Arch. Mech., 41, 6, pp. 895—908, Warszawa 1989

Description of micro-damage process by porosity parameter for nonlinear viscoplasticity

P. PERZYNA and A. DRABIK (WARSZAWA)

THE MAIN PURPOSE of this paper is to investigate the contribution implied by additional terms arising in the nonlinear viscoplastic response for the growth mechanism of microvoids in ductile solids. The dissipative mechanisms suggested by dynamics of dislocations are discussed. Experimental data are reviewed and compared. The evolution equation for the porosity parameter for the general nonlinear overstress viscoplastic theory is derived. Three different particular overstress functions are investigated, namely — logarithmic, power and linear. The approximation procedure for the three types of the evolution equation obtained shows that a description of the dynamic fracture phenomenon for dissipative solids can be the same in the case of logarithmic and linear functions.

Celem pracy jest zbadanie wpływu nieliniowych właściwości materiałów niesprężystych na mechanizm wzrostu mikropustek w procesie dynamicznego zniszczenia. Przedyskutowano mechanizmy dysypacyjne wynikające z dynamiki ruchu dyslokacji. Zestawiono i porównano wyniki badań doświadczalnych. Wyprowadzono równanie ewolucji dla parametru porowatości w nieliniowej lepkoplastyczności. Obliczenia wykonano dla trzech rodzajów funkcji nadwyżki: logarytmicznej, potęgowej i liniowej. Z przeprowadzonej aproksymacji otrzymanych równań ewolucji wynika, że w przypadku funkcji logarytmicznej i liniowej można zastosować ten sam opis procesu zniszczenia materiału lepkoplastycznego.

Целью работы является исследование влияния нелинейных свойств неупругих материалов на механизм роста микропустот в процессе динамического разрушения. Обсуждены диссипативные механизмы, вытекающие из динамики движения дислокаций. Составлены и сравнены результаты экспериментальных исследований. Выведено уравнение эволюции для параметра пористости в нелинейной вязкопластичности. Вычисления проведены для трех родов избыточных функций: логарифмической, степенной и линейной. Из проведенной аппроксимации полученных уравнений эволюции следует, что в случае логарифмической и линейной функций можно применять то же самое описание процесса разрушения вязкопластического материала.

Introduction

THE MAIN OBJECTIVE of the present paper is to describe ductile fracture in the dynamic process for a material characterized by linear and nonlinear overstress functions. The physical model for a ductile, porous material is such that we consider the aggregate composed of a matrix material and voids. The matrix material is assumed to be a work-hardening viscoplastic described by the constitutive equation as formulated by PERZYNA [17].

The evolution of voids during the dynamic process is described by the parameter ξ which gives the ratio of void volume to the material volume. It describes the evolution of internal imperfections caused by the nucleation, growth and linkage-up of voids.

The first chapter contains physical considerations for the viscoplastic flow process. The dissipative mechanisms suggested by dynamics of dislocations are discussed. As an example some experimental results are shown. Both, physical considerations and the results of the experimental investigations suggest the linear form of the constitutive equation for the viscoplastic porous material in the range of strain rate higher than 10^3 s^{-1}. For this range of strain rates, the most important dissipative mechanism in the matrix material is that of damping by phonon viscosity. For this mechanism the relation between inelastic strain rate and stress is linear.

The question arises whether this suggestion can be obtained directly from the investigation of the viscoplastic growth mechanism of microvoids in ductile material.

To answer this question, the evolution equation for the porosity parameter for the general nonlinear overstress viscoplastic theory is derived. Three different particular overstress functions are investigated, namely logarithmic, power and linear.

The last chapter contains the approximation procedure which shows that for the logarithmic function, the evolution equation for the porosity parameter ξ can be the same as in the case of a linear function. For the power function the result is different.

1. Physical foundations

The basic result of the microscopic investigations is that the elementary process of plastic deformation is the motion of dislocations. For a better understanding of the mechanisms of plastic deformation, we have to investigate the mechanisms which control the motion of dislocations.

It can be explained basing on the experimental observations obtained for mild steel. Taking into account ROSENFIELD and HAHNS [22] experimental results which show the strain rate and temperature dependence, it is possible to distinguish four regions characterized by the different mechanism of plastic deformation (Fig. 1), see also PERZYNA [18, 19, 20].

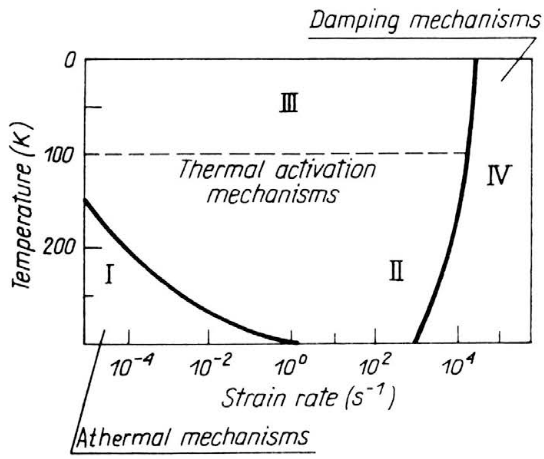

FIG. 1. The distinction of four regions on the temperature–strain rate plane characterized by different dissipative mechanisms (After ROSENFIELD and HAHN [22] and PERZYNA [18]).

Region I is characterized by a plastic flow governed by athermal mechanisms. The yield stress is relatively insensitive to strain rate and temperature.

A typical fact for Region II is that the yield stress is more markedly temperature- and rate-sensitive. The plastic strain rate is controlled by the thermal activation of dislocation motion. The thermal obstacles or mechanisms in pure metals are the Peierls–Nabarro stress, forest dislocations, the motion of jogs in screw dislocations and climb of edge dislocations.

