | 0.6 | 58.2 | 49.0 | 49.0 | 29.4 | 30.0 | 3.0 | 4.1 | 7.5 | 7.1 |
| 11.6 | 0 | 63.5 | 117.7 | 117.7 | 0 | 0 | 11.6 | 11.6 | 0 | 0 |
| 11.6 | 2.0 | 63.5 | 17.2 | 17.2 | 34.3 | 36.1 | 0 | 1.7 | 10.0 | 9.9 |
| 14.4 | 0.5 | 68.3 | 68.6 | 68.3 | 34.3 | 33.3 | 5.0 | 7.7 | 11.7 | 10.5 |
| 21.3 | 1.0 | 78.6 | 44.1 | 43.9 | 44.1 | 43.2 | 4.5 | 6.4 | 18.0 | 17.6 |
| 25.1 | 0.3 | 83.3 | 98.1 | 99.9 | 29.4 | 36.8 | 17.0 | 16.2 | 16.0 | 16.6 |
| 28.9 | ~ | 87.7 | 0 | 0 | 50.0 | 50.6 | 0 | 0 | 25.0 | 25.0 |
| 29.7 | 0.4 | 88.5 | 98.1 | 98.4 | 39.2 | 40.9 | 20.0 | 17.8 | 19.0 | 20.6 |
| 36.4 | 0 | 95.5 | 147.1 | 176.9 | 0 | 0 | 36.4 | 36.4 | 0 | 0 |
| 42.0 | 0 | 100.5 | 176.5 | 186.2 | 0 | 0 | 42.0 | 42.0 | 0 | 0 |
| 45.0 | ~ | 102.7 | 0 | 0 | 58.8 | 59.3 | 0 | 0 | 39.0 | 39.0 |
| 53.4 | 1.0 | 109.1 | 58.8 | 58.8 | 58.8 | 60.3 | 12.5 | 15.5 | 45.0 | 44.3 |
| 58.2 | 0.6 | 112.5 | 83.3 | 84.5 | 50.0 | 59.4 | 19.5 | 23.5 | 47.5 | 46.1 |
| 63.7 | 2.0 | 116.2 | 34.3 | 34.1 | 68.6 | 66.3 | 5.0 | 10.0 | 55.0 | 54.5 |
| 69.3 | ~ | 119.8 | 0 | 0 | 60.0 | 69.2 | 0 | 0 | 60.0 | 60.0 |

and the material response is expresed by the reduced form of Eqs. (4.18)–(4.20) as follows:

$$\dot{\varepsilon}_{11} = \frac{2}{9} L(\tau)^{m-1}\left(2\omega_{I}^{4}+\frac{1}{2}\omega_{II}^{4}+\frac{1}{2}\omega_{III}^{4}+\omega_{I}^{2}\omega_{II}^{2}+\omega_{I}^{2}\omega_{III}^{2}-\frac{1}{2}\omega_{II}^{2}\omega_{III}^{2}\right)\sigma_{11}, \tag{5.5}$$

$$\dot{\varepsilon}_{22} = -\frac{2}{9} L(\tau)^{m-1}\left(\omega_{I}^{4}+\omega_{II}^{4}-\frac{1}{2}\omega^{4}+\frac{5}{4}\omega_{I}^{2}\omega_{II}^{2}-\frac{1}{4}\omega_{I}^{2}\omega_{III}^{2}-\frac{1}{4}\omega_{II}^{2}\omega_{III}^{2}\right)\sigma_{11}, \tag{5.6}$$

$$\dot{\varepsilon}_{33} = -\frac{2}{9} L(\tau)^{m-1}\left(\omega_{I}^{4}-\frac{1}{2}\omega_{II}^{4}+\omega_{III}^{4}-\frac{1}{4}\omega_{I}^{2}\omega_{II}^{2}+\frac{5}{4}\omega_{I}^{2}\omega_{III}^{2}-\frac{1}{4}\omega_{II}^{2}\omega_{III}^{2}\right)\sigma_{11}, \tag{5.7}$$

where $\tau$ is defined in Eq. (5.2) or in Eq. (5.4), depending on the form of Eq. (5.1) or Eq. (5.3), respectively.

