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Unsteady viscous shock layer near permeable surface

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PROBLEM of a shock layer confined between an axisymmetric, vibrating blunt body and a detached shock wave is analyzed. Fields of velocity, pressure, density etc. are represented by power expansions with respect to the distance from the symmetry axis, coefficients of the expansion depending on time and on the distance measured in the transversal direction. Nonstationarity is the result of variable axial, tangential and angular velocities. The problem is solved numerically.

Przeanalizowano problem warstwy uderzeniowej zawartej między osiowo-symetrycznym tępym, wirującym ciałem, a odsuniętą falą uderzeniową. Pola prędkości, ciśnienia i gęstości przedstawia się w postaci rozwinięć potęgowych względem zmiennej odpowiadającej odległości od osi symetrii. Współczynniki rozwinięcia zależne są od czasu i od odległości w kierunku poprzecznym. Niestacjonarność jest wynikiem zmian prędkości w kierunku osiowym, stycznym i obwodowym. Problem rozwiązano numerycznie.

Проанализирована задача ударного слоя, содержавшегося между осесимметричным тупым, вращающимся телом и отошедшей ударной волной. Поля скорости, давления и плотности представляются в виде степенных разложений по отношению к переменной, отвечающей расстоянию от оси симметрии. Коэффициенты разложения зависят от времени и от расстояния в поперечном направлении. Нестационарность является результатом изменений скорости в осевом, касательном и периметрическом направлениях. Задача решена численно.

1. Introduction

IN RECENT PAPERS on the numerical calculation of viscous gas flow, it was shown that the calculation efficiency may be enhanced by using the boundary-layer methods and appropriate scales both in the complete Navier–Stokes equations and in simplified composite asymptotic equations [1–8].

The asymptotic theory of high-Reynolds-number flows yields the scales and mechanism of interaction between shear layers and outer locally-inviscid flow. For moderate values of the perturbation parameter, the composite equations often give better results than the higher-order asymptotic equations. On the other hand, if for high Reynolds numbers the correct scales of the quantities in question are not taken into account, then the solution obtained will not be sufficiently exact whatever equations are used, including complete Navier–Stokes equations.

In the present paper the flow in a $3D$ viscous shock layer is analysed asymptotically, and a composite system of equations of a $3D$ shock layer is derived for different flow regimes where $\alpha \to 0$, $\varepsilon \to 0$, $\delta \to 0$ ($\alpha^2 = (\gamma-1)/(\gamma+1)$, $\varepsilon^2 = 1/\mathrm{Re}_0$, $\delta^{-1} = (\gamma-1)\, M_\infty^2$).

The unsteady equations of a viscous shock layer are simplified in the neighbourhood of the stagnation point of a blunt body which rotates about its symmetry axis at an angular velocity of Ω_1 and is flown around by a hypersonic stream with a vorticity of Ω. We have also simplified the unsteady equations for the neighbourhood of the stagnation line on a wing of infinite span flown at the sweep angle β_0. The equations for a thin shock layer have been solved numerically using an implicit finite-difference iteration method of so-called subdivided iterations, based on the scalar "progonka" process. The solution thus obtained was employed as the initial approximation to solve general equations for a viscous shock layer, using an implicit finite-difference method with Newton iterations in conjunction with the vector "progonka" and for the equations written in the conservative form. The linearization error was estimated in the course of solution and did not exceed the approximation error. The partial derivatives were approximated on nonuniform networks that took into account the solution scales.

Calculation were performed for shock layers near a wing and near a rotating body, including unsteady effects caused by blowing-in, rotation of the body, and external vorticity. Calculations were confined to the neighbourhood of the wing critical line and to the neighbourhood of the stagnation point.

2. Asymptotic analysis of flow regimes

We shall write the Navier–Stokes equations in a curvilinear coordinate system $q = (q^1, q^2, q^3)$ attached to the body surface, assuming that the coordinate line q^3 is orthogonal to the body surface and to the coordinate lines q^1 and q^2. Let u^i denote contravariant velocity components in the system q, ϱ — density, and h — enthalpy.

Depending on the relation between the perturbation parameters $\alpha^2 = (\gamma-1)/(\gamma+1)$, $\varepsilon^2 = \mathrm{Re}_0^{-1}$, $\delta^{-1} = (\gamma-1)\, M_\infty^2$ and the body surface curvature K (see [3]), the shock layer is either completely viscous or contains an inviscid region and a viscous sublayer whose thickness depends on the relation between the perturbation parameters.

The thickness Δ of the shock layer is determined by the body geometry and by the parameters of the oncoming flow. Estimates [3] lead to the following result: $O(k) \leqslant \Delta \leqslant O\sqrt{k}$, where k is the ratio of the densities on either side of the bow shock. If the principal curvatures K_1 and K_2 of the body surface satisfy the inequalities $0 < K_i < \infty$, $i = 1, 2$ for $\alpha \to 0$, then $\Delta = O(k) = O(\alpha^2)$. In the case of very blunt bodies $K_i = \alpha K_i^*$, $K_i^* \neq 0$ we have $\Delta = 0\left(\sqrt{k}\right) = O(\alpha)$.

Consider a shock layer of thickness $\Delta = O(\alpha^D)$, $0 < D \leqslant 2$. The pressure gradients $\partial P/\partial q^i$, $i = 1, 2$ are appreciable in the sublayer $0 \leqslant q^3 \leqslant \Delta_p$ of thickness $\Delta_p = 0(\alpha^{D+1})$ (cf. [3] for $D = 1, 2$).

2.1. Regime $\varepsilon^2 = O(\alpha^{2D-2})$

The equations for the leading terms in the expansion of the vector function $f = (u^1, u^2, u^3, \varrho, h)^T$ with respect to a parameter α are similar to those for a boundary layer, but in this regime the longitudinal pressure gradients $\partial P/\partial q^i$, $i = 1, 2$ can be neglected. The equation for the normal velocity component is then reduced to the form $\dfrac{\partial P}{\partial q^3} =$

$= \frac{1}{2}\frac{\partial g_{lm}}{\partial q^3}\varrho u^l u^m$. For a given regime the entire shock layer is viscous, i.e., $\Delta_v = \Delta$ (Δ_v is the layer thickness within which the viscosity is essential) and $\Delta_v > \Delta_p$.

At the outer boundary the equations are closed by relations behind the bow shock, which contain terms with viscous stress and thermal flux [4].

2.2. Intermediate regimes $\varepsilon^2 = O(\alpha^{2D-2+L})$, $1 \leqslant L \leqslant 3$

For a given family of regimes L is a parameter. $\varepsilon^2 = N\alpha^{2D-2+L}$, $N > 0$.

