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On the role of strain concentrations in the mechanics of ductile fracture of metals

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SLIP-LINE SOLUTIONS are presented in order to demonstrate the role of internal strain concentrations in the mechanics of ductile fracture. Coalescence of voids and possible mechanisms of internal fracture are examined. Examples of simple experimental modelling substantiate theoretical analysis.

Przedstawiono kilka rozwiązań metodą linii poślizgu dla zademonstrowania roli, jaką spełniają wewnętrzne koncentracje odkształceń w materiale w procesie pękania ciągliwego. Zbadano procesy łączenia się pustek i inne możliwe mechanizmy pękania ciągliwego. Proste badania na modelach eksperymentalnych potwierdzają wnioski wynikające z analizy teoretycznej.

Представлено несколько решений методом линии скольжения для представления роли, какую исполняют внутренние концентрации деформаций в материале в процессе вязкого разрушения. Исследованы процессы соединения пустот и другие возможные механизмы вязкого разрушения. Простые исследования на экспериментальных моделях подтверждают следствия, вытекающие из теоретического анализа.

1. Introduction

STRAIN concentrations play an important role in the mechanics of ductile fracture of metals. For example, it is well known that if a thin sheet of a ductile metal is stretched uniaxially or biaxially there appear in it grooves with reduced thickness. This is connected with strong strain concentration in the grooves and leads finally to fracture along the grooves. This important problem was analysed by HILL [1, 2] for the case of uniaxial tension and by MARCINIAK and KUCZYŃSKI [3] for biaxial stretching in connection with the analysis of various methods of plastic forming of sheet metal. Strain localization in the form of shear bands in a bulk material was analysed by MARCINIAK and KUCZYŃSKI [4] in an attempt to obtain theoretical estimation of limit strain during cold working processes. In the latter paper a review of other attempts to examine processes leading to the strain localization has been presented.

Mechanisms of strain localization caused by growth and coalescence of voids have been analysed by several authors. RICE and TRACEY [5] and MCCLINTOCK [6] examined the process of the growth of a single spherical or cylindrical cavity as a function of strains in the surrounding material. In the latter work the coalescence of voids as a factor responsible for ductile fracture of metals has also been analysed. Experimental evidence for ductile fracture by the growth of holes has been known for a number of years (see for example [6] and [7]).

The role of strain concentration as a result of internal micronecking has been discussed in several works in connection with ductile fracture. In order to demonstrate the influence of the structure on the ductility of metals, ROGERS [8] compared the tensile ductility of two theoretical models. One is a cylindrical rod of a perfectly plastic material and the other is a cable of equal external diameter consisting of a bundle of cylindrical strands. Each of the strands behaves in the same manner as a single rod, necking down to a point. Thus total elongation of the rod just before separation into two pieces is many times larger than that for the cable. A similar mechanism of ductile fracture was discussed by BACKOFEN [9] and by HULT and TRAVNICEK [10].

These ideas have been extended in [11] and [12] in order to study the progressing process of ductile fracture in the presence of a system of cracks parallel to the direction of the uniaxial tensile stress. Theoretical analysis of the process of internal micronecking was based on the slip-line technique. Experiments on specimens made of a ductile metal with prepared slits simulating cracks substantiate theoretical analysis.

A theoretical and experimental study of interaction between differently oriented cracks in a plastic medium was given in another work [13] in an attempt to obtain a deeper insight into the process of ductile fracture of metals. Theoretical analysis was based on the slip-line technique. Experiments confirmed theoretical results. The present study constitutes a continuation of the latter work. Interaction of variously oriented voids and cracks is analysed under conditions of plane plastic flow by means of the slip-line technique. Some preliminary simple experiments are also discussed.

Note that a similar problem of interaction between regularly distributed cracks or voids was examined by means of the slip-line technique by NAGPAL *et al.* [14], in order to obtain approximate equations for fracture by the development of a sheet of localized flow at the front of a crack. The present paper which is, to a certain degree, connected with Nagpal's work is, however, mainly devoted to the study of the process of evolution of strain localization and concentration. They lead to local microfractures inside the material in the presence of defects existing in the material or nucleated during the deformation process.