In Region III the lower rate and temperature dependence of the yield stress is observed.

Region IV encompasses very high strain rates, 10^3 to 10^6 s^{-1}. The yield stress seems to be extremely sensitive to strain rate and for this range of strain rates damping of phonon is the most important dissipative mechanism.

The rate and temperature dependence of the flow stress of metals can be explained by different physical mechanisms of dislocation motion.

Let us discuss the most important ones.

1.1. Dislocation creep mechanism

The steady state creep of metals and other materials at a temperature of above one-third of the melting point is characterized by the power relation between strain rate and stress. At this range of temperature vacancies have sufficient mobility to allow dislocation to climb as well as glide (cf. ASHBY and FROST [1]).

The connection between the shear rate $\dot{\gamma}^{(\alpha)}$ and the resolved shear stress $\tau^{(\alpha)}$ on the α slip system for high temperatures can be described by the semi-empirical equation (cf. STEIN and LOW [27]; ASHBY and FROST [1]; HUTCHINSON [7]; PAN and RICE [15]; PEIRCE, ASARO, NEEDLEMAN [16]):

$$\dot{\gamma}^{(\alpha)} = \eta_c^{(\alpha)} \left[\frac{\tau^{(\alpha)}}{g_c^{(\alpha)}} \right]^{\frac{1}{m}} \operatorname{sgn} \tau^{(\alpha)}, \tag{1.1}$$

where $\eta_c^{(\alpha)}$ is a convenient reference creep rate for the slip system α, $g_c^{(\alpha)}$ denotes the reference stress. The functions $\eta_c^{(\alpha)}$, $g_c^{(\alpha)}$ and the exponent m depend on temperature.

1.2. Thermally-activated mechanism

The dislocation moving through the rows of barriers dissipates energy mainly due to the thermally-activated mechanism. This process depends on time, temperature and strain rate.

The average velocity v of a dislocation that surmounts obstacles with the assistance of thermal fluctuation is assumed to be an Arrhenuis-type relationship:

$$v = AL^{-1}\nu \exp\left(-\frac{\Delta G}{k\theta}\right), \tag{1.2}$$

where ν is the frequency of vibration of the dislocation, AL^{-1} is the distance moved after a successful fluctuation, ΔG is the activation energy (Gibbs free energy), k is the Boltzman constant and θ is the actual absolute temperature. For the α slip system, Eq. (1.2) can be written in the form

$$\gamma^{(\alpha)} = \eta_T^{(\alpha)} \exp\{-\psi[(\tau^{(\alpha)} - g_T^{(\alpha)})Lb]/k\theta\}, \tag{1.3}$$

where

$$\eta_T^{(\alpha)} = AL^{-1}\nu b\varrho_M,$$
$$\Delta G = \psi[(\tau^{(\alpha)} - g_T^{(\alpha)})Lb] = \psi(\tau^{\#(\alpha)}Lb),$$
$$g_T^{(\alpha)} = \tau_\mu^{(\alpha)},$$

and $\tau^{\#}$, τ_μ denote the thermal and athermal component of stress, respectively.

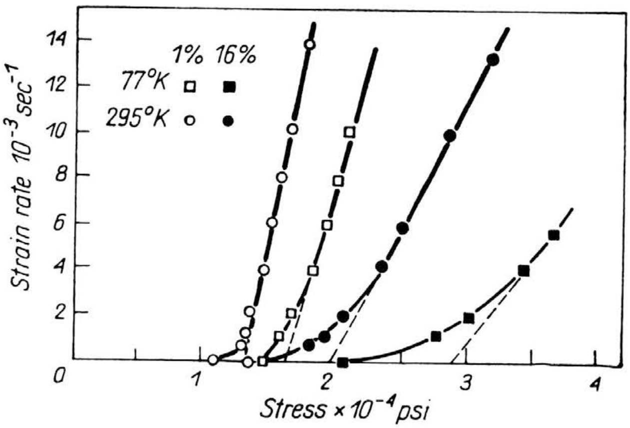

FIG. 2. The strain rate dependence of the yield stress for pollycrystaline aluminium. The data of HAUSER, SIMMONS and DORN [6] plotted on a linear scale.

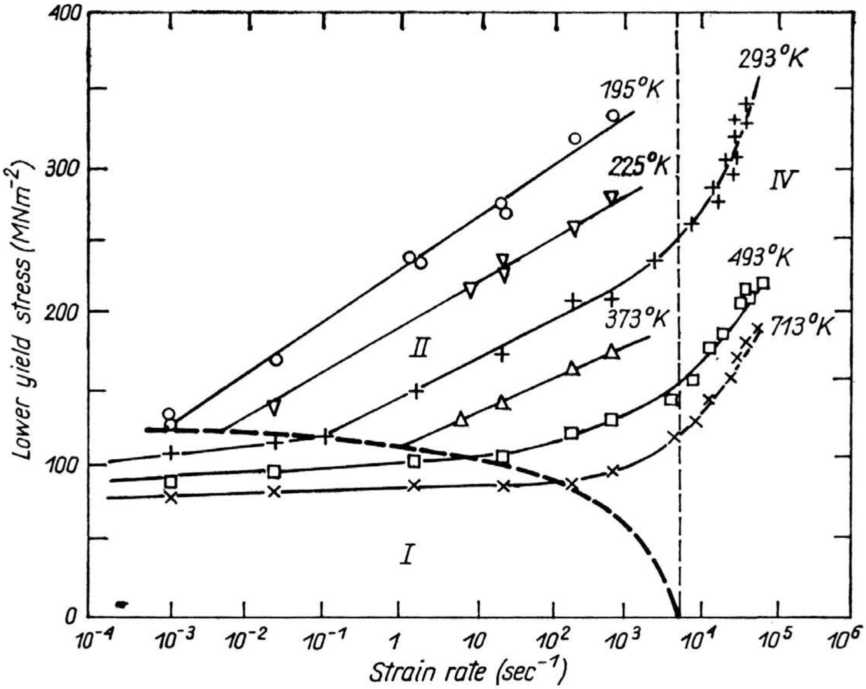

FIG. 3. Variation of lower yield stress with strain rate for mild steel at constant temperature (After CAMPBELL and FERGUSSON [2]).