The shock layer of thickness $\Delta = O(\alpha^D)$ consists of an inviscid part $\Delta_v \leqslant q^3 \leqslant \Delta$ and a viscous sublayer $0 \leqslant q^3 \leqslant \Delta_v$ of thickness $\Delta_v = O(\alpha^{D+L/3})$.

At the outer boundary we impose the Rankine–Hugoniot conditions. The pressure gradients $\partial P/\partial q^i$, $i = 1, 2$ have a relative order of $O(\alpha^{2-2L/3})$ and can be neglected for $L < 3$ but they become important for $L = 3$ when the viscous sublayer thickness $\Delta_v = O(\alpha^{1+D})$ and $\Delta_v = \Delta_p$.

2.3. Regime $\varepsilon^2 = O(\alpha^{2D-2+L})$, $L > 3$

For this limiting process $\Delta_v = O(\varepsilon\sqrt{\alpha})$ (cf. [3]) and $\Delta_v \ll \Delta_p$, so that $\Delta_v < \Delta_p < \Delta$. The shock layer is now subdivided into an inviscid external part G_1, $\Delta_p \leqslant q^3 \leqslant \Delta$, of thickness $O(\alpha^D)$, an inviscid transitional part G_2, $\Delta_v \leqslant q^3 \leqslant \Delta_p$, of thickness $O(\alpha^{1+D})$, and a viscous sublayer G_3, $0 \leqslant q^3 \leqslant \Delta_v$, of thickness $O(\varepsilon\sqrt{\alpha})$. In the region G_1 the pressure gradients $\partial P/\partial q^i$, $i = 1, 2$ can be neglected, in the first approximation, and variation of the pressure across G_1 depends on all the terms appearing in the equation of motion for the normal component for the inviscid flow. In the region G_2, the gradients $\partial P/\partial q^i$ must be taken into account, but the viscous effects can be neglected. In the first approximation, the pressure does not change across the layer $0 \leqslant q^3 \leqslant \Delta_p$ for regimes 2.2 and 2.3.

The asymptotic analysis performed in this paper justifies the composite system of equations for a shock layer of thickness $\Delta = O(\alpha^D)$ (see [3]) which contains all essential terms for the regimes considered above. Unlike classical equations for a 3-D boundary layer in our system of equations, the longitudinal pressure gradients $\partial P/\partial q^i$ $i = 1, 2$ are not given a priori, and pressure variations across the shock layer take into account all the terms in the equations for the inviscid flow projected onto the normal to the body surface.

3. Conservative form of viscous shock layer equations

We shall write the composite system of equations for the viscous shock layer (VSL) in a strictly conservative form that is convenient for numerical calculations of flows with internal shocks, shear layers and discontinuities of parameters of blowing. We shall introduce some notations. Let (y_1, y_2, y_3) be Cartesian coordinates, $(q^1, q^2, q^3) \equiv (\xi, \eta, \zeta)$ — curvilinear coordinates of a point M, $(u^1, u^2, u^3) \equiv (u, v, w)$ — contravariant components of velocity. Let us introduce the covariant basis $\mathbf{a}_i = \{\partial y_1/\partial q^i, \partial y_2/\partial q^i, \partial y_3/\partial q^i\}$ and the contravariant basis $\mathbf{a}^i = \{\partial q^i/\partial y_1, \partial q^i/\partial y_2, \partial q^i/\partial y_3\}$, then the components g^{ij}, g_{ij},

$i, j = 1, 2, 3$ of the metric tensor can be expressed as follows: $g_{ij} = \mathbf{a}_i \cdot \mathbf{a}_j$, $g^{ij} = \mathbf{a}^i \cdot \mathbf{a}^j$. Let us denote $g = \det ||g_{ij}||$ and consider the vectors $\mathbf{a}^i$, $\mathbf{a}_j$ at two points M and M_0, employing index zero for point M_0. Following the paper [8], we shall introduce the coefficients G_k^j:

$$G_k^j(M, M_0) = \mathbf{a}_0^j \cdot \mathbf{a}_k.$$

We have the following relations of orthogonality

$$\mathbf{a}^i \cdot \mathbf{a}_j = \delta_j^i, \qquad G_k^j(M_0, M_0) = \delta_k^j,$$

$\delta_j^i = 1$ for $i = j$ and $\delta_j^i = 0$ for $i \neq j$.

The VSL equations can be written in the following form for the point M_0 (index zero is omitted):

$$\begin{aligned}(3.1)\qquad &\frac{\partial}{\partial t}(\sqrt{g}\,\varrho u^l) + \frac{\partial}{\partial q^j}\{\sqrt{g}\,G_k^l(pg^{jk} + \varrho u^j u^k)\} - \frac{1}{\mathrm{Re}_0}\frac{\partial}{\partial \zeta}\left\{\sqrt{g}\,G_k^l\left[\frac{\varkappa g^{k3}}{\sqrt{g}}\frac{\partial}{\partial \zeta}(\sqrt{g}\,w)\right.\right.\\ &\left.\left. + \mu g^{33}\frac{\partial}{\partial \zeta}(G_s^k u^s - u^k) + \mu g^{k3}\frac{\partial}{\partial \zeta}(G_S^3 u^S - w)\right]\right\} = 0,\end{aligned}$$

$$(3.2)\qquad \frac{\partial}{\partial t}(\sqrt{g}\,\varrho) + \frac{\partial}{\partial q^j}(\sqrt{g}\,\varrho u^j) = 0,$$

$$\begin{aligned}(3.3)\qquad &\frac{\partial}{\partial t}[\sqrt{g}\,\varrho(e + e_k)] + \frac{\partial}{\partial q^j}(\sqrt{g}\,\varrho u^j H) - \frac{1}{\mathrm{Re}_0}\frac{\partial}{\partial \zeta}\left\{\sqrt{g}\left[\frac{\mu}{\mathrm{Pr}}g^{33}\frac{\partial h}{\partial \zeta}\right.\right.\\ &\left.\left. + \frac{\varkappa w}{\sqrt{g}}\frac{\partial}{\partial \zeta}(\sqrt{g}\,w) + \mu g_{kl}g^{33}u^l\frac{\partial}{\partial \zeta}(G_s^k u^s - u^k) + \mu w\frac{\partial}{\partial \zeta}(G_s^3 u^s - w)\right]\right\} = 0,\end{aligned}$$

$$(3.4)\qquad \begin{aligned}&H = h + e_k, \quad e_k = \frac{1}{2}g_{ij}u^i u^j, \quad e = \frac{h}{\gamma}, \quad p = \frac{(\gamma - 1)}{\gamma}\varrho h,\\ &\mu = h^w, \quad \varkappa = -\frac{2}{3}\mu + \mu_2, \quad \frac{1}{2} \leqslant \omega < 1,\end{aligned}$$

μ_2 is the second coefficient of viscosity ($\mu_2 \ll 1$).