2. Ductile fracture by the hole growth

As an introductory example let us analyse by means of the slip-line technique the process of the ideally ductile fracture of a rigid-ideally plastic body weakened by a single row of circular holes as shown in Fig. 1a. The problem will be examined under the assumption of plane strain conditions. It is evident that the presence of even very small holes causes remarkable strain localization along the band of holes. The initial slip-line system consisting of logarithmic spirals is shown in Fig. 1a. For the assumed ratio of dimensions $a/d = 4.75$, where a is the spacing between the holes and d stands for the diameter of the holes, the slip-lines do not overlap. The theoretical analysis presented below is kinematically admissible only, since the extension of the slip-line field into the rigid region has not been found.

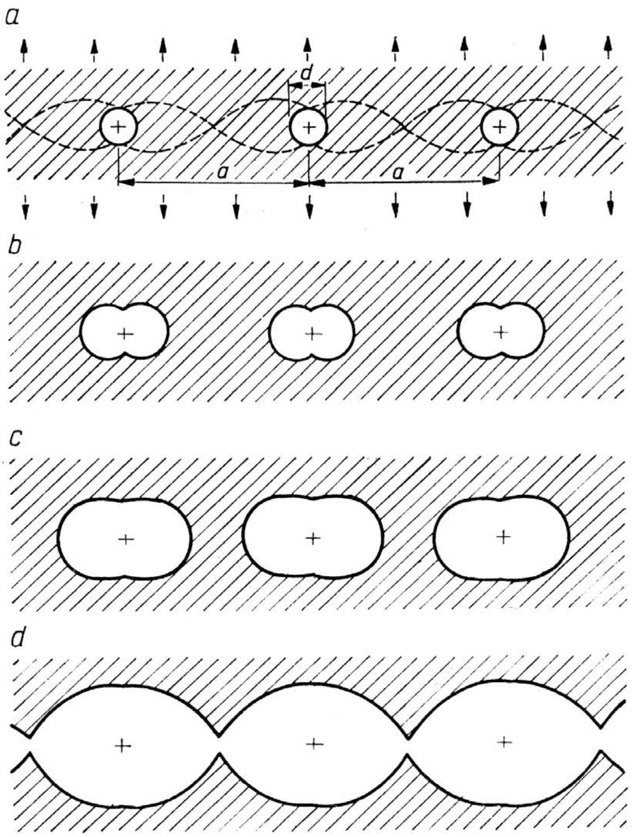

FIG. 1.

The left-hand side of Fig. 2 shows the initial configuration with the square grid assumed to exist in the non-deformed body. On the right the initial stage of deformation (cf. [15]) with the strong strain concentration along the band of holes has been presented. For

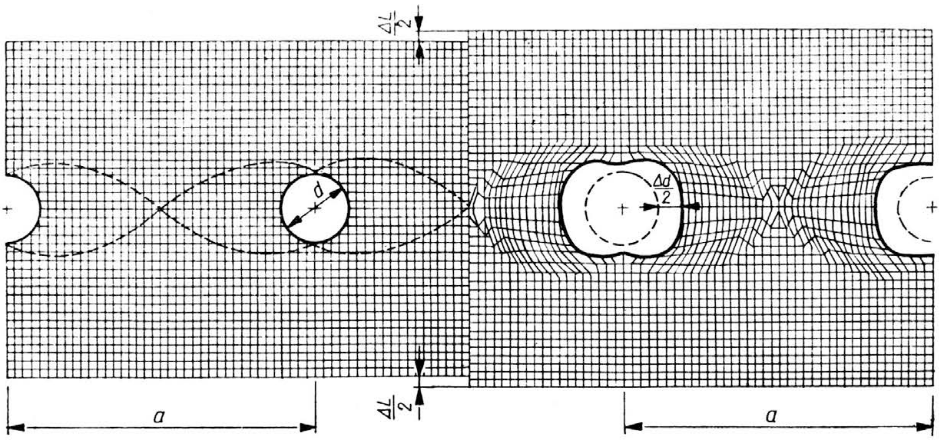

FIG. 2.