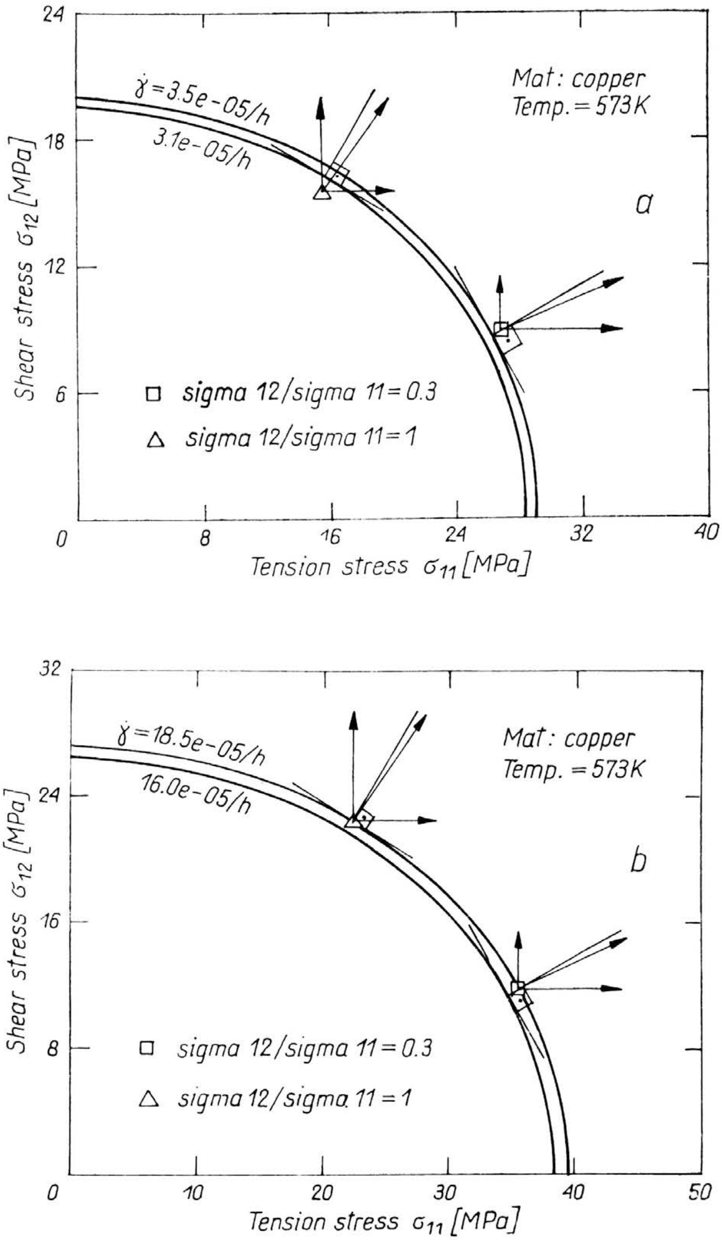
Moreover, the isochoric creep rate in torsion direction is represented by Eq. (4.21). The constants $\omega_{I}$, $\omega_{II}$, $\omega_{III}$ defined in Eq. (4.13)–(4.15) and material constants $L$, $m$ depend on the form of the equivalent stress $\tau$, i.e., $L = K_{12}$ and $m = n_{12}$ if $\tau = (\sigma)_{12}$, Eq. (5.2) or $L = K_{11}$ and $m = n_{11}$ if $\tau = (\sigma)_{11}$, Eq. (5.4), Fig. 4. In the case of intersection of the straight lines presented on Fig. 4 for anisotropic creep behaviour, we note the evolution of anisotropy during the creep process. On the contrary, the components of the tensor of anisotropy are independent of the changes of creep stress levels.