Summation over repeated indices in the range 1, 2, 3 is assumed.

4. Unsteady shock layer near stagnation stream line of rotating body

Consider the above equations for a shock layer near the stagnation point of an axisymmetric cooled surface rotating about the longitudinal axis with an angular velocity of Ω_1.

We shall assume that in the neighbourhood of the symmetry axis the parameters of the oncoming flow satisfy the conditions rot $V'_\infty = (\Omega, 0, 0)$, $V'_\infty = (u'_\infty, u'_{1\infty}, u'_{2\infty})$, $\varrho'_\infty =$ const, $p'_\infty = p'_{0\infty} + r'^2\varrho'_\infty\Omega^2/2$ (prime notation refers to dimensional quantities, r is the distance to the x axis). The system of equations is closed by the Rankine–Hugoniot relations behind the bow shock and by the relations $u^1 = u_*^1$, $u^2 = \Omega_1 q^1\sqrt{g_{22}}$, $u^3 = u_*^3$, $h = h_w$ at the body surface. The quantities $u_*^1(t)$, $\Omega_1(t)$, $u_*^3(t)$ are known functions of time. For $t = 0$ some steady-state distributions are also given, which correspond to a steady-state flow.

An approximate solution is sought in the form of expansions:

(4.1) $u(1) = \xi\big(u_0''(\xi, t)+ \dots\big), \quad u(2) = \xi\big(v_0''(\zeta, t)+ \dots\big), \quad u(3) = w_0''(\zeta, t)+ \dots,$

(4.2) $p = p_0''(\zeta, t)+\xi^2(p_1(\zeta, t)/2+ \dots), \quad \varrho = \varrho_0''(\zeta, t)+ \dots, \quad h = h_0''(\zeta, t)+ \dots.$

In these expansions the dots denote quantities vanishing for $\xi \to 0$; $u(1) = u^1\sqrt{g_{11}}$, $u(2) = u^2\sqrt{g_{22}}$, $u(3) = u^3$, $q^1 = \xi$, $q^2 = \eta$, $q^3 = \zeta$, $u(i)$, $i = 1, 2, 3$ are physical contravariant components of the velocity vector, ξ varies along the body surface, ζ, along the normal to the body, and η is the angular coordinate.

Introduce the notations

$$u_0 = u_0''/\sqrt{g_{11}}, \quad v_0 = v_0''/\sqrt{g_{11}}, \quad w_0 = w_0'', \quad H_0 = h_0+w_0^2/2,$$
$$\varrho_0 = g_{11}\varrho_0'', \quad \mu_0 = \mu_0'' g_{11}, \quad p_\beta = g_{11}p_\beta'', \quad \beta = 0, 1.$$

The system of equations for the leading terms of the expansions (4.1) and (4.2) can be written in the form (the index 0 is omitted)

$$\frac{\partial E(f)}{\partial t}+\frac{\partial A(f)}{\partial z}+B(f) = \frac{\partial}{\partial z}C(f)\frac{\partial f}{\partial z}, \tag{4.3}$$

$$\frac{1}{\Delta}\frac{\partial p_1}{\partial z} = 2K\varrho(u^2+v^2), \quad p = \frac{(\gamma-1)}{\gamma}\varrho h, \tag{4.4}$$

$f = (u, v, w, \varrho, h)^T$, $z = \zeta/\Delta(t)$, the vectors E, A, B and the matrix C are functions of the vector f whose components are dependent variables.

$$\begin{gathered}
E_1 = \varrho u, \quad E_2 = \varrho v, \quad E_3 = \varrho w, \quad E_4 = \varrho, \quad E_5 = \varrho(h/\gamma+w^2/2),\\
A_1 = \varrho uw, \quad A_2 = \varrho vw, \quad A_3 = \varrho w^2+p, \quad A_4 = \varrho w, \quad A_5 = \varrho wH,\\
B_1 = p_1+\varrho(3u^2+2uw-v^2), \quad B_2 = 2\varrho v(2u+w),\\
B_3 = 2\varrho w, \quad B_4 = 2\varrho u, \quad B_5 = 2\varrho uH-\frac{4}{3}\frac{\partial}{\partial z}\left(\frac{\mu}{\Delta^2}\frac{w}{\mathrm{Re}_0}\frac{\partial w}{\partial z}\right),\\
C_{mn} = 0, \quad m \neq n, \quad m = 1, \dots, 5, \quad n = 1, \dots, 5,\\
C_{33} = C_{11} = C_{22} = \mu\Delta^{-2}\mathrm{Re}_0^{-1}, \quad C_{44} = 0, \quad C_{55} = C_{11}/\mathrm{Pr}, \quad H = h+w^2/2.
\end{gathered} \tag{4.5}$$

The boundary conditions on the surface of the body are of the form

$$\zeta = 0, \quad u = u_*, \quad v = \Omega_1, \quad w = w_*, \quad h = h_w.$$

Here the quantities u_*, w_* determine gas blowing from the surface.

5. Modified shock conditions

Behind the surface of bow shock we employ the modified Rankine–Hugoniot (R–H) conditions (see [3]). Let us denote by $v_\infty(3)$, $v(3)$ the normal and $v_\infty(i)$, $v(i)$, $i = 1, 2$ the tangential components of velocity with respect to the surface S of bow shock; then the R—H conditions take the form

$$w(3)\varrho = w_\infty(3)\varrho_\infty = m,$$

$$v(i)-\mu_*\frac{\partial v(i)}{\partial n} = v_\infty(i), \quad i = 1, 2, \tag{5.1}$$

$$(5.1)\ [\text{cont.}]\quad p+mw(3)-\mu_*\frac{\partial v(3)}{\partial n} = p_\infty+mw_\infty(3),$$

$$v_f p+\varrho\left(h+\frac{1}{2}v^2(3)\right)w(3)-\mu_* v(3)\frac{\partial v(3)}{\partial n}-\lambda_*\frac{\partial h}{\partial n} = v_f p_\infty+\varrho_\infty\left(h_\infty+\frac{1}{2}v_\infty^2(3)\right)w_\infty(3);$$

$$\lambda_* = \frac{\mu}{\mathrm{Re}_0\mathrm{Pr}},\quad \mu_* = \frac{4}{3}\frac{\mu}{\mathrm{Re}_0},\quad \mathrm{Re}_0 = \mathrm{Re}_\infty\cdot T_\infty^\omega,\quad \varrho_\infty = 1,$$

$$(5.2)\qquad h_\infty = \frac{1}{(\gamma-1)M_\infty^2},\quad p_\infty = \frac{1}{\gamma M_\infty^2}+\frac{r^2}{2}\Omega^2,\quad T_\infty^{-1} = \frac{1}{2}(\gamma-1)M_\infty^2+1,$$

$$h = \left(\frac{1}{2}+\frac{1}{(\gamma-1)M_\infty^2}\right)\cdot T,\quad w(3) = v(3)-v_f,\quad w_\infty(3) = v_\infty(3)-v_f,$$

v_f is the velocity of bow shock, the coordinate n is varied on normal direction to the surface S.

