Arch. Mech., 41, 5, pp. 759-768, Warszawa 1989

Nonlinear stability for dusty fluids (*)

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THE NONLINEAR stability problem for the flows of dusty fluids in a bounded domain, subject to three-dimensional perturbations, is studied and an erroneous stability condition presented by two other authors in an earlier paper is corrected. By considering a family of equivalent Lyapunov energy measures, a new idea is introduced and new stability conditions are given and compared with the previous one. Moreover, the instabilizing effect of the dust on the fluid is recovered and a Lyapunov measure for the minimum instabilization is given. Finally, the best measure for the best stability of the equilibrium of dusty fluids is presented.

Przeanalizowano zagadnienie nieliniowe stateczności przepływów w płynach zapylonych zajmujących obszar ograniczony, prostując przy tym mylny warunek stateczności zaproponowany wcześniej przez innych autorów. Zaproponowano nową ideę polegającą na wprowadzeniu rodziny miar Lapunowa, a nowe warunki stateczności porównano międzq sobą a także z warunkiem stosowanym uprzednio. Wskazano na destabilizujący efekt zapylenia i podano miarę Lapunowa dla destabilizacji minimalnej. Podano także optymalną miarę dla optymalnej stateczności stanu równowagi płynu zapylonego.

Проанализирована нелинейная задача устойчивости течений в запыленных жидкостях, занимающих ограниченную область, исправляя при этом ошибочное условие устойчивости предложенное ранее другими авторами. Предложена новая идея, заключающаяся во введении семейства мер Ляпунова, а новые условия устойчивости сравнены между собой, а также с условием применяемым раньше. Указан дестабилизирующий эффект запыления и приведена мера Ляпунова для минимальной дестабилизации. Приведена также оптимальная мера для оптимальной устойчивости состояния равновесия запыленной жидкости.

1. Introduction

STABILITY problems for dusty gas, that is for viscous fluids with suspended particles, play an important role in many applications such as, for instance, in eliminating the pollution problems and in aerosol suspensions in the atmosphere. After the papers of KAZAKEVICH-KRAPIVIN [1] and SPROULL [2], several ones [3–15] have been published on the dynamics of dusty gases; but only in the papers [3–6] is the stability problem considered. The mathematical model adopted is the model introduced by SAFFMAN in [3]. In his paper Saffman considers only the linear stability of plane laminar dusty gas flows and the effects of the dust particles on the critical Reynolds number from laminar to turbulent flows. The conclusion was that coarse dust particles have a stabilizing effect on the dust-gas system. On the contrary, he proved that fine dust destabilizes the flow. Only linear stability is considered in [4]. Nonlinear stability aspects have, nevertheless, not received adequate

(*) This research was supported by GNFM-CNR, by a grant of the Italian Ministry of Education 60% and by 40% — "Problemi di Evoluzione nei Fluidi e nei Solidi".

attention. In fact, to our knowledge, the nonlinear stability was studied only in two papers [5, 6]. Unfortunately, in [5], indicated in the sequel with D. G., there is a mistake — as we shall prove — that questions the reliability of some results found there. In [6] we have investigated the nonlinear Lyapunov stability of some hydrodynamic laminar dusty flows in plane layers and some magnetohydrodynamic dusty flows in a bounded domain. In this paper [6] we have introduced a new idea that reverses, in some sense, the usual energy method — that gives sufficient conditions for nonlinear stability in the mean once the Lyapunov functionals, as a measure of the perturbations, are *prefixed*. In this paper we have considered some parametric families of equivalent energy measures depending on not *a priori prefixed* constants and we have investigated the stability, obtaining some best measures for the best, that is absolute unconditional stability.

In the present paper, adopting the Saffman's model, we investigate the nonlinear stability for dusty fluids in an arbitrary bounded domain subject to arbitrary three-dimensional perturbations.

