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Some remarks on the theory of irregular reflection of a shock wave from a surface

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DECAY of the initial discontinuity is interpreted as a mechanism of passage from a regular to an irregular phase in the problem of nonstationary reflection of a shock wave from a surface. Modification of the Mach triple point theory resulting from the hypothesis presented is considered.

Przedstawiono hipotezę o rozpadzie początkowej nieciągłości jako mechanizmie przejścia od fazy regularnej do nieregularnej w problemie niestacjonarnego odbicia fali uderzeniowej od powierzchni. Rozważono modyfikację teorii punktu potrójnego Macha, wynikającą z przyjęcia tej hipotezy.

Представлена гипотеза о распаде начального разрыва как механизме перехода от регулярной фазы к нерегулярной фазе в задаче нестационарного отражения ударной волны от поверхности. Рассмотрена модификация теории тройной точки Маха, вытекающая из принятия этой гипотезы.

1. Introduction

PERMANENT interest in the problem of irregular reflection of a shock wave from a surface is observed among scientists concerned with problems of gasdynamic flow, owing to the strong nonlinearity of the problem which makes impossible rigorous mathematical analysis. As a consequence of this difficulty, the problem is studied by combination of experimental [1, 2], numerical [3, 4] and approximate mathematical methods [5, 6].

By contrast with the problem of regular reflection (RR) of a shock wave, a complete theory of which has been formulated by VON NEUMANN [7], the two principal problems of irregular or Mach reflection (MR) remain still unsolved. They are the problem of the type of singularity of the triple point occurring in the Mach configuration and the problem of the angle of regular-to-irregular reflection transition.

In the case of irregular reflection there are three experimental situations which are possible, that is:

a) steady gas flow: such a situation occurs in supersonic wind tunnels;

b) the flow is unsteady, a stationary image can be obtained, however, by transforming the variables. This situation is that of interaction of a plane shock wave with a plane obstacle. There being no characteristic scale of lengths, selfsimilar variables can be introduced. Such an image is known in the literature as a pseudosteady flow;

c) the flow is unsteady, but selfsimilarity cannot be introduced. Such is the case of a plane shock wave acting on a curved obstacle or a curved shock wave acting on a plane obstacle.

It appears that the latter case differs essentially from the preceding ones, what is confirmed by the structure of the relevant mathematical models — the case a) and b) are described by partial differential equations of hyperbolic or elliptic type depending on whether the flow is super- or subsonic, nonstationary gas flow being described by a hyperbolic set of partial differential equations independently of the flow velocity.

The attention of the scientists was as yet concentrated on the cases a) and b). In Sect. 2 we present a survey of the literature on that subject and a summary of the results obtained.

It is only recently that some experimental and theoretical works appeared, devoted to (truly) unsteady reflection of a shock wave from a surface. The problem of interaction between plane shock waves and concave or convex cylindrical obstacles was studied by BEN-DOR, TAKAYAMA and KAWAUCHI [8]. Such a geometry ensures continuous variation of the incidence angle of the shock wave falling on an obstacle, therefore also observation of the transition from the regular to the irregular phase in the case of a convex cylindrical surface, or vice-versa, in the concave case.

The results of those experiments will be discussed in greater detail in Sect. 3. They have led the authors to a conclusion that the problem of criterion for the RR $\rightleftarrows$ MR transition, which appeared to be solved on the grounds of the experiments of the type a) and b), remains still open(1).

Another example of a truly unsteady reflection of a shock wave from a surface is the action of a spherical blast wave on a plane reflecting surface. This problem was formulated by von NEUMANN [7] in connection with the analysis of the effects of explosion of an atomic bomb, then studied by numerical means by PODLUBNY and FONARIEV [10] and by an approximate analytical method by VASILEV [11].

The Vasilev solution is constructed by expansion in double Taylor series, in the neighbourhood of the point of first contact of the shock wave with the surface. The method used by Podlubny and Fonariev was the Godunov numerical method which yields shock fronts smeared to a width of a few computation meshes. In neither case it was possible to study the transition from the regular to irregular reflection.

Some computation results obtained by the method of characteristics have recently been presented by GAŁKOWSKI [12] on the grounds of a model for which an asymptotic solution of the problem was used as well as the Vasilev result (in the case of regular reflection).

The computation method used in that paper takes into consideration all the singularities of the flow, that is the shock waves and contact discontinuities, what enables accurate study of the dynamics of the RR $\rightleftarrows$ MR transition. Starting out from the requirement of internal consistency of the mathematical model a criterion of RR $\rightarrow$ MR transition was formulated for the problem under consideration. This leads to a hypothesis that initial discontinuity decay is a cause of generation of the Mach wave. This hypothesis

(1) This class includes also problems of diffraction of a plane shock wave by a cylinder, a sphere or a cylindrical wedge, which are discussed in the monograph by WITHAM [9]. Due to the difference in the method of analysis they will not be discussed here, however. The method used by Witham is known as the CCW (Chester, Chisnell, Witham) method. It enables the dynamics of the shock front to be represented, but gives no information on the flow behind that wave front. In this connection it does not involve transition from regular to irregular reflection, the Mach wave occurring as a secondary (shock-shock) wave.

has some consequences in the theory of triple point. A study of those consequences is one of the aims of the present paper.

In Sect. 2 the criteria for the RR ⇄ MR transition will be formulated for a situation of the type a) and b). The nomenclature and the symbols will be established in agreement with the convention proposed by BEN-DOR [13].

The results obtained in [8] for truly unsteady action of a plane shock wave on a concave and a convex cylindrical surface will be discussed in Sect. 3. In the case of a convex cylindrical surface a wave pattern is obtained, topologically equivalent to that of interaction between a spherical blast wave and a surface.

The latter problem is analysed in Sect. 4, in which the consequences of the hypothesis adopted for the singularity type of the triple point are discussed.

Sect. 5 contains a summary of results.