Let us simplify the R—H conditions (5.1) for the small region near the stagnation stream line, using the expansions (4.1), (4.2). The following result can be obtained:

$$(5.3)\qquad \begin{gathered} \zeta = \zeta_s,\quad u_0-\mu_{0*}\frac{\partial u_0}{\partial\zeta} = K-w_0\cdot(K_s-K),\quad v_0-\mu_{0*}\frac{\partial v_0}{\partial\zeta} = \Omega,\\ \varrho_0(w_0-\dot\varphi) = -(1+\dot\varphi) = m_0,\\ p_0+m_0(w_0-\dot\varphi)-\mu_{0*}\frac{\partial w_0}{\partial\zeta} = \frac{1}{\gamma M_\infty^2}+m_0^2,\\ h_0+\frac{1}{2}(w_0-\dot\varphi)^2-\frac{\mu_{0*}}{m_0}(w_0-\dot\varphi)\frac{\partial w_0}{\partial\zeta}-\frac{\lambda_{0*}}{m_0}\frac{\partial h_0}{\partial\zeta} = h_{0\infty}+\frac{1}{2}(w_{0\infty}-\dot\varphi)^2, \end{gathered}$$

$$(5.4)\qquad \begin{gathered} p_1 = -\mu_{0*}(K_s-K)^2\frac{\partial w_0}{\partial\zeta}-2\mu_{0*}(K_s-K)\frac{\partial u_0}{\partial\zeta}+\Omega^2+\left(1-\frac{1}{\varrho_0}\right)(K_s^2+v_{2f}),\\ w_{0\infty} = -1,\quad p_{0\infty} = \frac{1}{\gamma M_\infty^2},\quad h_{0\infty} = h_\infty,\quad \mu_{0*} = \frac{4}{3}\frac{\mu_0}{\mathrm{Re}_0},\quad \lambda_{0*} = \frac{\mu_0}{\mathrm{Re}_0\cdot\mathrm{Pr}},\\ K_s^{-1} = K^{-1}+\Delta,\quad \dot\varphi = \frac{d\zeta_s}{dt} = \frac{d\Delta}{dt},\quad v_{2f} = \left.\frac{\partial^2 v_f}{\partial\xi^2}\right|_{\xi=0}. \end{gathered}$$

The quantity v_{2f} can be determined exactly only if calculation of the shape of bow shock has been done for $\xi \neq 0$. We use the approximation $v_{2f} = 0$ (see [3], [5–7]). The indices zero in the relations (5.3) and (5.4) will be omitted below.

6. Viscous shock layer near infinite swept wing

Consider the equations for a viscous shock layer near a smooth infinite-span wing with a slip angle β_0. Let $q^1 = \xi$, $q^3 = \zeta$ be coordinates along the profile of the wing and normal to it, respectively; the coordinate $q^2 = \eta$ is along the generator and $K(\xi)$ is profile curvature. The metric coefficients g_{ij}, g^{ij}, G^i_j depend on the parametrization of the wing surface. We assume

$$g_{11} = \big(1+\zeta K(\xi)\big)^2 a_{11}^2(\xi), \quad g^{11} = 1/g_{11}, \quad g^{ii} = g_{ii} = 1, \quad i = 2,3,$$
$$g_{ij} = 0, \quad i \neq j, \quad g = \det\|g_{ij}\| = g_{11}.$$

If the wing profile is given in polar coordinates by the equation $r = r_w(\theta)$, then we have $\xi = \theta$

$$a_{11} = \sqrt{r_w^2 + r_w'^2}, \qquad K = (r_w^2 - 2r_w'^2 - r_w r_w'')/a_{11}^3,$$

where $r_w' = dr_w/d\theta$, $r_w'' = d^2 r_w/d\theta^2$. The components $N(i)$, $i = 1, 2, 3$ of the vector N, normal to the wing surface, can be written as follows:

$$N(1) = 0, \quad N(2) = (r_w'\sin\theta + r_w\cos\theta)/a_{11}, \quad N(3) = (r_w\sin\theta - r_w'\cos\theta)/a_{11}.$$

For the angle of attack α and swept angle β_0, the components $u_\infty(i)$, $i = 1, 2, 3$ of upstream velocity take the form

$$u_\infty(1) = \cos\beta_0\big(\cos\alpha N(3) - \sin\alpha N(2)\big), \quad u_\infty(2) = \sin\beta_0,$$
$$u_\infty(3) = \cos\beta_0\big(\cos\alpha N(2) + \sin\alpha N(3)\big).$$

We consider below a simplification of equations and R—H conditions for the case $\alpha = 0$ and parabolic profile of wing. In the neighbourhood of the stagnation line, we represent the solution as expansions:

$$\begin{gathered} u(1) = \xi\big(u_0(\xi, t) + \ldots\big), \quad u(2) = v_0(\zeta, t) + \ldots, \quad u(3) = w_0(\zeta, t) + \ldots, \\ p = p_0(\zeta, t) + \xi^2\big(p_1(\zeta, t)/2 + \ldots\big), \quad \varrho = \varrho_0(\zeta, t) + \ldots, \quad h = h_0(\zeta, t) + \ldots. \end{gathered} \tag{6.1}$$

The system of equations for the principal terms of expansions can be written in the vector form (4.3) with the following equation for the pressure:

$$\frac{1}{\Delta}\frac{\partial p_1}{\partial z} = 2K\varrho u^2. \tag{6.2}$$

The components of $E(f)$, $A(f)$, $C(f)$ are given by the relations (4.5), and

$$B_1 = 2\varrho u^2 + p_1, \quad B_2 = \varrho uv, \quad B_3 = \varrho uw, \quad B_4 = \varrho u,$$
$$B_5 = \varrho uH - \frac{4}{3}\frac{\partial}{\partial z}\left(\frac{\mu v}{\Delta^2 \mathrm{Re}_0}\frac{\partial v}{\partial z}\right).$$

This system is closed by the initial conditions and the boundary conditions on the wing surface

$$\zeta = 0, \quad u = u_*(t), \quad v = v_*(t), \quad w = w_*(t), \quad h = h_w \tag{6.3}$$

and behind the shock $\zeta = \zeta_s(t)$, $\quad p_{1s} = -2K^2(\dot\varphi + \sin\beta)(w_s + \sin\beta), \quad \beta = \dfrac{\pi}{2} - \beta_0$

$$\begin{gathered} u_s = K\sin\beta + (K_s - K)w, \quad v_s = \cos\beta, \quad w_s = \dot\varphi - (\sin\beta + \dot\varphi)/\varrho_s, \\ \varrho_s^{-1} = \left(\gamma - 1 + \frac{2}{M_\infty^2(\sin\beta + \dot\varphi)^2}\right)\frac{1}{\gamma+1}, \quad p_s = \frac{2}{(\gamma+1)}\left[(\sin\beta + \dot\varphi)^2 + \frac{1-\gamma}{2\gamma M_\infty^2}\right]. \end{gathered} \tag{6.4}$$

The initial conditions are the solution of the steady-state problem.