After the model has been recalled in Sect. 2, in Sect. 3 — by the classical energy method — we correct the mistake of D. G. in [5], we obtain the correct stability conditions and we recover the instabilizing effect of the dust on the fluid. In Sect. 4, following the idea introduced in our last paper [6] and applying a stability theorem given in [16], we obtain new stability conditions with respect to families of Lyapunov functionals chosen as measures for the perturbations. Moreover, in Sect. 5 we discuss and compare all these conditions and we find a subfamily of measures that gives stability conditions better than our (correct) conditions given by D. G. in [5].

In a remark of Sect. 5, we give a measure leading to the minimum instabilizing effect. Finally, in Sect. 6 we give a stability condition, in the considered family of Lyapunov measures, for the equilibrium of a dusty fluid and we find the best measure for the best stability of this equilibrium, that is for absolute unconditional stability.

2. The mathematical model

The equations governing the hydrodynamics of a dusty fluid, following Saffman's incompressible model [3], are

$$\begin{aligned} \varrho\left(\frac{\partial \mathbf{V}}{\partial t}+\mathbf{V}\cdot\nabla\mathbf{V}\right) &= -\nabla p+\mu\Delta\mathbf{V}-KN(\mathbf{V}-\mathbf{v}),\\ \nabla\cdot\mathbf{V} &= 0,\\ mN\left(\frac{\partial \mathbf{v}}{\partial t}+\mathbf{v}\cdot\nabla\mathbf{v}\right) &= KN(\mathbf{V}-\mathbf{v}),\\ \nabla\cdot\mathbf{v} &= 0. \end{aligned} \tag{2.1}$$

In these equations $\mathbf{V}$ and $\mathbf{v}$ are the fluid and the dust velocities, respectively. Moreover, N is the number density of dust particles, each of mass m; K is the Stokes coefficient of resistance; p, ϱ, μ are pressure, density and viscosity of the fluid. In this model the simplifying assumptions made by SAFFMAN [3] also hold. The introduction of the following parameters will be useful:

$\nu = \mu/\varrho$, the kinematic viscosity of the clean fluid;

$r = KN/\varrho$, dimension of frequency;

$\tau = m/K$, dimension of time, relaxation time of dust particles;

$f = mN/\varrho = r\tau$ mass concentration of the dust;

$s = 1/\tau = K/m$.

Of course, to complete the evolution problem, suitable initial and boundary conditions must be added to Eq. (2.1).

3. Nonlinear stability problem and instabilizing effect of the dust

Referring to the D. G. paper [5], we intend to show and to correct a mistake in the stability conditions of Sect. 2., following the same classical technique of energy method adopted there.

Let us consider an arbitrary basic flow in a bounded domain S, i.e., let us consider a solution of the system (2.1) belonging to the class

$$\{\mathbf{V}(P,t); \mathbf{v}(P,t); p\} \tag{3.1}$$

that we assume regular — that is with $\mathbf{V}(P,t) \in C^{(2,1)}$: $\mathbf{v}(P,t) \in C^{(1,1)}$; $p \in C^{(1)}$ $\forall (P,t) \in S \times I$ — satisfying the boundary conditions:

$$\mathbf{V}(P,t) = \mathbf{v}(P,t) = 0, \quad \forall (P,t) \in \partial S \times I \tag{3.2}$$

and the initial conditions

$$\begin{aligned} \mathbf{V}(P,0) &= \mathbf{V}_0(P), \\ \mathbf{v}(P,0) &= \mathbf{v}_0(P), \quad \forall P \in S \end{aligned} \tag{3.3}$$

with given (divergence-free) $\mathbf{V}_0$, $\mathbf{v}_0$ and I denoting the time interval $[0, \infty]$ of the motions.

Let

$$\{\mathbf{U}(P,t); \mathbf{u}(P,t); \pi\} \tag{3.4}$$

be the class of three-dimensional perturbations of one of the flows Eq. (3.1) under the initial conditions (3.3); $\mathbf{U}$, $\mathbf{u}$, π denote the perturbed fluid velocity, dust velocity and pressure, respectively.

