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Creep scatter as an inherent material property

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STATIONARY creep with inhomogeneous material properties is studied. The inhomogeneity may arise owing to randomly varying material parameters along the specimen, e.g. due to the manufacturing process, or owing to random variations of the temperature along the specimen. The strain rate is described by a stochastic process and compared with experimental observations. This stochastic process is applied to an ordinary creep test specimen.

Rozpatrzono ustalone pełzanie materiału o niejednorodnych lokalnie własnościach. Niejednorodność ta może być wynikiem losowych różnic we własnościach materiału, np. w wyniku zastosowanego procesu technologicznego lub też losowej zmienności temperatury wzdłuż próbki. Opisaną stochastycznie prędkość deformacji porównano z danymi doświadczalnymi, otrzymanymi w typowych badaniach pełzania.

Рассмотрена установившаяся ползучесть материала с локально неоднородными свойствами. Эта неоднородность может быть результатом случайных разниц в свойствах материала, например, в результате примененного технологического процесса, или же случайного изменения температуры вдоль образца. Вычисленная стохастически скорость ползучести сравнена с экспериментальными наблюдениями. Рассуждения касаются образцов применяемых в типичных исследованиях ползучести.

1. Introduction

DURING creep testing at constant stress level the scatter in creep deformation rate and creep rupture time is large for most materials. Experimental observations of local variations in the deformation rate are presented by GAROFALO [5, 1965]. He observed different creep rates at different locations of the specimen. WALLES [11, 1967] studied the scatter between different test specimens. This scatter was given a thorough statistical treatment.

Tests presented in this paper show a good agreement with the tests carried out by Walles, although quite different materials were used.

The scatter in creep deformation rate may occur due to uncontrolled variations in load, temperature, specimen geometry or material creep properties or due to unaccounted effects such as bending or friction. HAYHURST [7, 1974] has shown that this scatter can be reduced, but not eliminated, through rigorous control of the test situation. The remaining scatter must be explained as an inhomogeneity in the material.

BJÖRKENSTAM [1, 2, 1973, 1974] studied the scatter due to load variations. He considered a load that consisted of a constant part and a superimposed small randomly varying part. The expected values and the variances of the stresses and the strain rates for some structural elements were determined. The material was assumed to obey the Hooke-Norton constitutive equation.

Creep bending of a circular plate in a random temperature field was analysed by SOONG and COZZARELLI [10, 1967]. They considered a temperature distribution that consisted of a constant part and a superimposed small part with random variations in the radial direction. The influence of the temperature variations on the moments and the lateral deflection was analysed.

In the present study the scatter will be assumed to be due to a spatial variation of material parameters. The material is assumed to obey a modified Hooke-Norton constitutive equation, as suggested by BROBERG [3, 1973]. The variation is expressed as a stochastic process in space. Steady state creep of an ordinary test specimen is analysed. The expected value and the variance of the creep strain are calculated.

2. Properties of creep scatter

Due to inhomogeneous material properties the local strain rate $\dot{\varepsilon}(x)$ is not consant (x being the axial coordinate). Experiments where the rate of elongation $\dot{\Delta}$ over a certain gage length L is measured give the mean strain rate

$$\dot{\varepsilon}_L = \frac{\dot{\Delta}}{L} = \frac{1}{L}\int_0^L \dot{\varepsilon}(x)\,dx. \tag{2.1}$$

For different specimens at the same stress level the mean strain rate shows a scatter that originates from variations in the local strain rate $\dot{\varepsilon}(x)$.

If the material is assumed to obey Norton's creep law (σ_n being a standard stress)

$$\dot{\varepsilon}_c(x) = \dot{\varepsilon}_0\left(\frac{\sigma}{\sigma_n}\right)^n, \tag{2.2}$$

then the scatter originates from $\dot{\varepsilon}_0$ and n.

Observations from a number of creep tests, c.f. WALLES [11, 1967] and BROBERG [3, 1973], are presented in Fig. 1. Here the mean, a confidence interval and distribution of the measured strain rate are presented. The shape of the distribution function shows no noteworthy

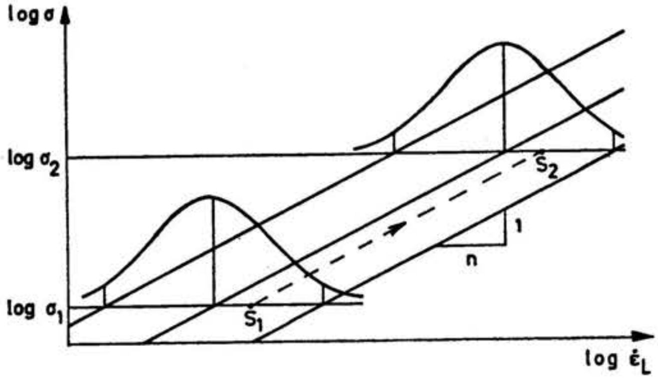

FIG. 1. Scatter under steady state creep conditions.

dependence on the stress level, i.e. at a change of stress level, state S_1 to state S_2, the strain rate is changed as a parallel translation with the mean characteristic.