The linear form of the activation energy ΔG was proposed by SEEGER [24, 25]:

$$\Delta G = \frac{U_0^{(\alpha)}}{k\theta} - \left[(\tau^{(\alpha)} - g_T^{(\alpha)}) \frac{v^*}{k\theta} \right], \tag{1.4}$$

where $U_0^{(\alpha)}$ denotes the constant value of the activation energy, m the α slip system and v^* is the activation volume.

The experimental justifications of the thermally-activated mechanism can be found in the paper of CAMPBELL and FERGUSSON [2] and HAUSER, SIMMONS, DORN [6]. Figure 2 shows the strain rate dependence of the yield stress for pollycrystalline aluminium for different temperatures. In Fig. 3 the variation of lower yield stress of the mild steel is shown.

1.3. Damping mechanism (phonon viscosity)

For the high strain rates the velocity of moving dislocation is controlled by the phonon damping mechanism. This theory has been developed by MASON [11, 12] and NABARRO [14]. The applied stress is high enough to overcome the dislocation barriers without the aid of any thermal fluctuations. In this region, the evolution equation for the plastic shearing has the form

$$\dot{\gamma}^{(\alpha)} = \frac{\varrho_M b^2 g_D^{(\alpha)}}{B} \left[\frac{\tau^{(\alpha)}}{g_D^{(\alpha)}} - 1 \right] \operatorname{sgn} \tau^{(\alpha)}, \tag{1.5}$$

where B is called the dislocation drag coefficient, $\tau^{(\alpha)}$ is the flow stress, $g_D^{(\alpha)}$ is attributed to the stress needed to overcome the forest dislocation barriers to the dislocation motion. This stress is called the back stress.

Experimental justification of the phonon damping mechanism was made by KUMAR, HAUSER and DORN [9], see Fig. 4.

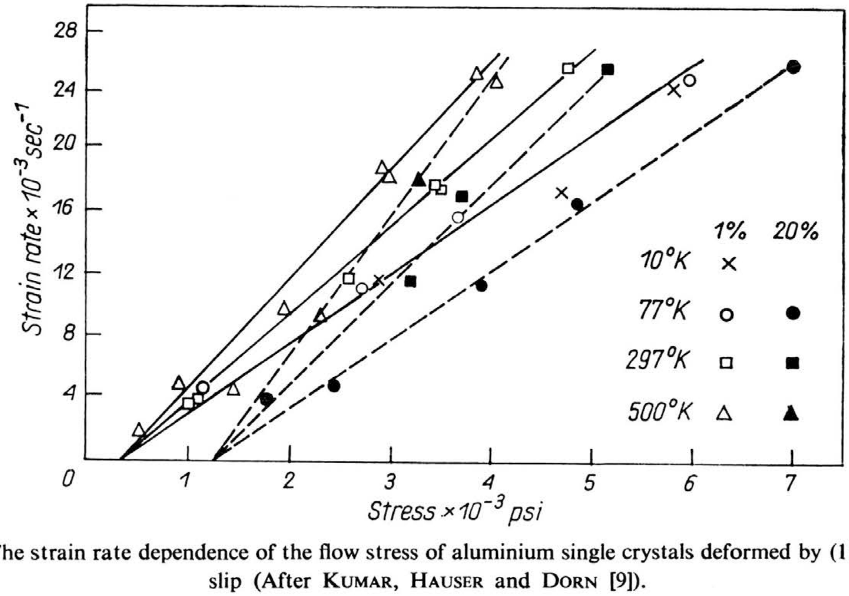

FIG. 4. The strain rate dependence of the flow stress of aluminium single crystals deformed by (111) (110) slip (After KUMAR, HAUSER and DORN [9]).

1.4. Interaction of the thermally-activated and phonon damping mechanisms

The velocity of the dislocation moving through the rows of barriers can be described by the expression

$$v = AL^{-1}/(t_s + t_B), \tag{1.6}$$

where AL^{-1} is the average distance of dislocation movement after each thermal activation, t_s is the time which the dislocation spends at the obstacle and t_B is the time of travelling between barriers.

The evolution equation for the plastic shearing is as follows:

$$\dot{\gamma}^{(\alpha)} = \eta_T^{(\alpha)}[\exp\{\psi[(\tau^{(\alpha)} - g_T^{(\alpha)})Lb]/k\theta\} + ABL^{-1}\nu/(\tau^{(\alpha)} - g_D^{(\alpha)})b]^{-1}, \tag{1.7}$$

where

$$\eta_T^{(\alpha)} = \varrho_M bAL^{-1}\nu,$$

$$\eta_T^{(\alpha)} \frac{b}{ABL^{-1}\nu} = \frac{\varrho_M b^2}{B} = \eta_D^{(\alpha)},$$

and the effective stresses $\tau_T^{\#(\alpha)} = \tau^{(\alpha)} - g_T^{(\alpha)}$ and $\tau_D^{\#(\alpha)} = \tau^{(\alpha)} - g_D^{(\alpha)}$ are defined for the thermally-activated and phonon damping mechanism, respectively.