From Eqs. (2.1)–(3.4), we obtain that the perturbations have to satisfy the following dimensionles equations:

$$\begin{aligned} \frac{\partial \mathbf{U}}{\partial t} &= -R(\mathbf{V}+\mathbf{U}) \cdot \nabla \mathbf{U} - R\mathbf{U} \cdot \nabla \mathbf{V} - \nabla \pi + \Delta \mathbf{U} - R_1(\mathbf{U}-\mathbf{u}), \\ \frac{\partial \mathbf{u}}{\partial t} &= -R(\mathbf{v}+\mathbf{u}) \cdot \nabla \mathbf{u} - R\mathbf{u} \cdot \nabla \mathbf{v} + R_2(\mathbf{U}-\mathbf{u}), \\ \nabla \cdot \mathbf{U} &= \nabla \cdot \mathbf{u} = 0. \end{aligned} \tag{3.5}$$

The dimensionless variables are introduced:

$$t = d^2/\nu t^*, \quad x = x^* d, \quad \mathbf{U} = W_0 \mathbf{U}^*, \quad \mathbf{u} = W_0 \mathbf{u}^*,$$

with d and W_0 being suitable reference length and velocity. In Eqs. (3.5), in which the stars have been omitted, the following dimensionless numbers appear:

$$R = W_0 d/\nu, \quad R_1 = rd^2/\nu, \quad R_2 = sd^2/\nu;$$

that is, the Reynolds number for the corresponding clean fluid and the two numbers R_1, R_2, of the Reynolds type, for the dust. Moreover, the perturbed velocities $\mathbf{U}$, $\mathbf{u}$ must satisfy the homogeneous boundary conditions

$$\mathbf{U}(P, t) = \mathbf{u}(P, t) = 0, \quad \forall (P, t) \in \partial S \times I \tag{3.6}$$

and the initial conditions

$$\mathbf{U}(P, 0) = \mathbf{U}_0(P), \quad \mathbf{u}(P, t) = \mathbf{u}_0(P), \quad \forall P \in S \tag{3.7}$$

with given (divergence-free) $\mathbf{U}_0$ and $\mathbf{u}_0$.

At this point the stability problem of a flow of class (3.1), that is (from the analytical point of view) the behaviour of the strong solutions of the system (2.1) or of the zero solution of the system (3.5) is reduced to the choice of a Lyapunov functional as a measure for the perturbations (3.4), and to the choice of the initial values (3.7) for these perturbations.

Proceeding like in the D. G. paper, let us introduce the energy measure of the perturbations:

$$E(t) = K_1(t) + K_2(t) = \frac{1}{2}\int_S U^2 dS + \frac{1}{2}\int_S u^2 dS. \tag{3.8}$$

Multiplying Eqs. $(3.5)_1$ and $(3.5)_2$ by $\mathbf{U}$ and $\mathbf{u}$, respectively, and integrating over S, we find:

$$\frac{dK_1}{dt} + 2R_1 K_1 = -\int_S [R\mathbf{U}\cdot\mathbf{D}\cdot\mathbf{U}^2 + (\nabla\mathbf{U})^2 - R_1\mathbf{U}\cdot\mathbf{u}]dS, \tag{3.9}$$

$$\frac{dK_2}{dt} + 2R_2 K_2 = -\int_S [R\mathbf{u}\cdot\mathbf{D}'\cdot\mathbf{u}^2 - R_2\mathbf{U}\cdot\mathbf{u}]dS, \tag{3.10}$$

where D and D' are the deformation tensors of the fluid and the dust, respectively. From Eqs. (3.6), (3.8), (3.9) and (3.10), by classical integral inequalities and by the divergence theorem, it follows that

$$\frac{dK_1}{dt} + 2R_1 K_1 \leqslant 2(R_f - \gamma^2)K_1 + 2R_1(K_1 K_2)^{1/2}, \tag{3.11}$$