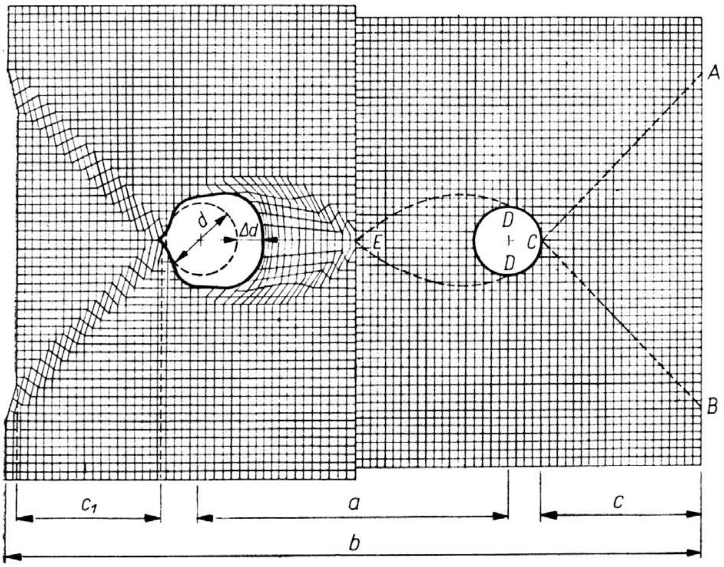
A
d
Δd
E
D
C
D
B
c_1
a
c
b

FIG. 3.

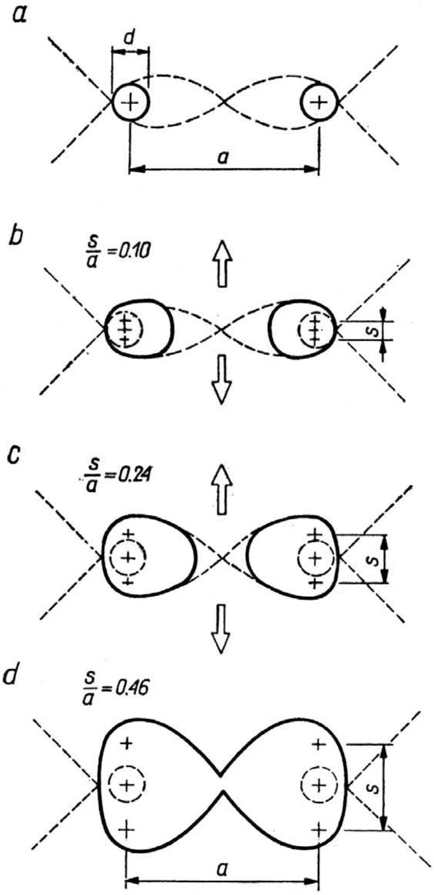
a
d
a
b
$\frac{s}{a}=0.10$
s
c
$\frac{s}{a}=0.24$
s
d
$\frac{s}{a}=0.46$
s
a

FIG. 4.

the relatively small total elongation ΔL of the body shown in the figure the conventional lateral strain within the band is as large as $\Delta d/(a-d) \sim 0.22$. For ready comparison this stage of deformation has also been shown in Fig. 1b. Figure 1c (cf. [16]) shows how quickly the process of the hole growth advances, leading finally to the total separation of the body along the band of holes as shown in Fig. 1d. Total separation of the body occurs when its elongation ΔL is rather small. Thus, assuming in our analysis the rigid-ideally plastic model of the material, we obtain the final result very close in effect to the brittle fracture of the body.

Figure 3 shows the theoretical plane strain solution to the problem of the growth of two holes prepared in a rigid-ideally plastic bar of the finite width b. The initial configuration of slip-lines is shown on the right-hand side of the figure. The left-hand side shows an advanced stage of the process of the hole growth and demonstrates the strain concentration in the band of holes. The deformation pattern in the ligament between the holes is the same as that shown in Fig. 2 (cf. [16]). It can be seen that the conventional lateral