Experimental justification can be found in the papers written by KUMAR, KUMBLE [10] and SHIOIRI, SATOH, NISHIMURA [26].

According to the ratio between t_s and t_B, the dislocation velocity can be controlled only by the thermally- activated mechanism or only by the phonon damping mechanism or by both of them together.

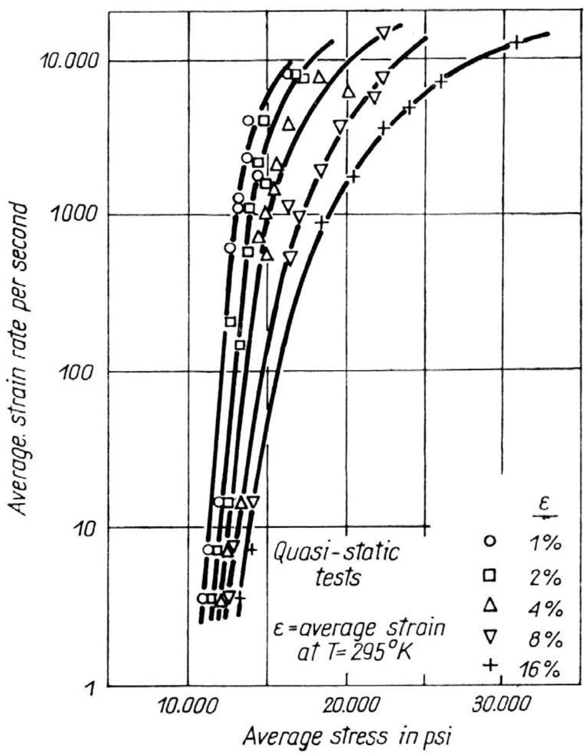

FIG. 5. The strain rate dependence of the yield stress for pollycrystaline aluminium plotted on a semilogarithmic scale (After HAUSER, SIMMONS and DORN [6]).

In this paper we describe a fracture phenomenon for a very high strain rate so our attention is focussed on Region IV which was characterized at the beginning of this chapter.

The experiments performed by F. E. HAUSER, J. A. SIMMONS and J. E. DORN [6] have shown the strain and yield stress dependence for pollycrystaline aluminium for the range of strain rates considered. The results plotted in semilogarithmic scale are shown in Fig. 5. Their transformation into the linear scale shows that in Region IV the yield stress and strain dependence is linear (Fig. 2, cf. Ref. [6]). The yield stress as a function of strain rate is also presented in Fig. 6 and Fig. 7 (cf. Ref. [13]). We intend to obtain similar results by directly considering the viscoplastic growth mechanism of microvoids in ductile metals.

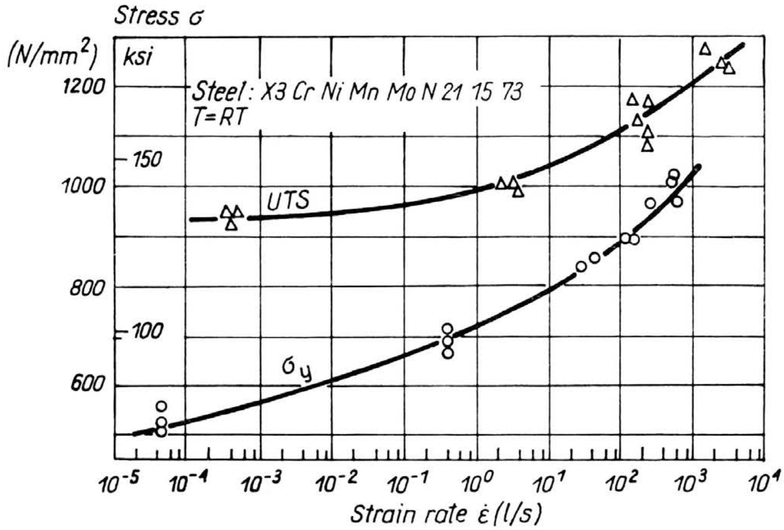

FIG. 6. Yield stress σ_Y and UTS as function of strain rate of 18 Ni-maraging steel.

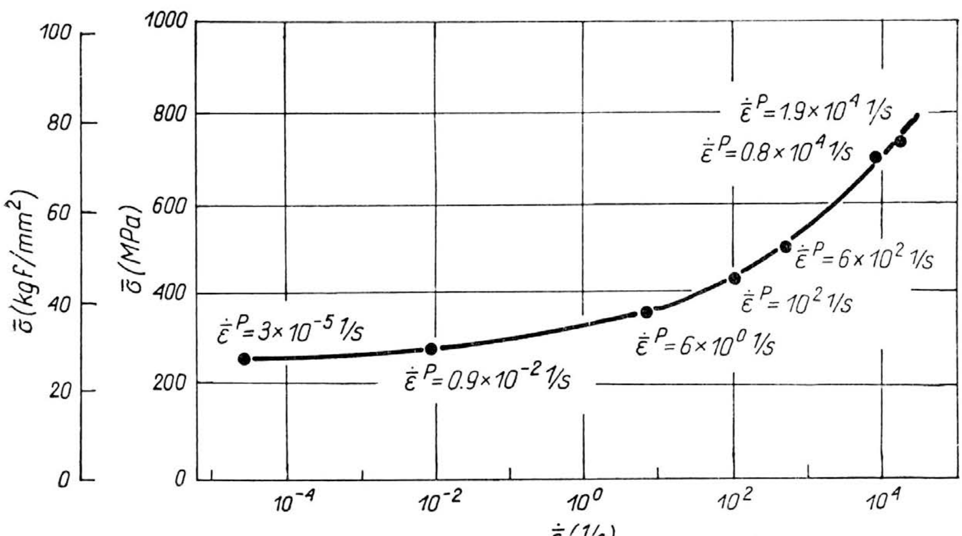

FIG. 7. Representative stress-strain rate relation.