$$\frac{dK_2}{dt} + 2R_2 K_2 \leqslant 2R_s K_2 + 2R_2(K_1 K_2)^{1/2}, \tag{3.12}$$

with γ^2 denoting the Poincaré constant for the domain S, and where two other dimensionless numbers appear, that is,

$$R_f = \frac{md^2}{\nu}, \quad R_s = \frac{m'd^2}{\nu}.$$

Here $-m$ and $-m'$ are the least (dimensional) characteristic values of D and D'. Putting $K_1 = \theta^2$ and $K_2 = \Gamma^2$, as in [5] of D. G., we have

$$\frac{d\theta}{dt}+(R_1+\gamma^2-R_f)\theta-R_1\Gamma \leqslant 0, \tag{3.13}$$

$$\frac{d\Gamma}{dt}+(R_{2-}R_s)\Gamma-R_2\theta \leqslant 0. \tag{3.14}$$

Summing up we find the relation

$$\frac{d(\theta+\Gamma)}{dt}+(R_1+\gamma^2-R_f-R_2)\theta+(R_2-R_1-R_s)\Gamma \leqslant 0 \tag{3.15}$$

which, for $R_1 = N_0$, $\gamma^2 = \delta$ and $R_2 = G$, is exactly the same as Eq. (30) of D. G. From Eq. (3.15), D. G. derives the conditions

$$R_2 > R_1+R_s, \tag{3.16}$$

$$\gamma^2 > R_2-R_1+R_f \tag{3.17}$$

that imply

$$\theta_+\Gamma \leqslant (\theta_+\Gamma)_0 e^{-Lt} \tag{3.18}$$

with $L = \min\ (R_1+\gamma^2-R_f-R_2;\ R_2-R_1-R_s)$

At this point D. G. gives the following theorem:

A sufficient condition for stability of the flow of a dusty gas is that

$$\begin{aligned} R_2 &\geqslant R_1+R_s,\\ \gamma^2 &> R_f+R_s. \end{aligned} \tag{3.19}$$

Now the error made in the D. G. paper consists in passing from the conditions (3.16) (3.17) — that really assure asymptotic exponential stability — to the conditions (3.19) that, as it will be shown, cannot assure stability! In fact, from the condition (3.16) it simply follows that

$$R_2-R_1 > R_s \quad \text{implies} \quad R_2-R_1+R_f > R_s+R_f, \tag{3.20}$$

therefore the condition $(3.19)_2$, that is $\gamma^2 > R_f+R_s$ is not sufficient to guarantee the condition (3.17), because from Eqs. $(3.19)_2$ and (3.20) it could occur that

$$R_f+R_s \leqslant \gamma^2 \leqslant R_2-R_1+R_f.$$

So, Eq. $(3.19)_1$ being true, the condition $(3.19)_2$ is not enough to assure Eq. (3.17), that is the positivity of the term $R_1+\gamma^2-R_f-R_2$. In fact,

$$\gamma^2 \geqslant R_s+R_f \Rightarrow R_1+\gamma^2-R_f-R_2 \geqslant R_s+R_f+R_1-R_f-R_2 = R_1+R_s-R_2$$

and the last term, by Eq. $(3.19)_1$ is not positive!

In conclusion, following the procedure adopted in D. G.'s paper, *the correct stability conditions in the measure* (3.8) *are*

$$\begin{aligned} R_2 &> R_1+R_s,\\ \gamma_2 &> R_2-R_1+R_f. \end{aligned} \tag{3.21}$$

Nevertheless, even if the correct stability conditions are not the inequalities (3.19) but the inequalities (3.21), the conclusion given in Sect. 4 (Discussion) of D. G.'s paper on the instabilizing effect of the dust on the fluid, remains valid because from the conditions (3.21), it follows that (provided $R_2 - R_1 > R_s$ holds) the flows are stable if

$$R_f < \gamma^2 - (R_2 - R_1), \; (< \gamma^2 - R_s).$$

From this inequality we obtain the condition $R_f < \gamma^2$ for a clean gas $(R_1 = R_2 = 0)$, and the instabilizing effect of the dust on the flows is proved.