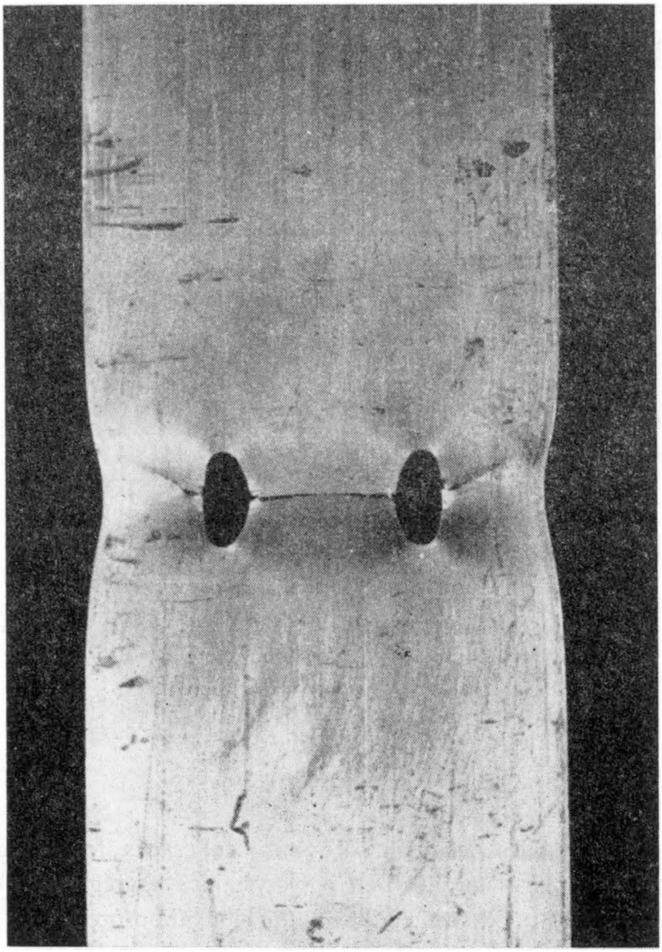

FIG. 5.

strain in the ligament between the holes defined as $\varepsilon' = 2\Delta d/(a-d)$ is in the particular stage of deformation shown in the figure approximately twice as large as the conventional lateral strain in the outer strips defined as $\varepsilon'' = (c-c_1)/c$. Further stages of the deformation process are presented in Fig. 4 (cf. [16]).

Figure 4a represents the initial configuration and Fig. 4b corresponds to the stage previously shown in Fig. 3. It can be seen that for the relatively small total elongation of the bar equal to $s = 0.46a$ the width of the ligament between the holes has been reduced to the point. Thus the theoretical analysis of the process of the ideally plastic deformation leads in the particular case under consideration to the local internal separation of the material. The successive appearance of such local fractures in the body with random distribution of voids may be interpreted as the evolution of the damage process.

Figure 5 shows the specimen with two holes made of a ductile AlMg2 aluminium alloy after plastic deformation. The dimensions of the specimen correspond to the theoretical solution discussed above. Fractures appearing in the regions of strong strain concentration (cf. Fig. 3) illustrate the significant role of strain concentration in the mechanics of ductile fracture of metals.

3. Interaction between linear defects

The significant role of strain concentrations is also clearly visible in the presence of certain systems of linear defects existing in the material. Such linear defects may appear in the material as the result of the previous cold working operations or may be produced in the course of the process of plastic deformation leading finally to the ductile fracture.

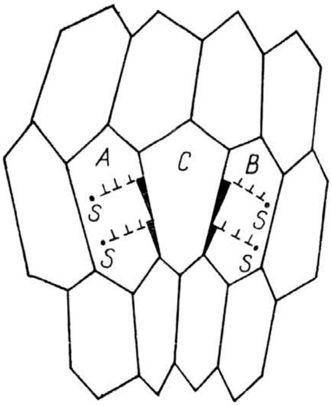

FIG. 6.

An example of the possible mechanism of formation of cracks along the grain boundaries during the uniaxial tension of an idealized polycrystalline aggregate is shown in Fig. 6. Randomly oriented grains are deformed during the elongation process to the state shown in the figure. Assume that there exist in some grains (say in grains A and B) dislocation sources S producing dislocations during the deformation process. Accumulation of the dislocations at the grain boundaries leads to the formation of cracks along these boundaries. Thus, at a certain level of plastic deformation there appear cracks separating the central grain from the grains *A* and *B*. In the following stages of plastic deformation of the aggregate the central grain deforms independently and may contract freely in the lateral direction.