2. Steady and pseudosteady flow

Figures 1a and 2a show the scheme of the process in the case of pseudosteady flow. An incident shock wave i falls on a wall inclined at an angle θ_w to the direction of wave propagation and undergoes reflection which is regular (Fig. 1a), if the angle θ_w is sufficiently large, and irregular (Fig. 2a) if it is small. In the latter case a typical wave configuration is formed. It is composed of a reflected wave r, a Mach wave m and a slipstream s. If there

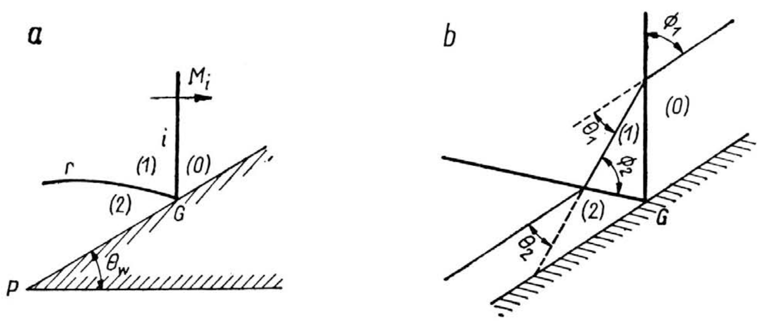

FIG. 1. Diagrammatic representation of regular reflection of a shock wave from a plane surface; a) pseudosteady image, b) steady image in a reference frame connected with the reflection point G.

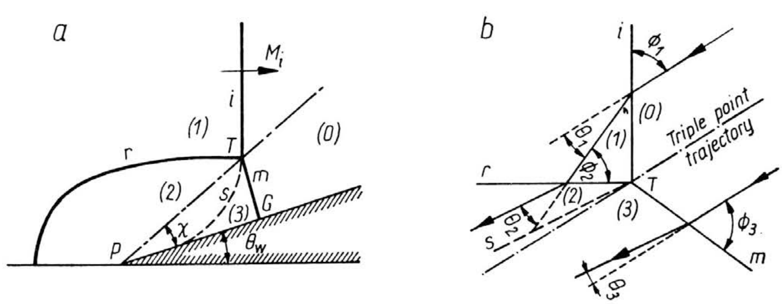

FIG. 2. Diagrammatic representation of irregular reflection of a shock wave from a plane surface; a) pseudosteady image, b) steady image in a reference frame connected with the triple point T.

are regions of homogeneous flow in the vicinity of the reflection point G (or the triple point T), the local flow may be reduced to a steady flow by Galilei transformation along the trajectory of the point G (or the triple point T). For a strong shock wave the flow behind the reflected wave is supersonic and homogeneous, therefore such transformation may be applied. If there are nonhomogeneous subsonic regions in the flow, a steady pattern can be obtained by similarity transformation, there occurs, however, a non-conservative field of external forces, in proportion to the local velocity field — and a source of mass in proportion to the local density [23].

The wave patterns in a reference frame connected with the point of reflection G or the triple point T are represented in Figs. 1b and 2b. The symbol ϕ_1 is used to denote the inclination angle of the shock wave i to the flow before its front. In the case of regular reflection we have $\phi_1 = \pi/2-\theta_w$ and in the case of irregular reflection $-\phi_1 = \pi/2-(\theta_w+\chi)$, where χ — the inclination angle of the virtual wall, that is the trajectory of the triple point T, to the real wall.

The remaining symbols are as follows:

ζ_i inverse strength of the incident wave i; $\zeta_i = p_0/p_1$,

ζ_r inverse strength of the reflected wave r; $\zeta_r = p_1/p_2$,

$\theta_{1,2,3}$ flow deflection after the passage of the shock wave i, r, m, respectively,

M_i, M_m Mach numbers of shock waves i, m, respectively, in unsteady reference frame (Figs. 1a and 2a),

$$M_i = D_i/a_0, \qquad M_m = D_m/a_0,$$

where D_i, D_m — speeds of shock waves, a_0 — speed of sound in undisturbed medium,

M_0, M_1, M_2, M_3 Mach numbers of flow in the regions 0, 1, 2, 3 (in a steady reference frame); $M_0 = M_i \operatorname{cosec}\phi_1$,

γ ratio of specific heats of the gas (at constant pressure and volume),

ϕ_2 inclination angle of the reflected shock wave r to the direction of flow before the wave front,

ϕ_3 inclination angle of the Mach wave to the direction of flow before the wave front.

The theory of regular reflection including the head-on reflection ($\theta_w = \pi/2$) has been formulated by von Neumann [7], the point of departure being the condition of $\theta_1+\theta_2 = 0$, from which a quadratic equation is found after some transformations. This equation determines the inclination angle of the reflected shock wave to the surface of the wall. Only one solution of that equation is realized physically, namely the solution for the weaker wave (greater ζ_r). For a certain value of the angle θ_{1d} the two solutions coincide, no real roots existing for $\phi_1 > \phi_{1d}$ due to negative determinant. A typical feature of the von Neumann solution is that the entire increase in pressure, $P_2/P_0 = (\zeta_i\zeta_r)^{-1}$ decreases with increasing angle ϕ_1, reaches a minimum for a certain value $\phi_{1\min}$, then increases to a value above that at head-on incidence. The latter effect occurs with weak shock waves (large ζ_i) and is referred to as an anomalous regime of regular reflection [15].

Further increase in the angle ϕ_1 leads to irregular reflection, which means formation of a triple point and a Mach shock wave.

The critical value of the angle ϕ_1, that is the angle of regular reflection–irregular reflection transition will be denoted by ϕ_1^*. There are the following criteria for the RR $\rightleftarrows$ MR transition:

a) *The von Neumann criterion.* This is the criterion of maximum angle for which regular reflection is possible. The RR $\rightleftarrows$ MR transition takes place for an angle $\phi_1^* = \phi_{1d}$.

b) *The Hornung–Oertel–Sandeman criterion* [16]. The RR $\rightleftarrows$ MR transition takes place for an angle $\phi_1^* = \phi_{1s}$ such that the flow behind the reflected shock wave becomes purely sonic, that is if $M_2 = 1$.

The relevant critical angle is, as shown by numerical means [14], somewhat smaller than ϕ_{1d}. We have $M_2 < 1$ for $\phi_{1s} < \phi_1 < \theta_{1d}$ and $M_2 > 1$ for $\phi_1 < \phi_{1s}$.