7. Numerical solution of the nonlinear finite-difference equations

The system of equations (4.3) and (4.4) with the boundary conditions (5.3), (5.4) and (6.3) was solved numerically by an implicit finite-difference iterative method. First, the steady-state problem for a thin shock layer $\partial P_0/\partial\zeta = 0$ was solved with specified linear distributions of u, v, w and constant $p = p_s$, $\varrho = \varrho_s$ in the first iteration. In each iteration the thickness Δ of the shock layer was determined from the continuity equation.

The solution of equations for a thin layer was used as initial data for calculating both the steady-state and unsteady shock layer for $\partial p_0/\partial\zeta \neq 0$.

The numerical solution to the problem (4.3), (4.4), (5.3), (5.4) and (6.3) was performed by the finite-difference method based on Newton's iterations in the form

$$(7.1) \qquad \frac{E_f^{(s)}\bar{f}_e^{(s+1)}}{\Delta t_n} + \left(\frac{\partial}{\partial z} A_f^{(s)}\bar{f}^{(s+1)}\right)_e + D_e^{(s)} = \left(\frac{\partial}{\partial z} C^{(s)} \frac{\partial \bar{f}^{(s+1)}}{\partial z}\right)_e,$$

$$(7.2) \qquad D_e^{(s)} = \frac{1}{\Delta t_n}\left(E_e^{(s)} - E(f_e^n)\right) + \left(\frac{\partial A}{\partial z}\right)_e^{(s)} + B_e^{(s)} - \left(\frac{\partial}{\partial z} C^{(s)} \frac{\partial f^{(s)}}{\partial z}\right)_e,$$

where $E_e^{(s)} = E(f_e^{(s)})$, $f_e^n = f(t_n, z_e)$, $f_e^{(s)}$ is a value of f for time level $t = t_{n+1}$ at iteration with number S for the grid point $z = z_e$. Increment on iteration $\bar{f}_e^{(s+1)}$ is equal $\bar{f}_e^{(s+1)} = f_e^{(s+1)} - f_e^{(s)}$. The matrices E_f, A_f are Jacobians $\partial E/\partial f$, $\partial A/\partial f$, respectively. The symbols $(\partial\varphi/\partial z)_e$, $\left(\frac{\partial}{\partial z}\psi\frac{\partial\varphi}{\partial z}\right)_e$ are finite-difference approximations for corresponding derivatives on a nonuniform grid $\{z_e\}$, which took into account the solution scales in accordance with asymptotic analysis.

Four to five iterations, on the average, were necessary for $\max\limits_e |\bar{f}_e^{(s+1)}| < \varepsilon$. The value of ε was chosen greater than the round-off error, but smaller than the approximation error. The vector $\bar{f}^{(s+1)}$ was determined by solving the linear system of algebraic equations (7.1) closed by the boundary conditions that follow from Eqs. (6.3) and (5.3), (5.4). An additional condition $\partial p/\partial z = 0$ for $z = 0$ was used, which was justified by the asymptotic analysis. The block tri-diagonal system of algebraic equations was solved by the vector "progonka" process.

8. Examples of calculations

The results of calculations presented below refer to the case where $\gamma = 1.4$, $\mathrm{Pr} = 0.71$, $M_\infty = 10$, $\mathrm{Re}_0 = 500$, 50 $h_w = 0.2$, 0.1.

Figure 1 illustrates a comparison of the present numerical results with the computations [4] of GERSHBEIN and KOLESNIKOV for the case $\Omega = \Omega_* = 0$, $R_\infty = 1100$, $M_\infty = 10$, $\mathrm{Pr} = 0.71$, $h_w = 0.3 \cdot h_0$ for two values of blowing: $(\varrho w)_* = 0$ (solid lines) $(\varrho w)_* = 0.1$ (dashed lines).

The profiles $10^{-2}\varrho(z)$, $u(z)$, $z = \zeta/\Delta$ for two values of blowing $w_* = 0.01$ (lines *1*, *3*) and $w_* = 0.03$ (line *2*) are presented in Fig. 2 for $\mathrm{Re}_0 = 500$ (lines *1*, *2*) and $\mathrm{Re}_0 = 50$ (line 3). The shock layer is fully viscous for $\mathrm{Re}_0 = 50$.

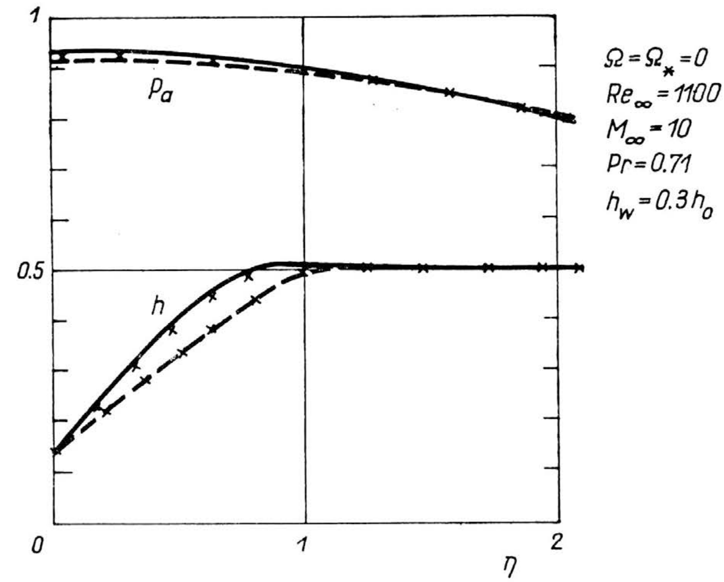

FIG. 1. xxxx — GERSHBEIN and KOLESNIKOV [4], $(\varrho w)_* = 0$ — solid lines, $(\varrho w)_* = 0.1$ — dashed lines;

$$\eta = 2\Delta \int_0^z ((1+z\Delta)\varrho dz).$$

Figures 4–13 for $\mathrm{Re}_0 = 50$, $h_w = 0.1$, $\Omega_* = 0.5$, $\Omega = 0$ illustrate the evolution of components of velocity, density, enthalpy and pressure for two cases of flow in a shock layer: 1) the flow with switching on the blowing, 2) the flow with interruption of blowing and beginning of suction. Switching the blowing on $u_* = w_* = 0.03$ t for $0 \leqslant t \leqslant 1$ and