In the previous paper [11] a particular case of such a mechanism was considered. The cracks separating grains were oriented along the direction of the tensile stress. For such a configuration of cracks the strong strain concentration is caused by internal micronecking taking place in central grain *C* separated by the cracks from the two adjacent grains. In this section we will analyse idealized systems of non-parallel slit cracks causing even stronger strain concentrations during the process of plastic deformation.

In order to demonstrate the role of internal strain concentrations caused by the existence of a certain system of defects, two specimens made of a rolled bar of a ductile AlMg2 aluminium alloy were tested. Pairs of narrow slits simulating cracks were prepared in the specimens as shown in Figs. 7 and 8. The specimens were pulled in tension up to the appearance of the fracture in the ligament between the slits.

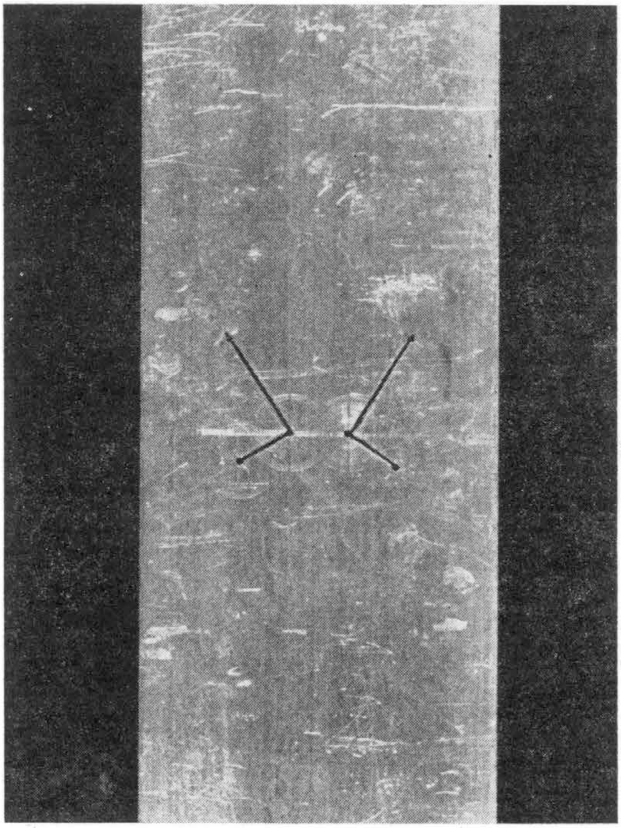

FIG. 7.

In the theoretical analysis we assumed the rigid-perfectly plastic model of the material and plane strain conditions in spite of the fact that the thickness of the specimens was

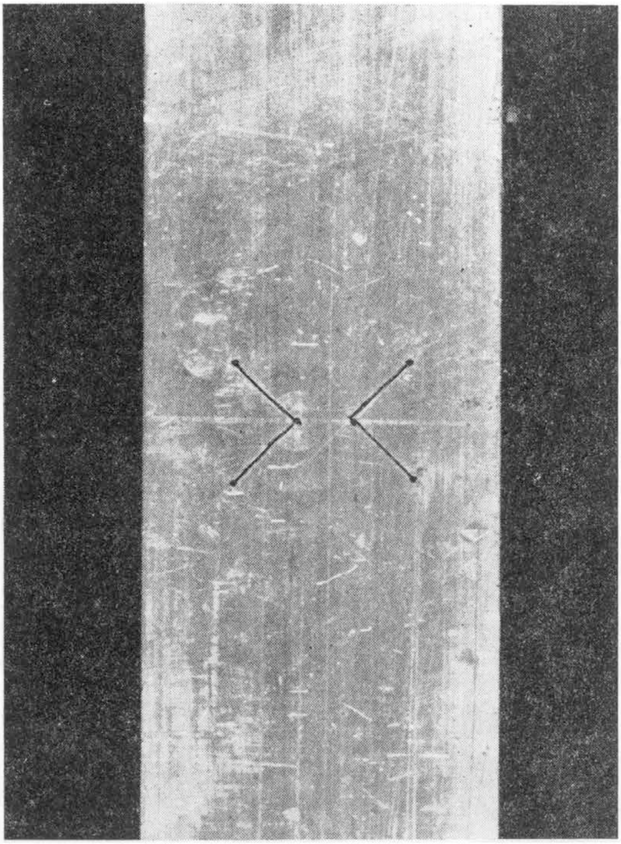

FIG. 8.

finite and equal $h = 10$ mm, while the distance between the slits in the narrowest cross-section was equal to 7 mm. Theoretical and experimental analysis (cf. DRUCKER [17], SZCZEPIŃSKI and MIASTKOWSKI [18]) indicates that even under such conditions the plane strain solutions constitute fairly good approximation of the unknown exact theoretical solutions.