The HOS criterion can be formulated in a more general manner, as a "length scale" criterion which tells that RR $\rightarrow$ MR transition occurs when the length scale of the process is available at the reflection point. In the case of pseudosteady flow this formulation leads to the condition of $M_2 = 1$. For steady flows it is equivalent to the Henderson–Lozzi criterion.

c) *The Henderson–Lozzi criterion* [17]. RR $\rightleftarrows$ MR transition occurs an angle $\phi_1^* = \phi_{1p}$ such that the pressure behind the reflected shock wave is equal to the pressure behind a single shock wave normal to the flow direction in the region (0). This situation enables formation of an infinitesimal Mach shock wave and further development of irregular reflection. Then the RR $\rightleftarrows$ MR transition process goes on without pressure jump, no additional waves with finite amplitude thus being generated.

The HL criterion yields critical angles ϕ_1 differing considerably from those resulting from the von Neumann and HOS criterion. Thus, for instance, for $\gamma = 7/5$ and $\zeta_i = 0$ we have:

$$\phi_{1d} = 39.971°, \quad \phi_{1s} = 39.910° \quad \text{and} \quad \phi_{1p} = 21.769°.$$

Another criterion has been put forward lately by Henderson [6], according to which the RR $\rightleftarrows$ MR transition occurs when the incidence angle of the shock wave is $\phi_1^* = \phi_{1\min}$ (minimum entire increase in pressure). Henderson argues that a hypothetical mechanical system in which the reflecting wall is kept at torque equilibrium about the point P owing to the moment of force due to the pressure of the shock wave being equilibrated by an opposite external moment, would not be stable for $\phi_1 > \phi_{1\min}$, because in such a case

$$\partial(P_2/P_0)/\partial\theta_w < 0.$$

This hypothesis is of a somewhat speculative character and has not been confirmed by experiment. As a result there are only three criteria which are considered in reality, namely the von Neumann, HOS and H—L criterion ([2]).

The three criteria above can most easily be illustrated by shock polar diagrams [18]. The letter i in Figs. 3, 4 and 5 marks the polar diagram of the incident wave and, possibly, the Mach wave (with fixed M_0) and r is used for the diagram of the reflected wave

([2]) Those criteria are sometimes given other names in the literature. Thus, the von Neumann criterion is termed the detachment criterion, the HOS criterion is known as sonic crition and the H-L criterion is referred to as mechanical equilibrium criterion.

(with fixed M_1). Because the total flow deflection must be zero for regular reflection, the final state lies at the point of intersection of the diagram r with the axis of abscissae.

Figure 3 illustrates the von Neumann criterion for three different strengths of the incident wave. Von Neumann classifies incident waves into strong and weak depending on whether the point of contact of the diagram r with the axis of obscissae lies inside or outside the diagram i. According to this classification the wave I is strong, III — weak and II is an intermediate wave.

For $\gamma = 7/5$ the separation condition for inverse strength of the incident wave is $\zeta_i^* = 0.433$, which corresponds to $M_i = 1.46$.

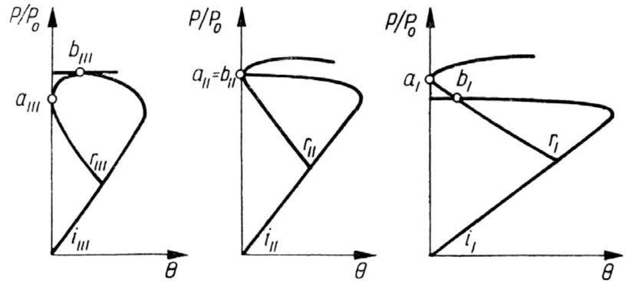

FIG. 3. Illustration of the von Neumann criterion, i — polar diagram of the incident wave, r — polar diagram of the reflected wave, a — critical points of regular reflection, b — critical points of irregular reflection.

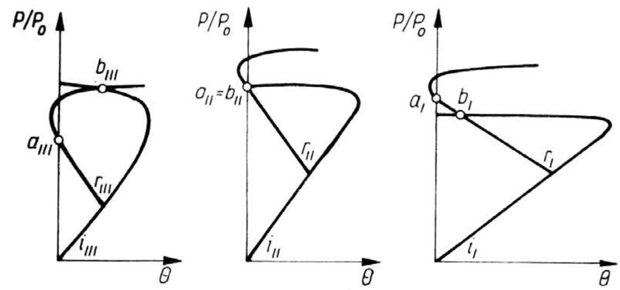

FIG. 4. Illustration of the Hornung–Oertel–Sandeman criterion.

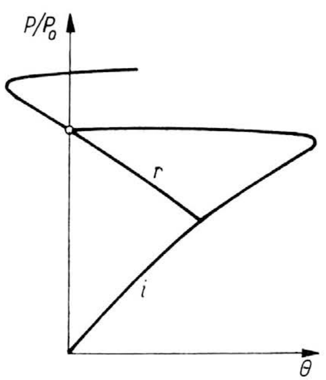

FIG. 5. Illustration of the Henderson–Lozzi criterion.

Figure 4 illustrates the HOS criterion, according to which the transition to the Mach reflection takes place for a somewhat smaller angle ϕ_1, such that the flow behind the reflected wave is strictly sonic. The final states of regular reflection are marked by the letter a and the three different diagrams r correspond to a weak, intermediate and strong shock wave. In this case the separation value of the inverse strength of the wave is different and is $\zeta_i^* = 0.375$ for $\gamma = 7/5$. In both cases, that is those of the von Neumann and HOS criterion (Figs. 3 and 4, respectively), the letter b marks the points corresponding to the Mach configuration for angles satisfying the criterion of transition.

If polar diagrams are used to illustrate truly unsteady reflection of a shock wave (according to the method of BEN-DOR and TAKAYAMA [24]), then, for a strong wave RR → MR transition occurs with decreasing pressure, therefore the transition should be accompanied by an unsteady rarefaction wave. The case of a weak wave is opposite, the transition. occurring with increasing pressure, therefore an unsteady shock wave is to be expected For intermediate strength the transition is not accompanied by any variation in pressure. Such an image of the RR → MR transition suggests an analogy with the Riemann problem (initial discontinuity decay), in which a combination of self-similar shock waves and rarefaction waves is also generated, depending on the parameters of initial discontinuity. This analogy will be studied later on.

It is required by the HL criterion (Fig. 5) that the RR → MR transition should occur with no pressure jump. The figure shows a shock polar diagram r for the critical angle. It is seen that this criterion can be satisfied only by a wave which is strong in the von Neumann sense.