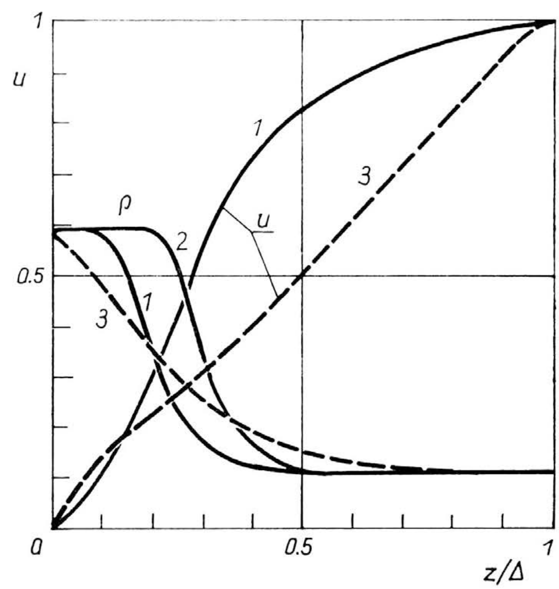

FIG. 2. The influence of blowing on profiles $10^{-2}\varrho z$, $u(z)$, $w^* = 0.01$ — curve *1*, ($\mathrm{Re}_0 = 500$) and curve *3* ($\mathrm{Re}_0 = 50$), $w^* = 0.03$ — curve *2* ($\mathrm{Re}_0 = 500$).

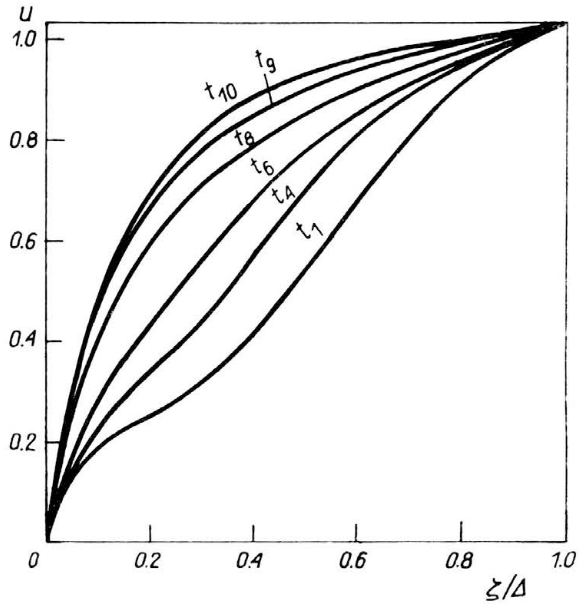

FIG. 3. The evolution of tangential components of velocity for switching the blowing off at the moments of time: $t_1 = 0.66$, $t_4 = 1.26$, $t_6 = 1.66$, $t_8 = 2.06$, $t_9 = 2.26$, $t_{10} = 2.46$.

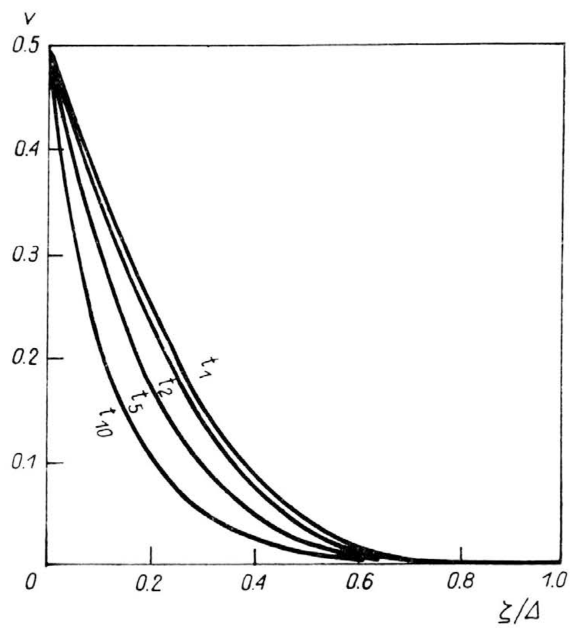

FIG. 4. The evolution of azimuthal components of velocity for switching the blowing off at the moments of time: $t_1 = 0.66$, $t_2 = 0.86$, $t_5 = 1.46$, $t_{10} = 2.46$.

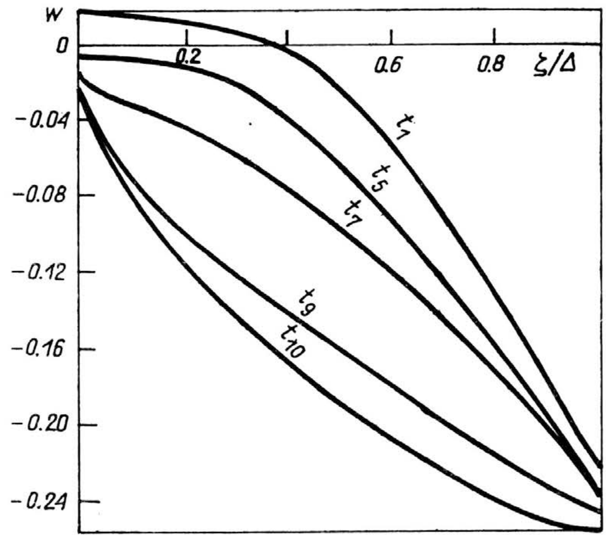

FIG. 5. The evolution of components of velocity normal to the body for switching off the blowing at the moments of the time: $t_1 = 0.66$, $t_{j+1} = t_j + 0.2$, $j = 1, \ldots, 9$.