Figure 9 shows the complete slip-line solution coresponding to the idealized configuration of defects shown in Fig. 7. The central part of the slip-line field is identical with that found by DIETRICH [19] for a notched bar problem. Regions of plastic flow are bounded by velocity discontinuity lines *DEFG*. In the two outer strips plastic deformation consists in shearing along the discontinuity lines *AC* and *BC*.

The right-hand side of Fig. 10 shows deformation of a square grid for small total elongation $\Delta L = 0.032$ b. At this stage the conventional lateral strain in the central narrowest cross-section reaches the value 0.22, while in the outer strips such a conventional lateral strain, defined as $(c-c_1)/c$ is more than four times smaller reaching the value 0.05. Strain concentration in the ligament between the two slits increases with the advancement of the process of deformation. For example, for more advanced total elongation $\Delta L = 0.81b$ (Fig. 11) conventional lateral strain in the central ligament is over five times

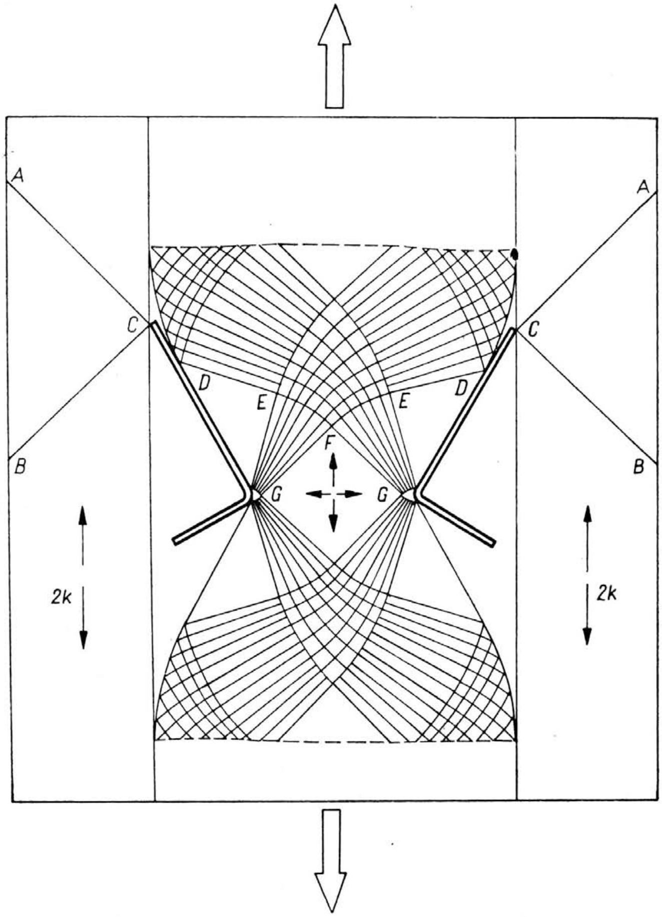

FIG. 9.

larger than that in the outer strips. The strain concentration factor, defined as the ratio of the conventional lateral strain in the central part to that in the outer strips, tends to infinity when the width of the ligament between the two slits reduces to a point. Thus present analysis based on the concept of ideally plastic material leads, similarly as in the previous case, to the local internal separation of the material.

In Fig. 12 is shown the specimen, presented previously in its initial state in Fig. 7, after deformation by simple tension causing fracture of the ligament between prepared slits simulating defects in the material. It can be seen that the deformation pattern is in general similar to the theoretical one shown in Figs. 10 and 11. Owing to the limited ductility of AlMg2 aluminium alloy used in the test, fracture occurred along the band of strong strain concentration clearly visible in Fig. 11.