The results of experiments and analyses of the case of steady and pseudosteady state can be summed up as follows [8]. In the case of steady flow and a strong wave in the von Neumann sense the Henderson–Lozzi criterion is valid. The HENDERSON LOZZI [17] and HOS criterion [16] are valid for pseudosteady flows and strong waves. For weak waves the question of correct criterion has been solved neither for steady nor pseudosteady flows. It should be stressed that the disagreement between the von Neumann criterion and the experiment still exists in the case of pseudosteady flow behind a weak shock wave and is known in the literature as the von Neumann paradox [1].

3. Unsteady flow

Truly unsteady gasdynamic flows with diffraction of shock waves were studied experimentally by BEN-DOR, TAKAYAMA and KAWAUCHI [8]. The subject of their studies was a plane wave acting on a cylindrical obstacle (Figs. 6a, b). In such a configuration the

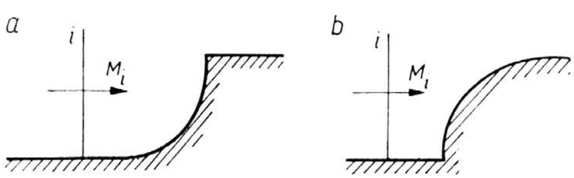

FIG. 6. Unsteady interaction between a plane shock wave and a cylindrical surface; a) concave cylindrical surface; MR → RR transition, b) convex cylindrical surface; RR → MR transition.

angle ϕ_1 varies in a continuous manner from 0° to 90° (for a convex cylindrical surface, Fig. 6b) or from 90° to 0° (for a concave cylindrical surface, Fig. 6a), direct observation of the dynamics of the RR ⇄ MR transition thus being possible. The results obtained led the authors to a conclusion that the existing criteria for RR ⇄ MR transition do not explain the phenomena occurring in the case of unsteady interaction between the shock wave and the obstacle. Another phenomenon observed by Ben-Dor, Takayama and Kawauchi was that of hysteresis, consisting in the fact that the RR → MR transition occurs for a different value of the angle ϕ_1 than the MR → RR transition.

With increasing angle ϕ_1 the RR → MR transition occurs in the following order of events (cf. Fig. 7): regular reflection (points *d*, *c*, *e*, *f*), RR → MR transition according to the von Neumann criterion ($f \rightarrow a$) and irregular reflection (point *a*). Conversely, if the angle ϕ_1 decreases in a continuous manner starting out from 90°, we have irregular reflection (point *a*, *b*, *c*), then MR → RR transition according to the Henderson–Lozzi criterion (point *c*) and, finally, regular reflection (point *d*).

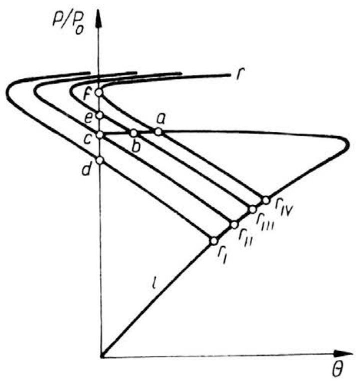

Fig. 7. Polar diagram for unsteady interaction between a shock wave and a surface. Illustration of the phenomenon of hysteresis.

Ben-Dor and the other authors conclude their work by stating that:

... "the question "what is the general criterion (concept) that explains whether the RR ⇄ MR transition represents steady, pseudosteady and non-stationary flows?" is reopened".

Some of the consequences of the facts observed (and some modifications of the theory) are considered in [19, 20]. The phenomenon of inverse Mach reflection [20] occurring if the angle ϕ_1 decreases in a continuous manner to a value corresponding to the MR → RR transition is analysed in particular.

The notion of inverse Mach reflection has been introduced by Courant and Friedrichs in their monograph [18].

In [19] Ben-Dor assumes that, during unsteady MR → RR transition, the phase of inverse Mach reflection occurs between the points *c* and *d*, that is transition to inverse Mach reflection occurs first at the point *c* and is followed by regular reflection, which starts at the point *d*.

Our principal subject being the RR → MR transition, we shall not consider that question in greater detail.

4. Interaction between a spherical shock wave and a plane reflecting surface

The results obtained in [12, 14] enable us to analyse the mechanism and the dynamics of the RR → MR transition in truly unsteady gas flow behind the front of a spherical blast wave reflected from a perfect plane.

Figure 8 represents the wave pattern of the phenomenon in the regular and irregular phase of reflection. It may be assumed that the irregular phase is a manifestation of an asymptotic property connected with the fact that the action of a spherical blast dipole is, for distances which are long as compared with the height of burst, equivalent to the action of a single blast of double strength (Fig. 9).

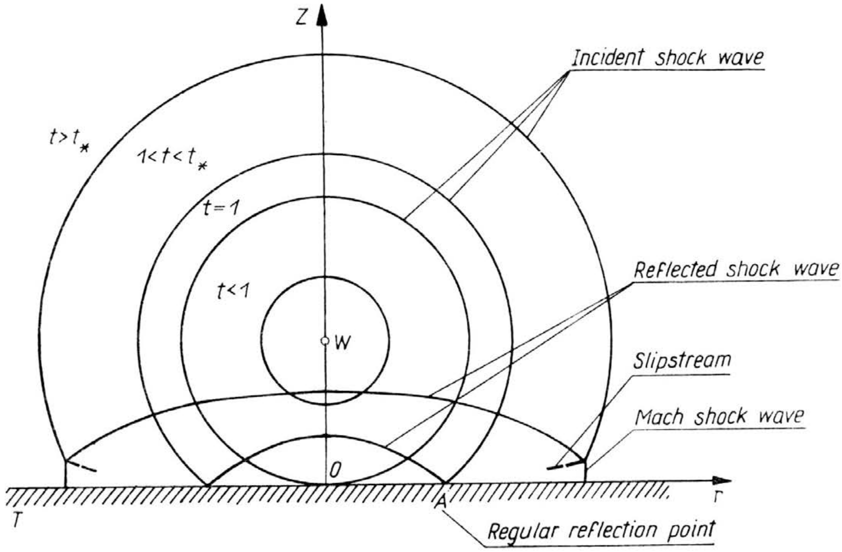

FIG. 8. Unsteady interaction between a spherical blast wave and a plane reflecting surface.