Comparisons of experimental data with numerical results are presented in Table 1 and Table 2 for two different materials. The values of equivalent stress $\tau$, Eq. (5.2) and the material constants $L$ and $m$ are identified. The values of the principal components

**Table 2.**

mat: pure copper $T = 573$ K
$m = 5.44$ $L = 1.5e-13$ $\alpha = 1.41$ $(\sigma)_p = (\sigma)_{12}/1.16$
$\omega_I = 1.09$ $\omega_{II} = 0.92$ $\omega_{III} = 1.20$
experimental values due to [27]

| $\dot{\varepsilon} = \dot{\gamma}$ $*10^{-5}$/h | $\sigma_{12}/\sigma_{11}$ | $(\sigma)_{xy} = \tau$ MPa | $\sigma_{11}$ MPa exp | $\sigma_{11}$ MPa Eq. (5.1) | $\sigma_{12}$ MPa exp | $\sigma_{12}$ MPa Eq. (5.1) | $\dot{\varepsilon}_{11}$ $*10^{-5}$/h exp | $\dot{\varepsilon}_{11}$ $*10^{-5}$/h Eq. (5.5) | $\dot{\varepsilon}_{12}$ $*10^{-5}$/h- exp | $\dot{\varepsilon}_{12}$ $*10^{-5}$/h- Eq. (4.21) |
|---|---|---|---|---|---|---|---|---|---|---|
| 2.3 | ~ | 32.0 | 0 | 0 | 17.9 | 18.5 | 0 | 0 | 2.0 | 2.0 |
| 3.1 | 1.0 | 33.9 | 15.5 | 15.8 | 15.5 | 16.2 | 1.8 | 1.7 | 2.2 | 2.2 |
| 3.5 | 0.3 | 34.6 | 26.8 | 26.3 | 8.9 | 8.6 | 3.2 | 3.2 | 1.2 | 1.3 |
| 5.0 | 0 | 36.9 | 31.0 | 31.1 | 0 | 0 | 5.0 | 5.0 | 0 | 0 |
| 9.0 | ~ | 41.1 | 0 | 0 | 23.7 | 23.7 | 0 | 0 | 7.8 | 7.8 |
| 13.0 | ~ | 44.0 | 0 | 0 | 26.0 | 25.4 | 0 | 0 | 11.3 | 11.3 |
| 13.5 | 1.0 | 44.4 | 20.5 | 20.8 | 20.5 | 21.2 | 7.5 | 7.5 | 9.8 | 9.8 |
| 16.0 | 0.3 | 45.8 | 35.5 | 34.8 | 11.8 | 11.3 | 14.5 | 14.5 | 6.2 | 6.0 |
| 18.5 | 1.0 | 47.0 | 22.5 | 22.4 | 22.5 | 22.3 | 10.5 | 10.5 | 13.2 | 13.3 |
| 21.0 | 0.3 | 48.2 | 39.0 | 36.9 | 13.0 | 11.6 | 19.2 | 19.1 | 7.3 | 7.6 |
| 22.1 | 0 | 48.5 | 41.0 | 40.8 | 0 | 0 | 22.1 | 22.1 | 0 | 0 |
| 38.0 | 0 | 53.5 | 45.0 | 45.1 | 0 | 0 | 38.0 | 38.0 | 0 | 0 |



Mat: copper
Temp. = 573K
$\dot{\gamma}$ = 3.5e-05/h
3.1e-05/h
a
□ sigma 12/sigma 11 = 0.3
△ sigma 12/sigma 11 = 1
Shear stress $\sigma_{12}$ [MPa]
Tension stress $\sigma_{11}$ [MPa]
0
8
16
24
32
40
6
12
18
24
Mat: copper
Temp. = 573K
$\dot{\gamma}$ = 18.5e-05/h
16.0e-05/h
b
□ sigma 12/sigma 11 = 0.3
△ sigma 12/sigma 11 = 1
Shear stress $\sigma_{12}$ [MPa]
Tension stress $\sigma_{11}$ [MPa]
0
10
20
30
40
50
8
16
24
32


[Fig. 5a,b]