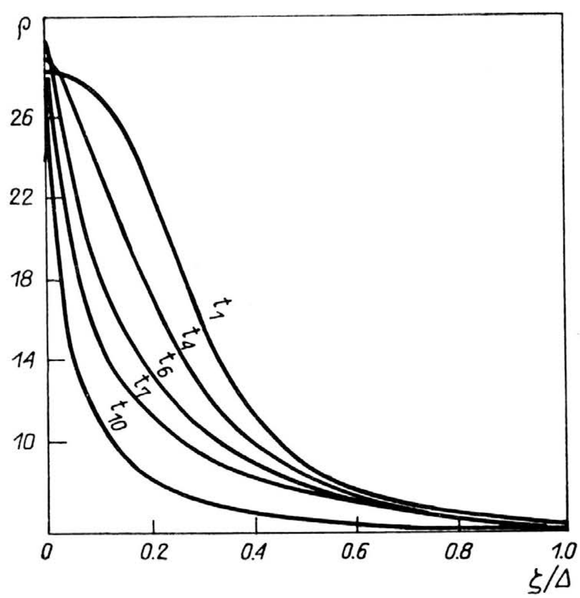

FIG. 6. The evolution of density profiles for switching off the blowing at moments of time: $t_1 = 0.66$, $t_{j+1} = t_j + 0.2$, $j = 1, \ldots, 9$.

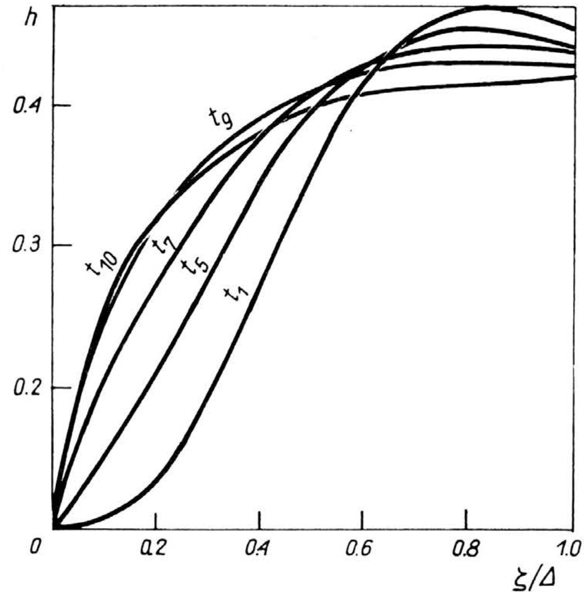

FIG. 7. The evolution of enthalpy profiles for switching off the blowing at moments of time: $t_1 = 0.66$, $t_{j+1} = t_j + 0.2$, $j = 1, \ldots, 9$.

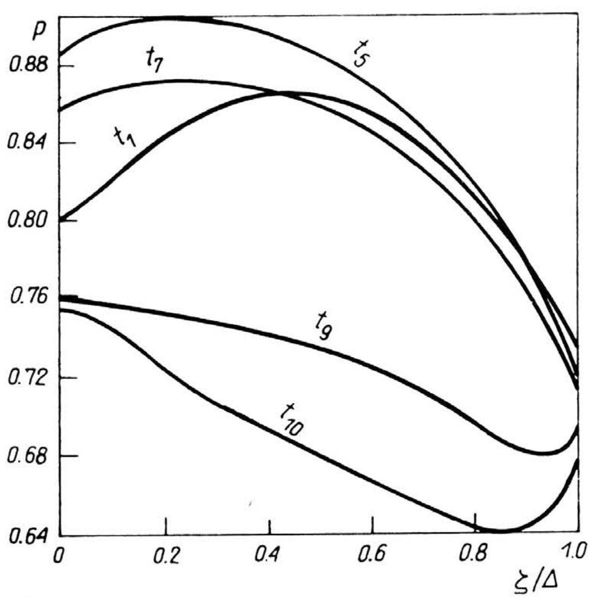

FIG. 8. The evolution of pressure profiles for switching off the blowing at the moments of time $t_1 = 0.66$, $t_{j+1} = t_j + 0.2$, $j = 1, \ldots, 9$.

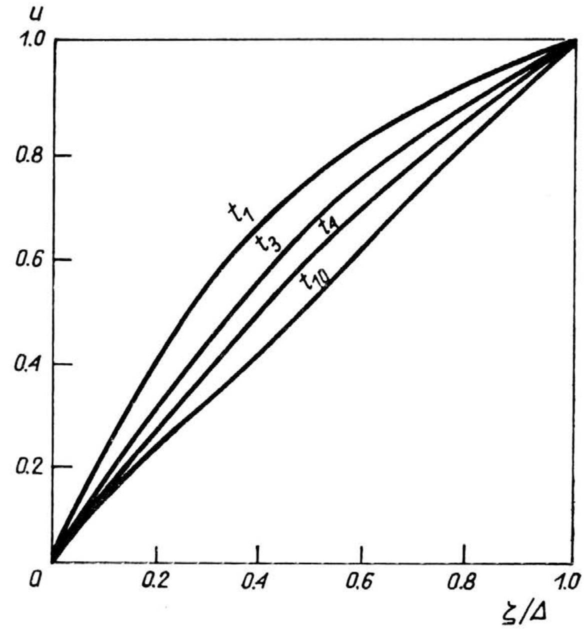

FIG. 9. The evolution of pressure profiles for switching on the blowing at moments on time: $t_1 = 0.66$, $t_{j+1} = t_j + 0.2$, $j = 1, \ldots, 9$.

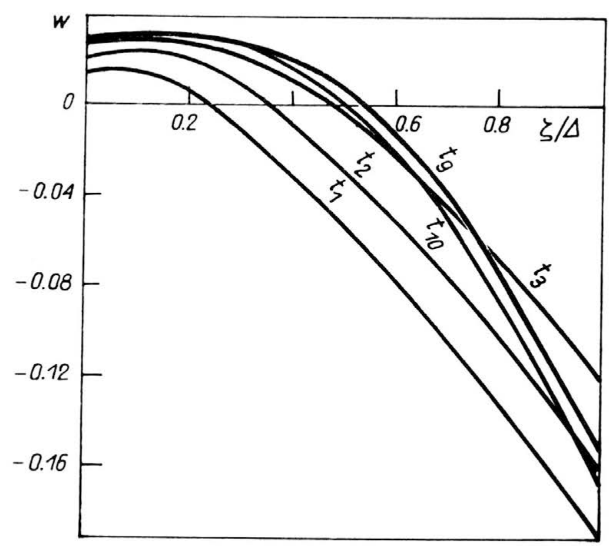

FIG. 10. The evolution of normal to the body component of velocity for switching on the blowing at moments of time: $t_1 = 0.66$, $t_{j+1} = t_j + 0.2$, $j = 1, \ldots, 9$.