4. New stability conditions

Following the idea introduced in our recent paper [6], we shall introduce *not a prefixed* Lyapunov functional as a measure for the perturbations, but a family of Lyapunov measures depending on arbitrary positive parameters, looking for stability conditions better than he inequalities (3.21). Let us introduce the family of equivalent energy measures

$$E(t) = \frac{1}{2} \int_S (c_1 U^2 + c_2 u^2) dS \tag{4.1}$$

with c_1, c_2 being arbitrary positive constants. Multiplying Eqs. $(3.5)_1$ and $(3.5)_2$ by $c_1 \mathbf{U}$ and $c_2 \mathbf{u}$, respectively, adding and integrating over S, we find

$$\frac{dE}{dt} = -\int_S [c_1 R \mathbf{U} \cdot \mathbf{D} \cdot \mathbf{U} + c_1 (\nabla \mathbf{U})^2 + c_1 R_1 U^2 - c_1 R_1 \mathbf{U} \cdot \mathbf{u} + c_2 R \mathbf{u} \cdot \mathbf{D}' \cdot \mathbf{u} - c_2 R_2 \mathbf{U} \cdot \mathbf{u} + c_2 R_2 u^2] dS \tag{4.2}$$

whence, by introducing the functionals

$$X_1^2 = \int_S U^2 dS, \qquad X_2^2 = \int_S u^2 dS \tag{4.3}$$

and by suitable classical integral inequalities, it follows

$$\frac{dE}{dt} \leqslant -\phi \tag{4.4}$$

with

$$\phi = c_1 (\gamma^2 - R_f + R_1) X_1^2 - (c_1 R_1 + c_2 R_2) X_1 X_2 + c_2 (R_2 - R_s) X_2^2. \tag{4.5}$$

From Eqs. (4.4) and (4.5), by the Cauchy inequality, it follows that the conditions

$$\begin{aligned} R_f &< \gamma^2 - \frac{1}{2}\left(\frac{c_2}{c_1} R_2 - R_1\right), \\ R_s &< \frac{1}{2}\left(R_2 - \frac{c_1}{c_2} R_1\right), \end{aligned} \tag{4.6}$$

assure the asymptotical exponential stability of the dusty flows in the class (3.1), with respect to the family of Lyapunov measures (4.1).

From the same Eqs. (4.4) and (4.5), by the theorem given in [16], once we have imposed the definite positivity of the quadratic form (4.5), we conclude that also the conditions

$$R_f < \gamma^2 - \frac{4c_1c_2R_1R_s + (c_1R_1 - c_2R_2)^2}{4c_1c_2(R_2 - R_s)}, \quad R_s < R_2 \tag{4.7}$$

assure the asymptotical exponential stability of the dusty flows of (3.1), in the same family of Lyapunov measures (4.1).

Moreover, from the conditions (4.7) we have, in particular, that the conditions

$$R_f < \gamma^2 - \frac{R_1R_s}{R_2 - R_s}, \quad R_s < R_2 \tag{4.8}$$

assure the asymptotic exponential stability of these dusty flows in the measure (4.1), with $c_1 = R_2$, $c_2 = R_1$.

5. Discussion, comparison and improvement

Finally, let us now discuss and compare the stability conditions (3.21) of D. G. and our conditions (4.6), (4.7) and (4.8) and find the measures that guarantee the stability conditions better than the conditions (22). Moreover, we want to choose, among these conditions, the best one.

For the sequel it is useful to take into account that

$$\frac{R_1}{R_2} = f,$$

where f is the mass concentration of the dust. We stress the role that f will play in following comparisons.