2. Model of the porous material

The physical model of the porous material considered by us was firstly described by CARROL and HOLLT [3] and then developed by JOHNSON [8] and PERZYNA [21]. The base is a rectangular material volume element which contains a representative distribution of voids, as it is shown in Fig. 8. A uniform hydrostatic tension $\bar{p}$ acts over the surface of this element. For the calculations, the simplified model of the porous material is taken into account.

Let us consider a spherical void of a radius a in a material sphere of a radius b subject to internal pressure p_g and external stress σ_r (see Fig. 9 and Ref. [8]). We define the imperfection parameter ξ, namely porosity, by the relation

$$\xi = \frac{a^3}{b^3}. \tag{2.1}$$

It describes the ratio of voids volume to the solid volume.

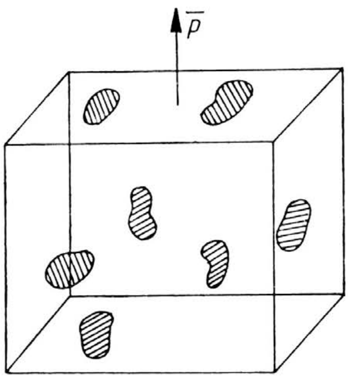

FIG. 8. Element of the material with representative distribution of voids.

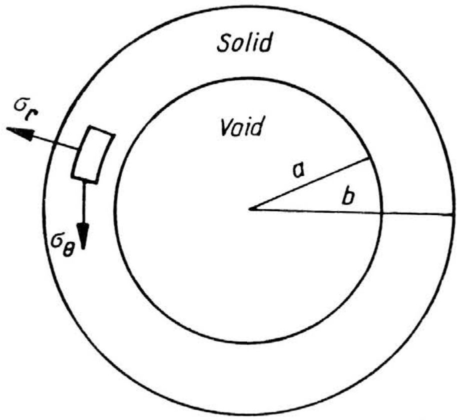

FIG. 9. Simplified model of the porous material.

3. Constitutive relation

The constitutive equation for a viscoplastic material with internal imperfections was proposed by PERZYNA [21]. This equation for the rate of the inelastic deformation tensor $\mathbf{E}^p$ has the form

$$\dot{\mathbf{E}}^p(t) = \frac{\gamma}{\varphi}\left\langle \Phi\left[\frac{f(\cdot)}{\varkappa} - 1\right]\right\rangle \partial_{\mathbf{T}(t)} f, \tag{3.1}$$

where γ denotes the temperature-dependent viscosity coefficient, φ is the control function dependent on $(I_2/I_2^s)^{-1}$ where I_2 is the second invariant of the rate of the deformation tensor $\dot{\mathbf{E}}^p$, I_2^s is its static value and Φ denotes the viscoplastic overstress function, $\varkappa$ is a material function describing work-hardening effects. The symbol $\langle[\,]\rangle$ is defined as

$$\langle[\ ]\rangle = \begin{cases} 0 & \text{if} \quad f \leqslant \varkappa \\ [\ ] & \text{if} \quad f > \varkappa. \end{cases} \tag{3.2}$$

The yield function for the damaged solid is postulated in the form

$$f(\cdot) = \left\{ J_2'[1-(n_1+\xi n_2)\frac{J_3'^2}{J_2'^3} + (n_3+\xi n_4)\frac{J_1^2}{J_2'}\right\}^{1/2}, \tag{3.3}$$

where n_i $(i = 1, 2, 3, 4)$ denote material constants; J_1 is the first invariant of the Cauchy stress tensor; J_2', J_3' the second and third invariants of the stress deviator.

4. Evolution equation for the porosity parameter

JOHNSON [8] derived the relation between the parameter ξ and external pressure $\bar{p}$ as follows:

$$(\bar{p}(t)-p_g)\frac{1}{1-\xi} - 2\int_a^b \frac{\Delta s}{r}\, dr = 0, \tag{4.1}$$

where $\Delta s = \sigma_r - \sigma_\theta$, $\bar{p}$ is a uniform hydrostatic pressure, i.e., $\bar{p} < 0$ in tension, p_g is the internal gas pressure, $p_g > 0$; σ_r and σ_θ denote the radial and circumferential deviatoric stresses, respectively.

The relation (4.1) was obtained from the equation of motion for the material surrounding the void.

4.1. Linear overstress function

This chapter contains the derivation of the evolution equation for the parameter ξ for the linear overstress viscoplastic function Φ.

The yield condition of the matrix material is given by the following relation:

$$\sqrt{J_2'} = Y\left\{1+\Phi^{-1}\left(\frac{\dot{\bar{\varepsilon}}^p}{\gamma}\right)\right\}, \tag{4.2}$$

where $Y = \pm Y_0 + H(\bar{\varepsilon}^p)$ is the plastic strain dependent yield stress due to work-hardening effects and the upper signs correspond to voids growth, whereas the lower signs — to voids compaction; $\bar{\varepsilon}^p$ is the equivalent plastic deformation (cf. Ref. [21]), $\bar{\varepsilon}^p = \int_0^t \left(\frac{2}{3} D^p : D^p\right)^{1/2} dt$.