The slip-line solution for the configuration of slits corresponding to that shown in Fig. 8 is presented in Fig. 13. The left-hand side of the figure shows the initial pattern of slip-lines. The central zone of plastic flow is bounded by lines of velocity discontinuity

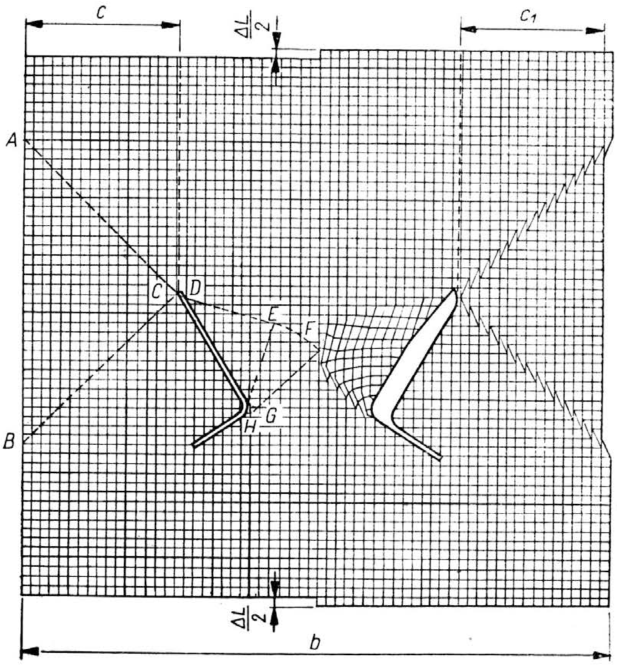

FIG. 10.

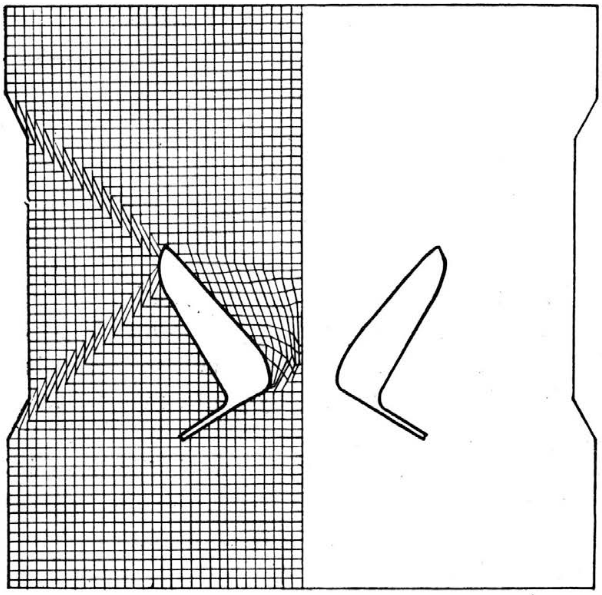

FIG. 11.

DEFG. The right-hand side of the figure shows the deformation pattern after relatively small total elongation (cf. [20]). The strain concentration in the central ligament can be clearly visible. The conventional lateral strain for this stage of deformation equals 0.31