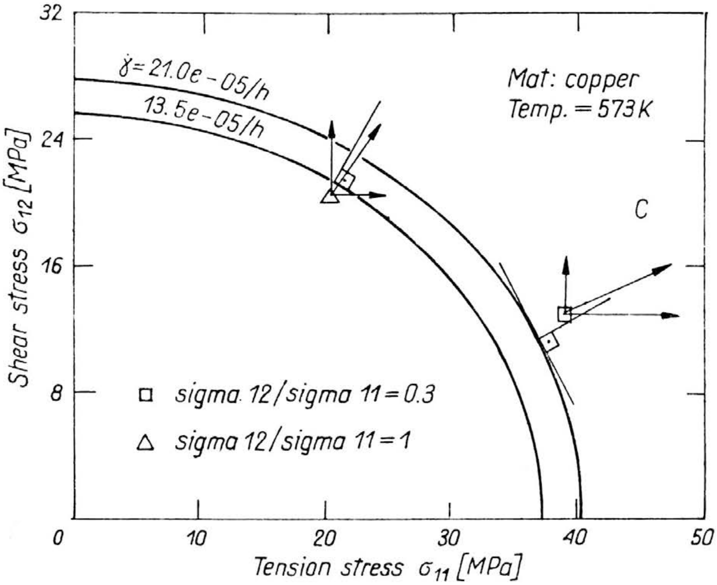
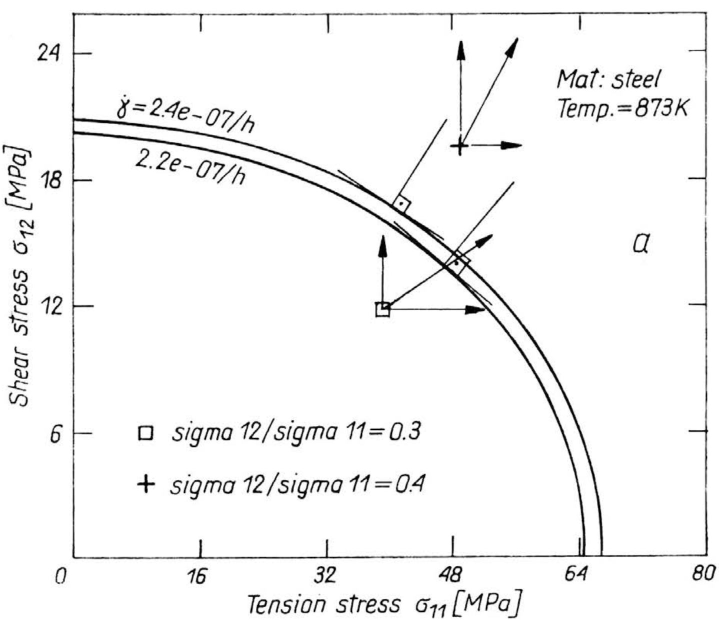



FIG. 5. Surfaces of constant steady-state creep rate.