The linearized form of Eq (4.2) is as follows:

$$\Delta s = \sigma_r - \sigma_\theta = \mp Y_0 + H'\bar{\varepsilon}^p + \eta\dot{\bar{\varepsilon}}^p, \qquad \eta = \frac{Y_0}{\gamma}. \tag{4.3}$$

Putting the expression (4.3) into Eq. (4.1), we obtain the final form of the evolution equation for the parameter ξ in the case of the linear overstress function

$$\dot{\xi} = \frac{F(\xi, \xi_0)}{\eta}\left[-\bar{p}(t)+p_p \mp \frac{2Y_0}{3}(1-\xi)\ln|\xi| + \frac{2H'}{3}F_1(\xi, \xi_0)\right], \tag{4.4}$$

where

$$F_1(\xi, \xi_0) = 3\left(\frac{1-\xi}{1-\xi_0}\right)^{1/3}\left[1-\left(\frac{\xi_0}{\xi}\right)^{1/3}\right],$$

$$F(\xi, \xi_0) = \frac{3\xi}{2}\left(\frac{1-\xi}{1-\xi_0}\right)^{1/3}\left[\xi-\left(\frac{\xi_0}{\xi}\right)^{1/3}\right]^{-1}.$$

The equilibrium state is reached in the case when $\dot{\xi} \equiv 0$, so assuming that $p_g \equiv 0$, we obtain

$$p_{\text{equ}} = \mp \frac{2Y_0}{3}(1-\xi)\ln|\xi| + \frac{2H'}{3}(1-\xi)F_1(\xi, \xi_0). \tag{4.5}$$

Equation (4.4) describes the growth of voids during a dynamical deformation process for $\bar{p} < p_{\text{equ}}(\xi)$ in tension or $\bar{p} > p_{\text{equ}}(\xi)$ in compression. In the other cases $\dot{\xi} \equiv 0$.

4.2. Logarithmic overstress function

We take into consideration the first nonlinear case of the function Φ. For $\Phi^{-1} = \ln(\dot{\varepsilon}^p/\gamma+1)$, Eq. (4.2) takes the form

$$\sqrt{J_2'} = Y\left\{1+\ln\left(\frac{\dot{\varepsilon}^p}{\gamma}+1\right)\right\}, \tag{4.6}$$

so

$$\Delta s = \sigma_r - \sigma_\theta = \mp Y_0 + H'\varepsilon^p + Y_0 \ln\left(\frac{\dot{\varepsilon}^p}{\gamma}+1\right). \tag{4.7}$$

It is impossible to find the analytical form of the integral $\int\limits_a^b \frac{\Delta s}{r}\,\mathrm{d}r$ for Δs described by Eq. (4.7). To obtain the approximate value, the power series expression procedure of the $\ln(\dot{\varepsilon}^p/\gamma+1)$ was used:

$$\ln\left(\frac{\dot{\varepsilon}^p}{\gamma}+1\right) = \sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}\left(\frac{\dot{\varepsilon}^p}{\gamma}\right)^n. \tag{4.8}$$

For the small values of $\dot{\varepsilon}^p$, the power series is converging and $\ln(\dot{\varepsilon}^p/\gamma+1)$ can be approximated by its first term

$$\ln\left(\frac{\dot{\varepsilon}^p}{\gamma}+1\right) \simeq \frac{\dot{\varepsilon}^p}{\gamma} \quad \text{and} \quad \Delta s = \mp Y_0 + H'\varepsilon^p + \eta\dot{\varepsilon}^p, \quad \eta = \frac{Y_0}{\gamma}. \tag{4.9}$$

Putting this result into Eq. (4.1), we obtain the same evolution equation for the porosity parameter as in the linear case:

$$\dot{\xi} = \frac{F(\xi, \xi_0)}{\eta}\left[-\bar{p}(t)+p_g \mp \frac{2Y_0}{3}(1-\xi)\ln|\xi| + \frac{2H'}{3}F_1(\xi, \xi_0)\right]. \tag{4.10}$$

Equation (4.10) applies for $\bar{p} < p_{\text{equ}}(\xi)$ in tension or $\bar{p} > p_{\text{equ}}(\xi)$ in compression, where $p_{\text{equ}}(\xi)$ is defined by Eq. (4.5).

4.3. Power overstress function

The last case of the nonlinear overstress function which is mainly used in the description of the viscoplastic material behaviour is the power function case.

For $\Phi^{-1}(\dot{\varepsilon}^p/\gamma) = (\dot{\varepsilon}^p/\gamma)^{1/n}$, Eq. (4.2.) is as follows.

(4.11) $$\sqrt{J_2'} = Y\left\{1+\left(\frac{\dot{\varepsilon}^p}{\gamma}\right)^{\frac{1}{n}}\right\},$$

so

(4.12) $$\Delta s = \sigma_r - \sigma_\theta = \mp Y_0 + H'\varepsilon^p + Y_0\left(\frac{\dot{\varepsilon}^p}{\gamma}\right)^{\frac{1}{n}}.$$

Here we encounter the same difficulty as in the previous chapter. The integral from Eq. (4.1) with Δs defined by Eq. (4.12) has no analytical form. To obtain the approximate solution, the power series expansion of $\left(\frac{\dot{\varepsilon}^p}{\gamma}\right)^{1/n}$ is applied:

(4.13) $$\left(\frac{\dot{\varepsilon}^p}{\gamma}\right)^{\frac{1}{n}} = \left[\left(\frac{\dot{\varepsilon}^p}{\gamma}\right) - 1 + 1\right]^{\frac{1}{n}} = \sum_{k=0}^{\infty}\begin{pmatrix}\frac{1}{n}\\ k\end{pmatrix}\left(\frac{\dot{\varepsilon}^p}{\gamma} - 1\right).$$

For small values of $\dot{\varepsilon}^p$, the following approximation is possible:

(4.14) $$\left(\frac{\dot{\varepsilon}^p}{\gamma}\right)^{\frac{1}{n}} = 1 - \frac{1}{n} + \frac{1}{n}\frac{\dot{\varepsilon}^p}{\gamma}.$$

Hence, from Eqs. (4.12) and (4.14) with $\eta = \dfrac{Y_0}{\gamma}$ it follows that

(4.15) $$2\int_a^b \frac{\Delta s}{r}\,dr = \pm\frac{2Y_0}{3}\ln|\xi| - \frac{2H'}{3}F_1(\xi,\xi_0) - \frac{\eta\dot{\xi}}{n\xi}F_2(\xi,\xi_0) + \frac{2Y_0}{3}\left(\frac{1}{n} - 1\right)\ln|\xi|,$$

where the function $F_1(\xi, \xi_0)$ is the same as in the linear case and

$$F_2(\xi,\xi_0) = \frac{2}{3(1-\xi_0)}\left(\frac{1-\xi}{1-\xi_0}\right)^{-\frac{2}{3}}\left[\xi - \left(\frac{\xi_0}{\xi}\right)^{\frac{1}{3}}\right].$$

The final form of the evolution equation for the parameter ξ is obtained by putting the expression (4.15) into Eq. (4.1)

(4.16) $$\dot{\xi} = \frac{nF(\xi,\xi_0)}{\eta}\left[-p(t) + p_g \mp \frac{2Y_0}{3}(1-\xi)\left(2 - \frac{1}{n}\right)\ln|\xi| + \frac{2H'}{3}(1-\xi)F_1(\xi,\xi_0)\right].$$

It is easy to observe that for $n = 1$ it is possible to obtain directly from Eq. (4.16) the linear case. For the other values of n, the forms of the evolution equation for linear and power function cases are different. For greater values of n the difference between them increases.

From the experimental observations it is obtained that for the material descriptions the following values of $n = 3, 5, 7$ are mainly used. Taking into account one of these values, Eq. (4.16) can be written in such a way

(4.17) $$\dot{\xi} = \frac{3F(\xi,\xi_0)}{\eta}\left[[-\bar{p}(t) + p_g \mp \frac{10Y_0}{9}(1-\xi)\ln|\xi| + \frac{2H'}{3}(1-\xi)F_1(\xi,\xi_0)\right]$$

for $n = 3$ and $\eta_2 = Y_0/\gamma$.

5. Discussion of the approximation procedure

Let us return to the case of the linear overstress function. The integral evaluated from Eq. (4.1) has the form

$$(5.1)\qquad 2\int_a^b \frac{\Delta s}{r}\,dr = \pm\frac{2Y_0}{3}\ln|\xi| + \frac{2H'}{3}F_1(\xi,\xi_0) - \frac{\eta\dot{\xi}}{\xi}F_2(\xi,\xi_0).$$

It is convenient to introduce the distension parameter α, firstly defined by J. N. JOHNSON [8]. The dependence between the distension parameter α and the void volume fraction parameter ξ is as follows:

$$(5.2)\qquad \alpha = \frac{1}{1-\xi}.$$

The transformation of the functions $F_1(\xi,\xi_0)$ and $F_2(\xi,\xi_0)$ into the new variable is the following:

$$(5.3)\qquad \begin{aligned} F_1(\xi,\xi_0) &\to F_1(\alpha,\alpha_0) = 3\left[\left(\frac{\alpha_0}{\alpha}\right)^{1/3} - \left(\frac{\alpha_0-1}{\alpha-1}\right)^{1/3}\right],\\ F_2(\xi,\xi_0) &\to F_2(\alpha,\alpha_0) = \frac{2(\alpha-1)}{3}\left(\frac{\alpha_0}{\alpha}\right)^{1/3} - \frac{2\alpha}{3}\left(\frac{\alpha_0-1}{\alpha-1}\right)^{1/3},\end{aligned}$$

and

$$\begin{aligned} F_1(\alpha,\alpha_0) &\cong 3\left(\frac{\alpha_0}{\alpha}\right)^{1/3},\\ F_2(\alpha,\alpha_0) &\cong \frac{2(\alpha-1)}{3}\left(\frac{\alpha_0}{\alpha}\right)^{1/3}, \qquad \text{for} \quad \alpha_0 \cong 1, \quad \alpha_0 - 1 \ll 1.\end{aligned}$$

Thanks to this approximation, some parts of the functions $F_1(\alpha,\alpha_0)$ and $F_2(\alpha,\alpha_0)$ could be neglected and this leads to the simpler form of Eq. (4.4):

$$(5.4)\qquad \dot{\xi} = \frac{\tilde{F}(\xi,\xi_0)}{\eta}\left[-\bar{p}(t) + p_g \mp \frac{2Y_0}{3}\ln|\xi|(1-\xi) + \frac{2H'}{3}(1-\xi)\tilde{F}_1(\xi,\xi_0)\right],$$

where

$$\begin{aligned}\tilde{F}(\xi,\xi_0) &= \frac{3}{2}\left(\frac{1-\xi_0}{1-\xi}\right)^{1/3},\\ \tilde{F}_1(\xi,\xi_0) &= 3\left(\frac{1-\xi}{1-\xi_0}\right)^{1/3}.\end{aligned}$$

As an application of this theory, the plate impact experiment on copper was calculated — a 0.6 mm thick copper plate strikes a 1.6 mm target backed by a relatively thick plate of PMMA (polymethylmethacrylate). The full description of the experiment is presented by SEAMAN, BARBEE, CURRAN [23] and from their paper the following values are taken:

$$\bar{p} = 3.0\times10^8\,\mathrm{N/m^2},\quad Y_0 = 26\times10^7\,\mathrm{N/m^2},\quad H' = 9.8\times10^7\,\mathrm{N/m^2},\quad \eta = 4.25\ \mathrm{Ns/m^2},$$
$$\xi_0 = 0.001,\quad p_g = 0.$$

The solution of Eq. (5.4) was found according to the Runge–Kutta–Gill method with the initial conditions $\xi(0) = \xi_0$. The result is shown in Fig. 10. It presents the dependence between the time and the porosity parameter. The experiments performed by SEAMAN *et al.* [23] specified the critical value of ξ at which spallation occurs — for $t = 0.3$ μs $\xi_s = 0.32$. From our calculations we obtained the value $\xi_s = 0.3136$.