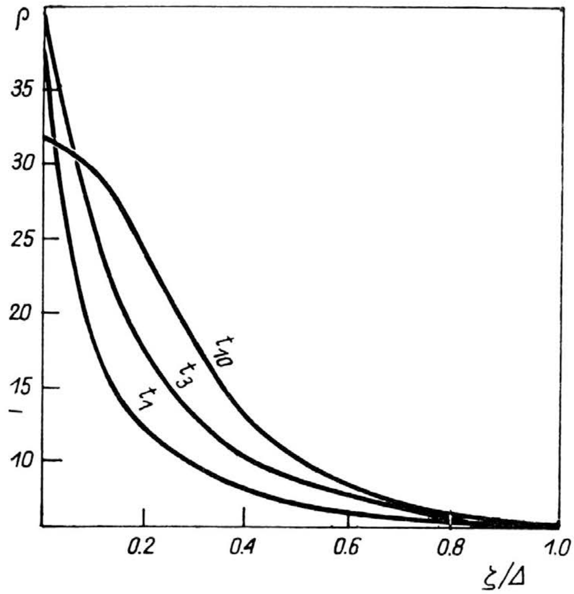

FIG. 11. The evolution of density profiles for switching on the blowing at moments of time: $t_1 = 0.66$, $t_{j+1} = t_j + 0.2$, $j = 1, \ldots, 9$.

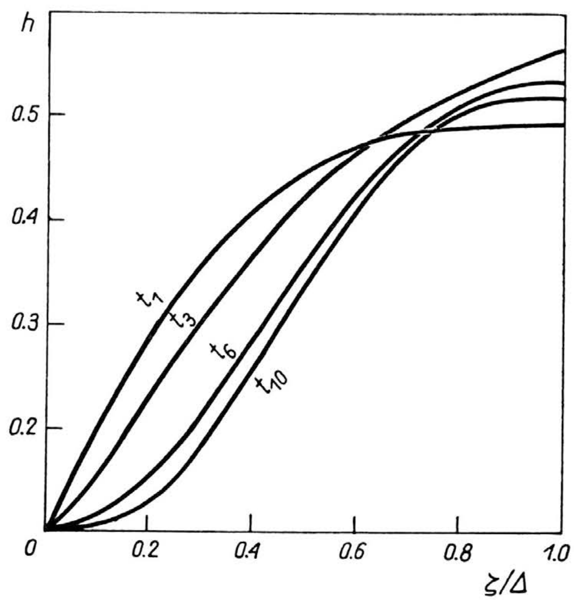

FIG. 12. The evolution of enthalpy profiles for switching on the blowing at moments of time: $t_1 = 0.66$, $t_{j+1} = t_j + 0.2$, $j = 1, \ldots, 9$.

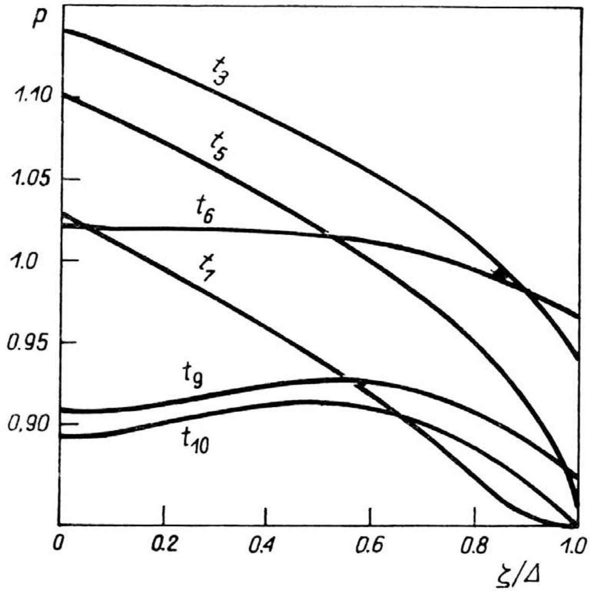

FIG. 13. The evolution of pressure profiles for switching on the blowing at moments of time: $t_1 = 0.66$, $t_{j+1} = t_i + 0.2$, $j = 1, \ldots, 9$.

$u_* = w_* = 0.03$ for $t \geqslant 1$. Switching the blowing off with suction is presented for the parameters: $u_* = 0.03(1-t)$ for $0 \leqslant t \leqslant 1$ and $u_* = 0$, $t \geqslant 1$, $w_* = 0.03(1-t)$ for $0 \leqslant t \leqslant 2$, $w_* = -0.03$ for $t \geqslant 2$. The results of calculations are given in Figs. 4-13 for the moments of time: $t_1 = 0.66$, $t_{j+1} = t_j + 0.2$, $j = 1, \ldots, 9$.

Figure 14 shows the dependence of dimensionless heat flux C_H on time for the cases: $\Omega \equiv 0$, $u_* = w_* = 0.03\,t$, $0 \leqslant t \leqslant 1$, $u_* = w_* = 0.03$, $t \geqslant 1$ (line *1*); $\Omega \equiv 0$, $u_* = 0.03\,(1-t)$, $0 \leqslant t \leqslant 1$, $u_* = 0$, $t > 1$, $w_* = 0.03\,(1-t)$, $0 \leqslant t \leqslant 2$, $w_* = -0.03$, $t \geqslant 2$ (line *2*); $\Omega = 0$, $u_* = w_* = 0.03\,(1-t)$, $t \leqslant 1$, $u_* = w_* = 0$, $t > 1$ (line *3*); $\Omega = 0.3$;

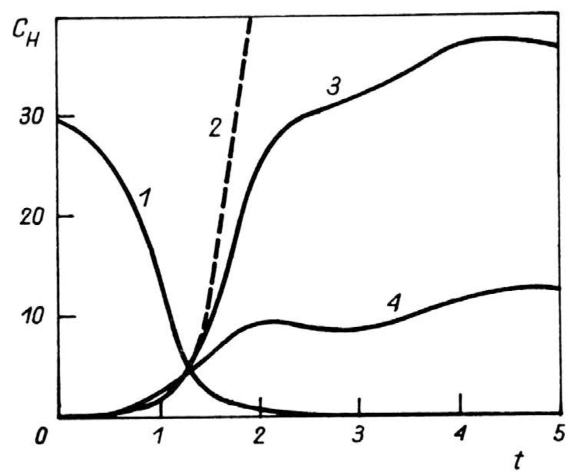

FIG. 14. Dependence of heat flux on time for switching blowing during rotation.

$u_* = 0.03\,(1-t),\ t \leqslant 1,\ u_* = 0,\ t \geqslant 1,\ w_* = 0.03\,(1-t),\ t \leqslant 2/3,\ w_* = 0.01,\ t > 2/3$ (line 4) where C_H is given by the relation (prime notation refers to dimensional quantities)

$$C_H = \frac{\mu w}{\mathrm{Re}_0 \mathrm{Pr}} \left(\frac{\partial h}{\partial \zeta}\right)_w \sqrt{\mathrm{Re}_0}\,(\gamma-1) M_\infty^2 = \left(\lambda' \frac{\partial T'}{\partial \zeta'}\right)_w \sqrt{\mathrm{Re}_0}\,(\gamma-1) M_\infty^2.$$

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