1) Comparison between the condition (3.21) (D. G. corrected) and (4.6):
if the mass concentration of the dust satisfies the condition

$$f < 2,$$

the stability conditions (4.6) can be better than (3.21) in the subfamily of measures (4.1) with

$$\frac{c_2}{c_1} < \min(1/2;\ 2-f).$$

2) Comparison between the conditions (3.21) and (4.8):
the condition (4.8) can be better than (3.21) if

$$f > 1 - R_s/R_2 \tag{5.1}$$

holds, that is if the condition $(4.8)_1$ is replaced with the condition $(3.21)_1$. We note that the stability condition (3.21) of D. G. has been obtained in the measure of the family (4.1) with $c_1 = c_2 = 1$. Therefore if the inequality (5.1) holds, with the measures $c_1 = c_2 = 1$

and $c_2/c_1 = f$ adopted in the conditions (4.8), in this second one, better stability conditions are obtained.

3) Comparison between the conditions (3.21) and the condition obtained from (4.7) in the same measure ($c_1 = c_2 = 1$) adopted by D. G.:

if the mass concentration of the dust satisfies the inequality

$$f < 2\sqrt{1-R_s/R_2}-1$$

from the conditions (4.7), with $c_1 = c_2 = 1$, our stability condition

$$(5.2) \qquad R_f < \gamma^2 - \frac{4fR_s+(f-1)^2}{4(1-R_s/R_2)}, \qquad R_s < R_2$$

is better than the conditions (3.21).

4) Comparison between the conditions (3.21) and (4.7):

if the same stability condition (5.1) holds, there exists a subfamily of measures

$$\delta_1 < \frac{c_1}{c_2} < \delta_2$$

with positive roots δ_1, δ_2 of the equation

$$f^2c_1^2-2[2(1-R_s/R_2)-f]c_1c_2+c_2^2 = 0,$$

in which our conditions (4.7) are better than (3.21).

5) Comparison between our conditions (4.6) and (4.7):

the condition (4.6) is better than (4.7) in the subfamilies

$$\frac{c_2}{c_1} < f.$$

On the contrary, the condition (4.7) is better than (4.6) in subfamilies

$$\frac{c_2}{c_1} > f.$$

Of course, in the measure $c_2/c_1 = f$, the condition (4.6) cannot exist.

6) Finally, if it is

$$f < \frac{1}{2}$$

among the conditions (3.21) of D. G., and our conditions (4.6) and (4.7), the (4.7) in all the measures such that best stability condition is

$$f < \frac{c_2}{c_1} < \frac{1}{2}.$$

REMARK. *From the condition* (4.7), *the instabilizing effect of the dust on the fluid is obvious, with any measure of the family* (4.1) *we consider*. But it is possible to minimize this instabilizing effect. In fact, this occurs when

$$R_f < \gamma^2+R_1-Z_{\min}$$

with

$$Z = \frac{(c_1 R_1 + c_2 R_2)^2}{4c_1 c_2 (R_2 - R_s)}.$$

We emphasize that such a minimum ((c_1, c_2) varying in $]0, \infty[\times]0, \infty[$), occurs exactly in the measure

$$\frac{c_2}{c_1} = f.$$

6. Unconditional stability of the equilibrium

Let us consider now the dusty fluid in equilibrium in S. In this case we have $\mathbf{V} = \mathbf{v} = m = m' = R_f = R_s = 0$. From the conditions (4.7) we have the following stability condition:

$$\frac{(c_1 R_1 - c_2 R_2)^2}{4c_1 c_2 R_2} < \gamma^2 \tag{6.1}$$

in all the measures of the family (4.1).

For this particular measure of the above minimum instabilizing effect,

$$\frac{c_2}{c_1} = f \tag{6.2}$$

we clearly have absolute unconditional asymptotic exponential stability.

Therefore the measure (6.2) is the best measure for the best stability of the equilibrium.

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Received January 16, 1989.