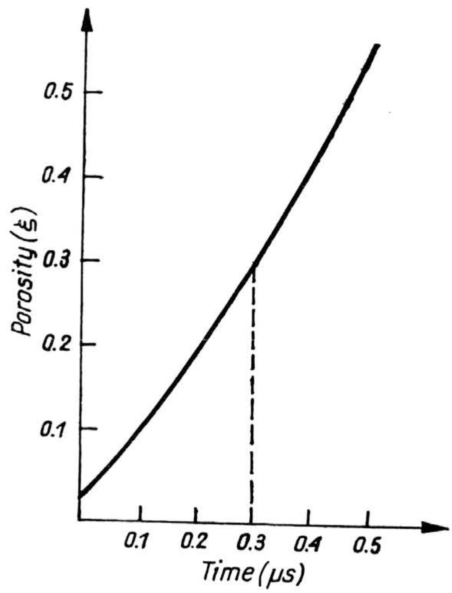

FIG. 10. Time and porosity dependence in plate-impact experiment on copper.

4. Final comments

The main purpose of this paper was to find a simple description of the dynamic fracture phenomenon by means of the evolution equation for the porosity parameter ξ when the viscoplastic properties of the material are described by different overstress functions, namely linear, logarithmic and power. The evolution equation for the parameter ξ derived for nonlinear cases contains the term which is similar to that for the linear case. The approximation proceduce developed shows that additional terms in the evolution equation can be neglected only in the case of the logarithmic overstress function. For the power function the analysis is more difficult and it does not give satisfactory results.

References

1. M. F. ASHBY and H. J. FROST, in: Constitutive Equations in Plasticity (ed. A. S. ARGON), MIT Press, Cambridge, Mass, 1975.
2. J. D. CAMPBELL and W. G. FERGUSSON, Phil. Mag., **81**, 63–82, 1970.
3. M. M. CARROLL and A. C. HOLT, J. Appl. Phys., **43**, 4, 1626–1636, 1972.
4. J. J. GILMAN, J. Appl. Phys., **36**, 2772–2777, 1965.

5. J. J. GILMAN, *Micromechanics of flow in solids*, McGraw-Hill, New York 1969.
6. F. E. HAUSER, J. A. SIMMONS and J. DORN, in: Response of Metals to High Velocity Deformations, pp. 93–114, Wiley, New York 1960.
7. J. W. HUTCHINSON, Proc. R. Soc. Lond., *A*. **348**, 101–127, 1976.
8. J. W. JOHNSON, J. Appl. Phys., **52**, 2812–2825, 1981.
9. A. KUMAR, F. E. HAUSER and J. E. DORN, Acta Metall., **16**, 1189–1191, 1968.
10. A. KUMAR and R. G. KUMBLE, J. Appl. Phys., **40**, 3475–3480, 1969.
11. W. P. MASON, Acoustical Soc. Amer., **32**, 458–472, 1960.
12. W. P. MASON, J. Appl. Phys., **35**, 2779–2781, 1964.
13. L. W. MEYER, H. D. KUNZE and K. SEIFERT, *Shock waves and high-strain-rate phenomena in metals*, Plenum Press, New York 1981.
14. F. R. N. NABARRO, *Theory of crystal disclocations*, Oxford Univ. Press, 1967.
15. J. PAN and J. R. RICE, Int. J. Solids Struct., **19**, 973, 1983.
16. D. PEIRCE, R. J. ASARO and A. NEEDLEMAN, Acta Metall., **30**, 1087–1119, 1982.
17. P. PERZYNA, Quarterly Appl. Math., **20**, 321–332, 1963.
18. P. PERZYNA, Arch. Mech., **29**, 4, pp. 607–624, 1977.
19. P. PERZYNA, *Thermodynamics of inelastic materials*, PWN, 1978 [in Polish].
20. P. PERZYNA, *Thermodynamics of dissipative materials*, Recent Developments in Thermomechanics, CISM Courses and Lectures, 262, 1980.
21. P. PERZYNA, Int. J. Solids Struct., **22**, **7**, 797–818, 1986.
22. A. R. ROSENFIELD and G. T. HAHN, Trans. ASM, **59**, 962–980, 1966.
23. L. SEAMAN, T. W. BARBEE and D. R. CURRAN, Stanford Res. Int. Tech. Rep. No A FWL-TR-71-156, December 1971.
24. A. SEEGER, Phil. Mag., **46**, 1194–1217, 1955.
25. A. SEEGER, in: Handbuch der Physik VII/2 (ed. S. FLÜGGE), Springer 1958.
26. J. SHIORI, K. SATOH and K. NISHIMURA, in: High Velocity Deformations in Solids, IUTAM Symp. Proc., 50–56, 1979.
27. D. F. STEIN and J. R. LOW Jr., J. Appl. Phys., **31**, 362–369, 1960.

POLISH ACADEMY OF SCIENCES
INSTITUTE OF FUNDAMENTAL TECHNOLOGICAL RESEARCH.

Received May 21, 1987; *new version February* 22, 1989.