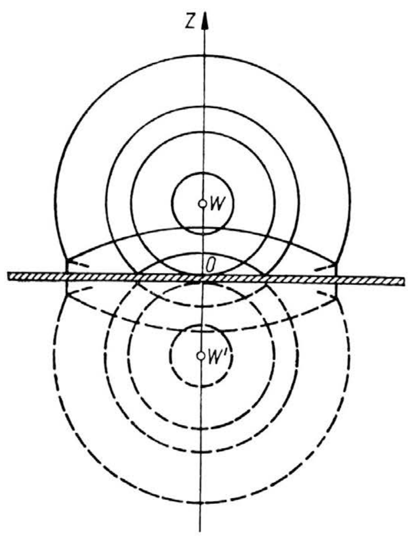

FIG. 9. Reflection of a spherical shock wave from a surface is equivalent to the action of a blast dipole.

Making use of this asymptotic property a one-dimensional model was proposed in [12] which enabled the problem to be solved in the neighbourhood of the reflecting plane and, therefore, the mechanism of the RR → MR transition to be analysed. According to the results of that work the RR → MR transition is a consequence of the decay of the initial discontinuity which was formed in the regular phase of the process. From the theory of the Riemann problem [21] it is known that waves of various types may then be generated, depending on the parameters of initial discontinuity. As regards the case under consideration, a shock wave, a contact discontinuity and a rarefaction wave are generated. Figures 10 and 11 which have been taken from [12] show diagrams of pressure and density at a few instants of time just after the decay of the initial discontinuity and the RR → MR transition. The rarefaction region is represented by two dashed lines. The rapidity of decay of that wave is seen. Figure 12 shows the wave pattern of the entire phenomenon of interaction between the spherical shock wave and the surface. The diagram concerns the close neighbourhood of the reflecting plane.

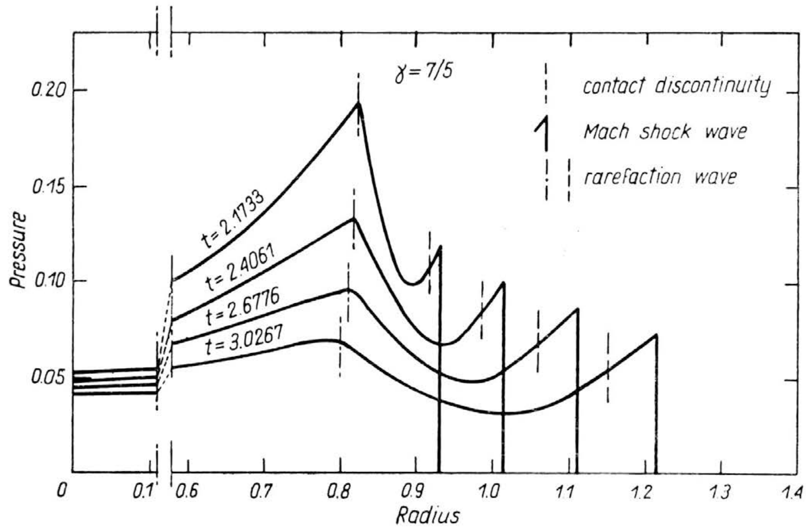

FIG. 10. Pressure diagrams at consecutive instants of time in the irregular phase of reflection of a spherical shock wave from a surface.

In the classical configuration of the triple point [7] the shock wave corresponds to the Mach wave and the contact discontinuity — to the slipstream. The rarefaction wave has no counterpart. Von Neumann contemplated the idea of the rarefaction wave being introduced into his theory of triple point but he did not find sufficient reasons for such a modification of that theory. He tried to explain the observed disagreement between the theory and the experiment in another way, independent of the origin of the Mach configuration, what may be justified for steady or even pseudosteady flow, but is not justified for unsteady flows.

In the subsequent sections we shall consider the consequences of introducing a rarefaction wave into a Mach configuration of shock waves, then the criterion for RR → MR transition will be analysed in the light of the results obtained.

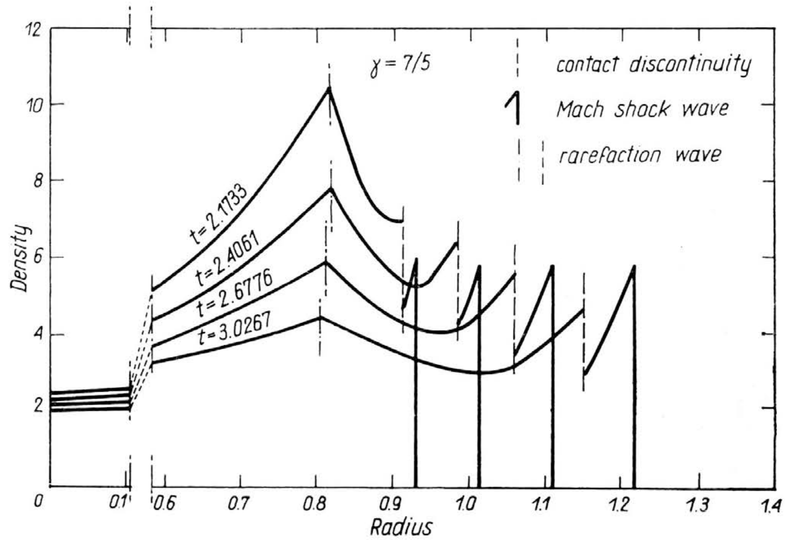

FIG. 11. Density diagrams at consecutive instants of time in the phase of irregular reflection of a spherical shock wave from a surface.