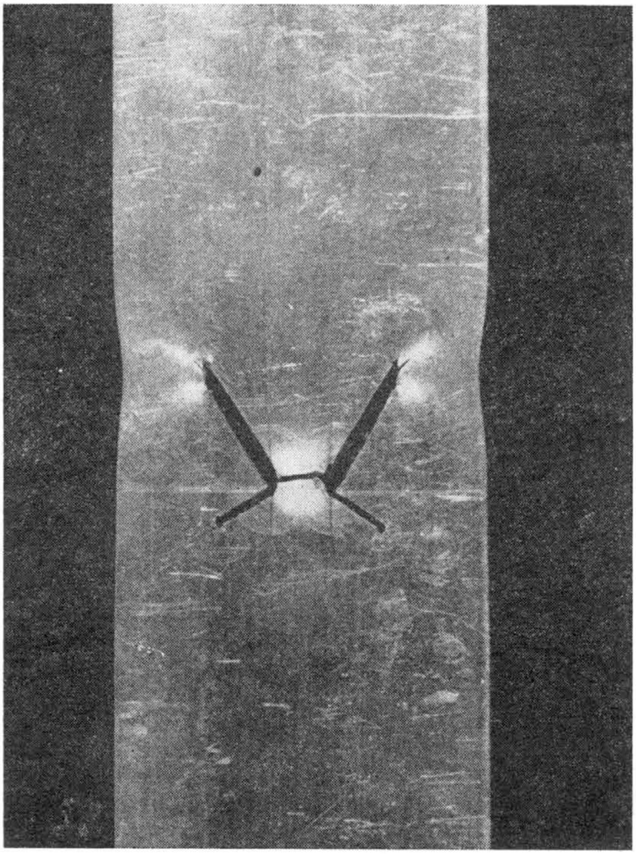

FIG. 12.

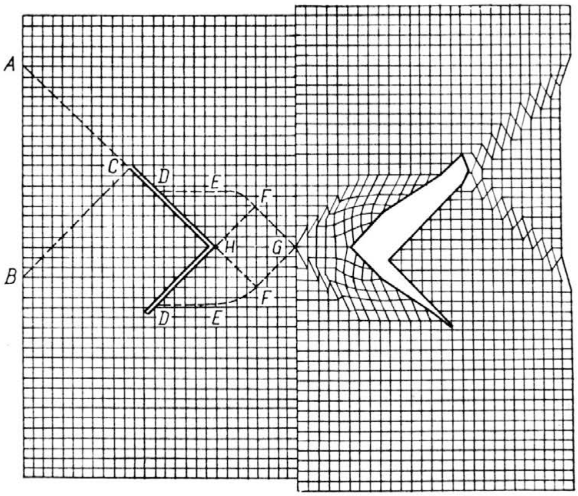

FIG. 13.

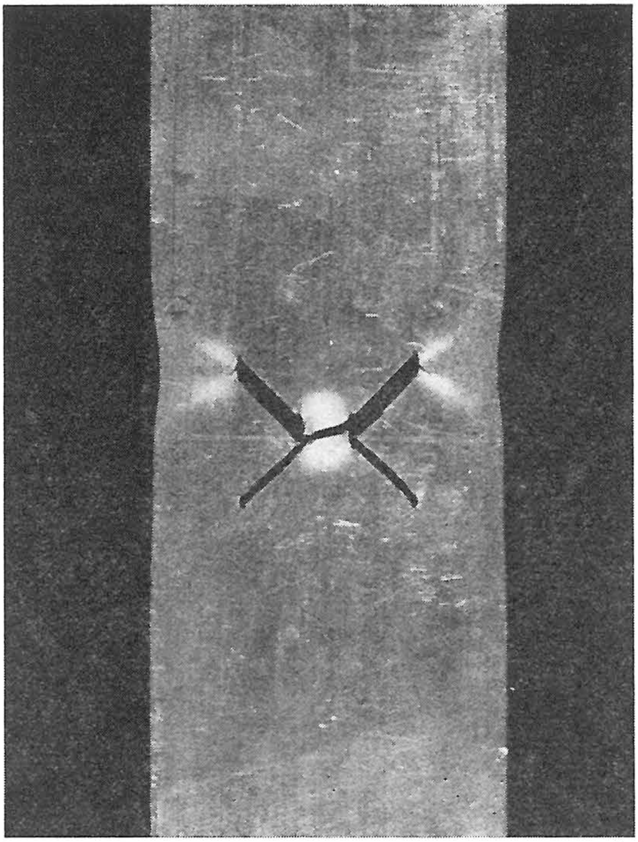

Fig. 14.

in this ligament, while in the outer strips it is 0.136. Thus the strain concentration factor equals approximately 2.3.

In Fig. 14 is presented the deformed specimen made of an AlMg2 aluminium alloy, shown previously in its initial state in Fig. 8. The deformation pattern is in general similar to the plastic deformation mode predicted by the theoretical solution shown in Fig. 13. The fracture of the central part is connected with the limited plastic deformability of the material.

4. Final remarks

The theoretical slip-line analysis of the mechanisms of ductile fracture in the presence of variously oriented systems of defects shows that strain concentrations play an important role in the process of forming local internal decohesion of the material. This phenomenon has been confirmed experimentally using models corresponding in macroscale to the configurations of defects analysed theoretically.

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