**[Fig. 6a]**

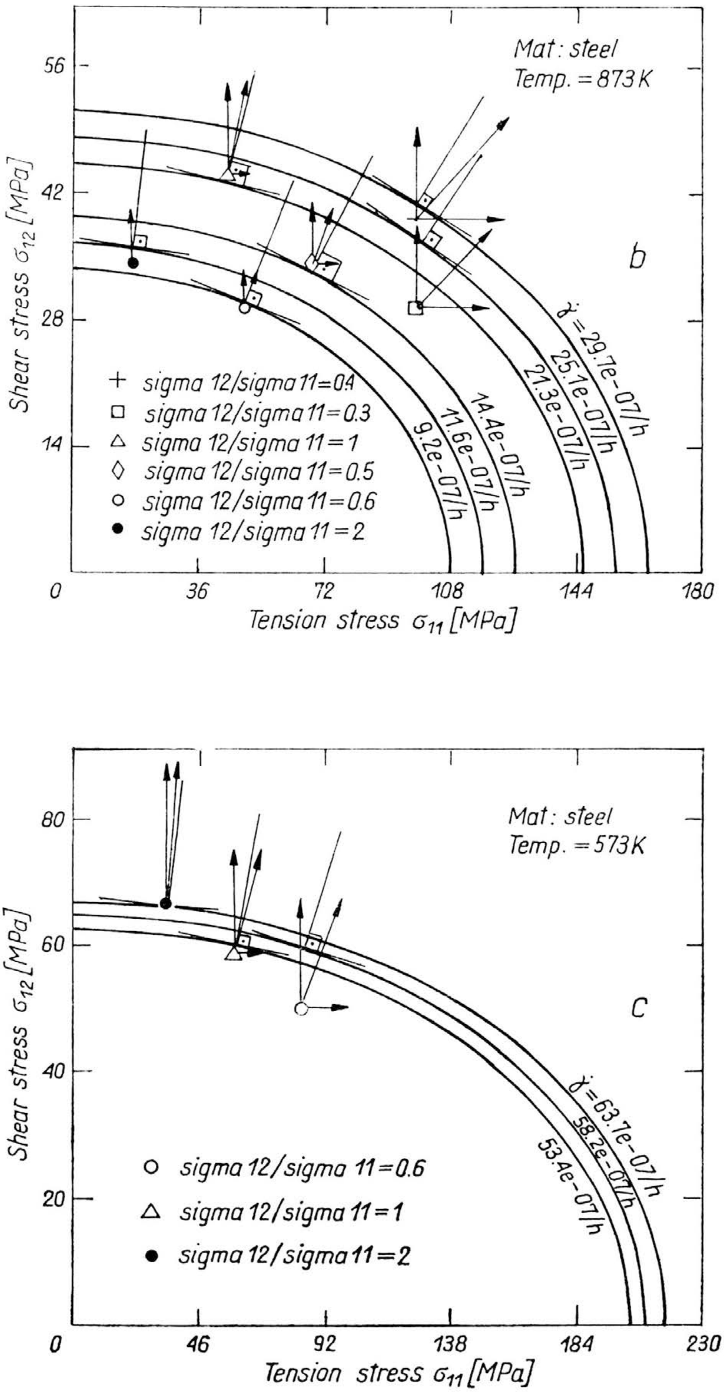



FIG. 6. Surfaces of constant steady-state creep rate.

of $\boldsymbol{\omega}$, Eq. (4.13)–(4.15) are calculated and the dependence between the stress components $\sigma_{xx} \equiv \sigma_{11}$ and $\sigma_{xy} \equiv \sigma_{12}$ is defined basing on Eq. (5.1). The components $\dot{\varepsilon}_{11}$, Eq. (5.5) and $\dot{\varepsilon}_{12}$, Eq. (4.21) of isochoric creep rate are specified and compared with values obtained from the experiment. Then the normality condition of the isochoric creep rate to the surfaces defined by Eq. (5.1) is proved in appropriate stress points, Figs. 5 and 6.

## 6. Conclusions

The experimentally obtained variation of mechanical properties, especially of the isochoric creep rate under constant but directionally variable creep stress level, is investigated by introducing a simple tensorially-linear transformation, Eq. (2.11). By assuming the existence of a flow potential as an Odqvist–Mises form, Eq. (3.1), further restriction is introduced to make impossible the second order effect studing. The theoretical approach is closely adapted to the investigation of the influence of the anisotropy effect on the secondary creep behaviour of thin-walled tubes subjected to the combined tension, torsion and internal pressure. Moreover, the "$\tau$-concept" is very useful to describe the creep process under non-proportional multiaxial load paths, assuming different inclination $\theta$, Eq. (4.4), between load directions and the orthonormal frame of the material.