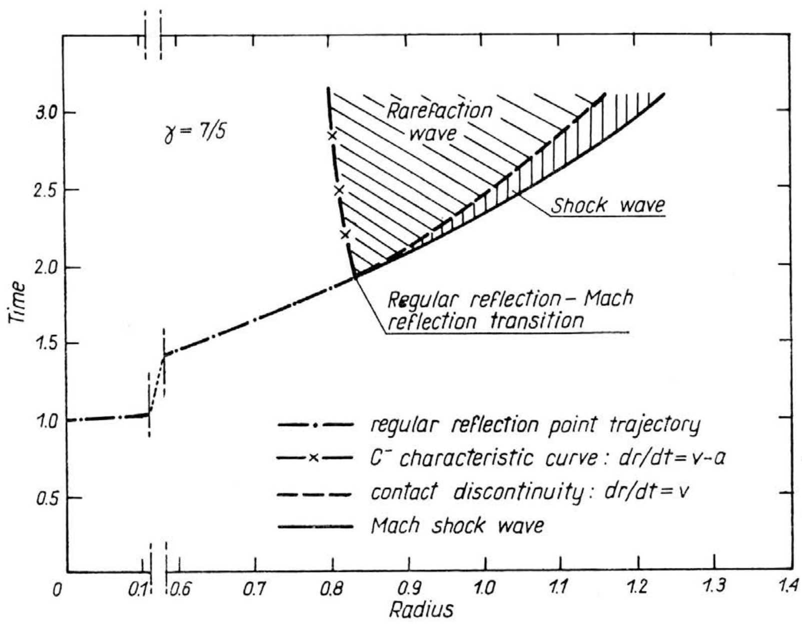

FIG. 12. Wave pattern of the phenomenon of reflection of a spherical shock wave from a surface.

4.1. Theory of Mach configuration of shock waves

The subject of our considerations will be a configuration of waves in a reference frame connected with the triple point T (Fig. 2b).

In the local theory it is assumed that the wave fronts have the form of rays and the flows in regions bounded by those rays are homogeneous. It is assumed, in addition, that on both sides of the slipstream s the pressure is the same and the flow velocities have the same direction.

The theory is based on the complete set of Rankine–Hugoniot conditions:

$$\varrho_j u_j = \varrho_i u_i,$$
$$p_j + \varrho_j u_j^2 = p_i + \varrho_i u_i^2,$$
$$v_j = v_i,$$
$$\frac{u_j^2+v_j^2}{2} + \frac{a_j^2}{\gamma-1} = \frac{u_i^2+v_i^2}{2} + \frac{a_i^2}{\gamma-1},$$
$$p_k = R_k \varrho_k T_k,$$

where p — pressure, a — speed of sound, $a^2 = \gamma p/\varrho$, ϱ, T — density and temperature, respectively, u — mass velocity component normal to the shock wave, v — tangential velocity component. The indices i, j denote states on the two sides of the discontinuity. The R–H conditions can be expressed in the following form convenient for further transformations:

$$\alpha_{ij} = \frac{\varrho_j}{\varrho_i} = \frac{u_i}{u_j} = \frac{\Gamma+\beta_{ij}}{\Gamma\beta_{ij}+1}, \tag{4.1}$$

$$\frac{M_i^2}{M_j^2} = \frac{\Gamma\beta_{ij}+1}{(\Gamma+\beta_{ij})\beta_{ij}} + \frac{2(\Gamma\beta_{ij}+1)}{\beta_{ij}(\Gamma+\beta_{ij})(\gamma-1)M_j^2} - \frac{2}{(\gamma-1)M_j^2}, \tag{4.2}$$

where

$$\Gamma = (\gamma+1)/(\gamma-1),$$
$$\beta_{ij} = p_i/p_j,$$
$$M_k^2 = (u_k^2+v_k^2)/a_k^2.$$

It in assumed in the von Neumann theory that there are three shock waves in a Mach configuration (Fig. 2b), that is the incident wave i, the reflected wave r, the Mach wave m and the slipstream s. The angles θ in Fig. 2b denote the deflection of the flow after passage across the shock wave, and the angle ϕ — the orientation of the shock waves with reference to the undisturbed flow.

From the Rankine–Hugoniot conditions we obtain the following relation between the angles θ and ϕ:

$$\mathrm{tg}\theta = 2\,\mathrm{ctg}\phi \frac{M_j^2 \sin^2\phi - 1}{M_j^2(\gamma+\cos 2\phi)+2}. \tag{4.3}$$

From geometrical relations we find, in addition,

$$\alpha_{ij} = \frac{\mathrm{tg}(\phi-\theta)}{\mathrm{tg}\phi} \tag{4.4}$$

because

$$\frac{\mathrm{tg}(\phi-\theta)}{\mathrm{tg}\phi} = \frac{u_i}{v}\frac{v}{u_j} = \frac{u_i}{u_j} = \alpha_{ij}.$$

Since the flow direction is the same in the regions (2) and (3), the following relation is obtained for the angles

$$\theta_1+\theta_2 = \theta_3, \tag{4.5}$$

where, by virtue of (4.3),

$$\mathrm{tg}\,\theta_1 = 2\,\mathrm{ctg}\,\phi_1 \frac{M_0^2\sin^2\phi_1-1}{M_0^2(\gamma+\cos 2\phi_1)+2},$$

$$\mathrm{tg}\,\theta_2 = 2\,\mathrm{ctg}\,\phi_2 \frac{M_1^2\sin^2\phi_2-1}{M_1^2(\gamma+\cos 2\phi_2)+2},$$

$$\mathrm{tg}\,\theta_3 = 2\,\mathrm{ctg}\,\phi_3 \frac{M_0^2\sin^2\phi_3-1}{M_0^2(\gamma+\cos 2\phi_3)+2}.$$

Because M_1 is expressed in terms of M_0 by the formula (4.2) and β_{10} is expressed in terms of the angles ϕ_1 and θ_1, by the formulae (4.1) and (4.4), therefore, finally, the equation (4.5) is a relation between the orientations of the shock waves and the slipstream.

From the condition of equality of pressure on both sides of the slipstream s we find:

$$\beta_{30} = \beta_{21}\beta_{10}, \tag{4.6}$$

where

$$\beta_{ij} = \frac{\Gamma-\alpha_{ij}}{\Gamma\alpha_{ij}-1}, \quad \text{by virtue of (4.1),}$$

$$\alpha_{ij} = \frac{\mathrm{tg}(\phi-\theta)}{\mathrm{tg}\,\phi}, \quad \text{by virtue of (4.4).}$$

Thus, the orientations of the rays s, i, r, m must satisfy in agreement with the von Neumann theory, the two conditions, (4.5) and (4.6).

4.2. Theory of Mach configuration with Prandtl–Meyer expansion

If the rarefaction wave is taken into consideration in the Mach configuration, its counterpart in the local theory is a Prandtl–Meyer expansion fan described by a function ν relating the flow deflection with the Mach number of the flow (cf. Fig. 13):

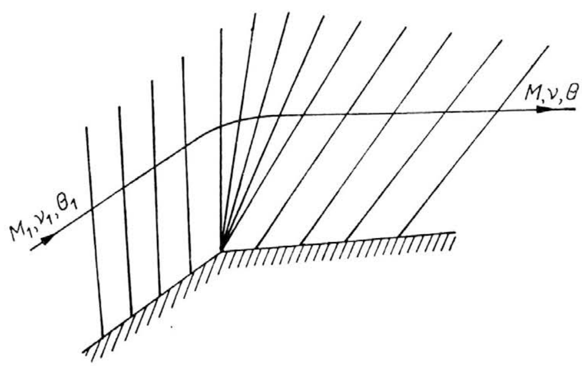

FIG. 13. Prandtl–Meyer expansion fan.

(4.7) $$\nu(M) = \sqrt{\Gamma}\,\mathrm{tg}^{-1}\sqrt{\frac{M^2-1}{\Gamma}}-\mathrm{tg}^{-1}\sqrt{M^2-1}.$$

The function ν is termed Prandtl–Meyer function [22]. In addition, as a consequence of the law of isentropic expansion, we have:

(4.8) $$p^{-1} \sim \left[1+\frac{\gamma-1}{2}M^2\right]^{\frac{\gamma}{\gamma-1}}.$$

If the rarefaction wave is taken into account in the triple configuration of shock waves, the pattern described in Sect. 4.1 changes in an essential manner due to the fact that the Prandtl–Meyer discontinuity is of the point type, what means that it is only at the point T (Fig. 14) that the velocity changes in direction and modulus in a jump-like manner. The variation in the remaining region of the Prandtl–Meyer expansion fan is of the con-

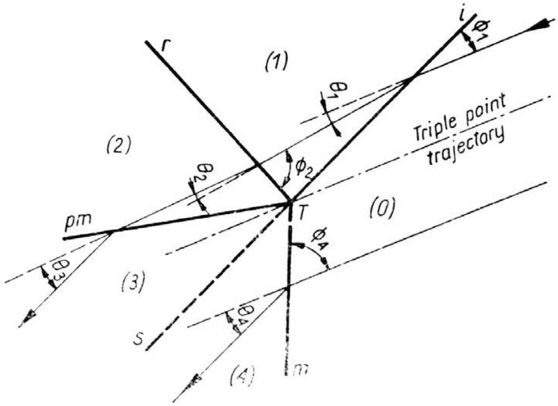

FIG. 14. Mach configuration of shock waves with Prandtl–Meyer expansion fan.

tinuous type. If, therefore, we consider the local theory of triple configuration, then, strictly, we should have, instead of the single line pm corresponding to the Prandtl–Meyer discontinuity, two lines bounding the expansion region. If the rarefaction wave is weak, the line pm may be interpreted as a shock rarefaction wave, the variation in entropy being a third-order quantity in wave strength, therefore the error committed will be insignificant.

In the theory of triple configuration with Prandtl–Meyer expansion taken into account, the conditions at the line of the slipstream s have the form:

(4.9) $$\theta_1+\theta_2+\theta_3 = \theta_4,$$

where

$$\mathrm{tg}\theta_1 = 2\,\mathrm{ctg}\phi_1\,\frac{M_0^2\sin^2\phi_1-1}{M_0^2(\gamma+\cos2\phi_1)+2},$$

$$\mathrm{tg}\theta_2 = 2\,\mathrm{ctg}\phi_2\,\frac{M_1^2\sin^2\phi_2-1}{M_1^2(\gamma+\cos2\phi_2)+2},$$

$$\mathrm{tg}\theta_4 = 2\,\mathrm{ctg}\phi_4\,\frac{M_0^2\sin^2\phi_4-1}{M_0^2(\gamma+\cos2\phi_4)+2}.$$

$$\theta_3 = \nu(M_3)-\nu(M_2),$$

the symbol ν denoting the Prandtl–Meyer function (4.7), and

$$\beta_{40} = \beta_{32}\beta_{31}\beta_{10}, \tag{4.10}$$

where

$$\beta_{ij} = \frac{\Gamma - \alpha_{ij}}{\Gamma\alpha_{ij} - 1},$$

$$\alpha_{ij} = \frac{\mathrm{tg}(\phi - \theta)}{\mathrm{tg}\theta}, \qquad \text{for} \quad ij = 40, 10, 21.$$

For $ij = 32$ we have, according to (4.8)

$$\beta_{32} = \frac{\left(1 + \frac{\gamma - 1}{2} M_2^2\right)^{\gamma/\gamma - 1}}{\left(1 + \frac{\gamma - 1}{2} M_3^2\right)^{\gamma/\gamma - 1}}.$$

All the quantities (except M_3) involved in the conditions (4.9) and (4.10) are expressed in terms of the orientations of the rays i, r, s and m and, of course, the Mach number of undisturbed flow and the characteristics of the gas. As regards the quantity M_3 we have no relation of the type (4.4) which would enable us to relate that quantity with some known quantities. Thus, a new, unknown variable occurs in the set of equations (4.9) and (4.10). By eliminating it we obtain a single equation, which must be satisfied by the orientations of the rays i, r, s and m, by contrast with two equations in the von Neumann theory. Thus, the image of the Mach phenomenon and the limitations imposed by the theory on the experimental results are modified, if the Prandtl–Meyer expansion is taken into account.

It is worthwhile to observe that if the hypothetical rarefaction wave is weak, it may be unobservable by the usual diagnostic methods, but its presence influences the theoretical interpretation of the experiment by introducing an additional degree of freedom or, to express it in a different manner, by eliminating one limitation interrelating the parameters of the wave pattern of the problem.

As already mentioned, a rarefaction wave occurs as a result of decay of the initial discontinuity, which was formed during the regular phase of reflection. It follows that this wave would be characteristic for truly unsteady flows and would have a transitory character connected with the establishment of a new regime after decay of the initial discontinuity. Such a transition wave was observed in numerical experiments described in [12].