## References

1. J. R. RICE, *On the structure of stress-strain relations for time-dependent plastic deformation in metals* J. Appl. Mech. **37**, 728–737, 1970.
2. C. R. CALLADINE, D. C. DRUCKER, *Nesting surfaces of constant rate of energy dissipation in creep*, Q. Appl. Math., **20**, 79–84, 1962.
3. YU. N. RABOTNOV, *Creep problems in structural members*, ed. F. A. LECKIE Norh-Holland, Amsterdam/London 1969.
4. A. R. S. PONTER, F. A. LECKIE, *Constitutive relationships for the time-dependent deformation of metals*, J. Engin. Mat. and Technology, Trans. ASME, **98**, 47–51, 1976.
5. F. K. ODQVIST, *Mathematical theory of creep and creep rupture*, Oxford University Press, 1974.
6. J. BETTEN, *Zur Aufstellung von Stoffgleichungen in der Kriech-mechanik anisotroper Körper*, Rheologica Acta, **20**, 527–535, 1981.
7. J. BETTEN, *Materialgleichung zur Beschreibung des sekundären und tertiären Kriechverhaltens anisotroper Stoffe*, ZAMM **64**, 211–220, 1984.
8. J. BETTEN, *Integrity basis for a second-order and a fourth-order tensor*, Int. J. Math. and Math. Sci., **5**, 87–96, 1982.
9. J. P. BOEHLER, A. SAWCZUK, *On yielding of oriented solids*, Acta Mechanica, **27**, 185–206, 1977.
10. J. BETTEN, *On the representation of the plastic potential of anisotropic solids*, Proceeding of CNRS Int. Colloqium 319 on the "Plastic bhaviour of anisotropic soids", Villard-de-Lans, June 1981, Presses du CNRS, Paris, France 213–228.
11. J. BETTEN, *Elastizitäts-und Plastizitätslehre*, Vieweg Verlag, Braunschweig/Wiesbaden 1986.
12. J. BETTEN, *Creep theory of anisotropic solids*, J. of Rheology, **25**, 565–51, 1981.
13. Z. SOBOTKA, *Theorie des plastischen Fliessens von anisotropen Körpern*, ZAMM, **49**, 25–32, 1969.
14. J. BETTEN, *Ein Beitrag zur Invariantentheorie in der Plasto-mechanik anisotroper Werkstoffe*, ZAMM, **56**, 557–559, 1976.
15. R. v. MISES, *Mechanik der plastischen Formänderung von Kristallen*, ZAMM, **8**, 161–185, 1928.

16. F. K. G. ODQVIST, Proc. IV Int. Congress on Applied Mechanics 1934, Cong. Proc. 228, 1935.
17. B. SÖDERQUIST, Acta Polytech. Scand., Series (f), **53**, 1–36, 1968.
18. J. BETTEN, *Tensorrechnung fur Ingenieure*, B. G. Teubner, Stuttgart 1987.
19. B. C. НАМЕСТНИКОВ, *О ползучести при постоянных нагрузках в условиях сложново напряженного состояниа*, Изв. АН СССР. ОТИ, 141–146. 1957.
20. И. А. ОДИН, В. С. ИВАНОВА, В. В. БУРДУКСКИЙ, В. Н. ГЕМИНОВ, *Теория ползучести и длительной прочности металлов*, Москва 1959.
21. A. E. JOHNSON, *Complex stress creep, relaxation and fracture of metallic alloys*, Proc. Int. Mech. Engrs., **164**, 1–76, 1962.
22. G. M. BROWN, *Inelastic deformation of an aluminium alloy under combined stress at elevated temperature*, J. Mech. Phys. Solids, **18**, 383–396, 1970.
23. J. J. BRASS, W. N. FINDLEY, *Short-time, biaxial creep of an aluminium alloy with abrupt changes of temperature and state of stress*, J. Appl. Mech., **38**, 489–501, 1971.
24. R. MARK, W. N. FINDLEY, *Concerning a creep surface derived from a multiple integral representation for 304 stainless steel under combined tension and torsion*, J. Appl. Mech., **45**, 773–779, 1978.
25. M. ANISIMOWICZ, A. SAWCZUK, *On the tensor functions approach to steady creep*, Int. J. Struc. Mech. and Math. Sci., **3**, 35–45, 1981.
26. C. OYTANA, P. DELOBELLE, A. MERMET, *Constitutive equations study in biaxial stress experiments*, J. Eng. Mat. Tech., **104**, 1–11, 1982.
27. Z. KOWALEWSKI, *The surface of constant dissipation energy rate under creep and its experimental determination*, Arch. Mech., **39**, 5, 445-454, 1987.
28. *Applications of tensor functions in solid mechanics* [ed. J. P. BOEHLER], (CISM No. 292), Springer-Verlag, Wien 1987.

TECHNICAL UNIVERSITY AACHEN, FRG
and
POLISH ACADEMY OF SCIENCES
INSTITUTE OF FUNDAMENTAL TECHNOLOGICAL RESEARCH.

*Received June* 20, 1988.