The occurrence of a rarefaction wave is in agreement with the HENDERSON–LOZZI postulate [17] which tells that if RR → MR transition is accompanied by a jump in pressure, an unsteady wave with constant amplitude or a sequence of such waves is generated in the flow. An identical conclusion was formulated in Sect. 2, in which it was stated that RR → MR transition should be accompanied, in the case of a strong shock wave (according to an appropriate criterion), by an unsteady rarefaction wave.

Similarly, the case of a weak shock wave is in agreement with the assumed hypothesis that initial discontinuity decay is a mechanism of formation of a Mach configuration. For appropriate values of the gas parameters on both sides of the initial discontinuity it is possible that two shock waves propagating in opposite directions are generated.

By using the term of "weak shock wave" we mean that a finite counterpressure occurs before the front of the incident shock wave. Under such conditions the blast wave is no more selfsimilar, which makes the analysis much more difficult. The theory of the Riemann problem [21] tells that a configuration with two waves is generated if the following condition is satisfied:

$$\frac{v}{a_0} > \frac{z}{\gamma\left[1+\frac{\gamma+1}{2\gamma}z\right]^{1/2}}, \tag{4.11}$$

where v — mass velocity behind the initial discontinuity front, a_0 — speed of sound in the medium at rest before the wave, $z = \frac{p-p_0}{p_0}$ — strength of the initial discontinuity, p — pressure behind the discontinuity, p_0 — pressure before the discontinuity.

By transforming the inequality (4.11) we obtain the following condition for the strength of the discontinuity:

$$z < \frac{\gamma+1}{4\gamma}\left[\frac{v\gamma}{a_0}\right]^2\left[1+\sqrt{1+4/\left[\frac{v(\gamma+1)}{2a_0}\right]^2}\right]. \tag{4.12}$$

If the pressure behind a regularly reflected shock wave, referred to the pressure before the incident wave, satisfies the relation (4.12) at the moment of RR → MR transition, four shock waves should be expected in the wave pattern of the Mach configuration (not three as in the classical image). This suggestion requires some quantitative studies. A particular question to be answered is as to whether there exists a subdivision into strong and weak waves (cf. Sect. 2) that is identical with the condition (4.12) and what is that subdivision. This problem will be the subject of a separate publication.

5. Summary and conclusions

The results of some recently published papers concerned with the problem of interaction between shock waves and obstacles have shown that the problem of universal criterion for the RR ⇄ MR passage has been reopened. In particular some specific features of unsteady flows other than those of steady or pseudosteady flows were observed. In this connection it may be of interest to study the interaction of a spherical shock wave with a plane reflecting surface. This problem is multidimensional and strongly nonlinear, which makes difficult the obtainment of accurate solution. From the point of view of the problem of dynamics of the RR ⇄ MR transition, in which we are interested, the numerical solution obtained in [12] by using one-dimensional model, in which the asymptotic properties for long times and distances are made use of, is sufficient. Thus, the problem is reduced to that of solving the Cauchy problem with initial conditions prescribed on the trajectory of the reflection point obtained on the ground of the theory of regular reflection. Correct formulation of the Cauchy problem requires the inclination of the trajectory, at any point, not to be identical with the characteristic direction. Such a requirement leads to the Hornung–

Oertel–Sandeman sonic criterion for the RR → MR transition. Thus, the HOS criterion is found to be justified by the logical consistency of the mathematical model describing the process.

If we continue the above considerations we arrive at the conclusion that the configuration of shock waves characteristic for irregular reflection known as the Mach configuration is formed as a result of decay of a discontinuity (which does not satisfy the Rankine–Hugoniot conditions), which had been formed in the regular phase of reflection. When the Cauchy problem discussed above becomes ill-posed, the reflection point moves away from the wall and the decay of the discontinuity leads to the formation of a Mach wave. This hypothesis implies that there exists a connection between the RR → MR transition and the Riemann problem. Other facts confirm the existence of an analogy between those two problems. Depending on the strength of the incident shock wave (the difference between a strong and a weak wave was discussed in Sect. 2) there is a possibility of transition with generation of either a shock wave or a rarefaction wave. This statement is implied by the analysis of polar diagrams illustrating the von Neumann criterion and the HOS criterion (Figs. 3 and 4). Also in the Riemann problem the occurrence of a configuration with a rarefaction wave or a shock wave is possible depending on the initial discontinuity parameters. An answer to the question as to whether the difference between a weak and a strong wave (according to the HOS criterion, for instance) is in agreement with the condition (4.12) requires a more detailed numerical analysis. It was shown in Sect. 4 that such an assumption necessitates a modification of the theory of the triple point originally developed by von Neumann. In the case of a RR → MR transition for a strong shock wave the Prandtl–Meyer expansion should additionally be taken into consideration for the configuration of the triple point. As a result, the number of limitations imposed by the theory on the experimental results is reduced to one (there being two in the von Neumann theory). It remains to analyse the configuration of four shock waves resulting from the reflection of a weak wave. This problem will be considered separately in future.

Summing up the above remarks it may be said that:

The analysis presented above shows that the Hornung–Oertel–Sandeman criterion is valid for the RR → MR transition in the problem of reflection of a spherical shock wave from a surface.

RR → MR transition occurs in the problem considered as a result of decay of a discontinuity (which does not satisfy the Rankine–Hugoniot conditions) formed during the regular phase of the process.

A hypothesis has been put forward that the analogy between the Riemann problem and the RR → MR transition problem may be of universal character for unsteady flows.

This leads to a modification of the theory of triple point. In the case of strong shock waves this modification means that Prandtl–Meyer expansion should be taken into account in the local theory of triple point.

As a result, the number of limitations imposed by the theory on the experimental result is reduced (a single equation instead of two interrelating the orientations of shock and contact discontinuity lines).

It should be stressed that the above considerations do not justify acceptation of the HOS criterion and the theory of decay of initial discontinuity as obligatory for steady or pseudosteady flows. They show rather that the cause of formation of a triple configuration and the dynamics of the process as a whole cannot be neglected for truly unsteady flows.

It follows that the von Neumann triple point theory may be of limited application and may not be suitable for any experimental situation.

The analysis just made shows that there is essential difference between a truly unsteady flow and a steady or pseudosteady flow. In the former case RR ⇄ MR transition is possible in the course of a single experiment, what gives rise to transitory processes and associated unsteady rarefaction and shock waves, which are not observed in relaxed states corresponding to steady flows [25].

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