Walles has shown that the scatter is log-normal distributed, i.e. normal distributed in $\log \dot{\varepsilon}_L/\log \sigma$. The notation log in the present paper designates the natural logarithm.

From these observations it follows that the scatter originates from variations in σ_n, and that n can be treated as a constant.

3. Model material

It is assumed that experimentally-observed steady state creep rates can be represented by the relation

$$\dot{\varepsilon}_L = \dot{\varepsilon}_0 C_L \left(\frac{\sigma}{\sigma_n}\right)^n. \tag{3.1}$$

Here $\dot{\varepsilon}_0$ and n are material constants and C_L is a random variable that represents the scatter between different tests.

If the same creep test machine and the same batch of materials are used for all the creep tests, then C_L will be dependent on the local properties of the specimen only. Since the strain rate is observed to be a log-normal distributed, then C_L is considered to be log-normal distributed as well.

For constant temperature of the specimen the scatter in creep data is considered to originate from randomly varying material parameters in creep. The material is assumed to obey the modified Norton cıeep law

$$\dot{\varepsilon}_c(x) = \dot{\varepsilon}_0 C(x) \left(\frac{\sigma}{\sigma_n}\right)^n, \tag{3.2}$$

where $C(x)$ is a stochastic process along the specimen. For the ideal scatter-free material $C(x) \equiv 1$.

From Eqs. (2.1), (3.1) and (3.2) it follows, for steady state creep,

$$C_L \doteq \frac{1}{L} \int_0^L C(x)\,dx. \tag{3.3}$$

Hence

$$E[C(x)] = E[C_L], \tag{3.4}$$

$$\mathrm{Var}[C(x)] \neq \mathrm{Var}[C_L]. \tag{3.5}$$

Here E represents the expected value, and Var the variance, of the stochastic process and the random variable.

The stochastic process $C(x)$ is assumed to be log-normal distributed, i.e. $\log C(x)$ is assumed to be normal distributed. Moreover, $C(x)$ is assumed to be stationary and ergodig, hence the expected value and variance of $\log C(x)$ are independent of x and the autocor relation of $\log C(x)$ is a function of the difference between the two x-values considered.

The ratio $\dot{\varepsilon}_0/\sigma_n^n$ is chosen such that

$$E[\log C(x)] = 0, \tag{3.6}$$

$$\mathrm{Var}(\log C(x)] = s^2. \tag{3.7}$$

It is assumed that the stochastic process is of Markov type and that autocorrelation, as a consequence, can be expressed as

$$R_{\log C}(x_0) = s^2 e^{-\beta|x_0|}. \tag{3.8}$$

Here x_0 is the distance between the two points considered and β is a material constant.

The statistical properties of $C(x)$ are deduced from Eqs. (3.6), (3.7) and (3.8) as

$$E[C(x)] = e^{s^2}/2, \tag{3.9}$$

$$\mathrm{Var}[C(x)] = e^{s^2}(e^{s^2}-1), \tag{3.10}$$

$$R_C(x_0) = \exp[s^2(1+e^{-\beta|x_0|})]. \tag{3.11}$$

All the calculations are performed in the appendix.

Typical values of the variance s^2 are 0.1 or less for specimens from the same material batch. Thus the exponential functions in Eqs. (3.9), (3.10) and (3.11) can be expanded in a Taylor series

$$E[C(x)] = 1+s^2/2+0(s^4), \tag{3.12}$$

$$\mathrm{Var}[C(x)] = s^2+0(s^4), \tag{3.13}$$

$$R_C(x_0) = 1+s^2(1+e^{-\beta|x_0|})+0(s^4). \tag{3.14}$$

Moreover it is suitable to express $C(x)$ as

$$C(x) = 1+\alpha H(x)+0(\alpha^2), \tag{3.15}$$

where $H(x)$ is a stochastic process and α is a constant chosen as half the variance, i.e. $\alpha = s^2/2$. The statistical properties of $\alpha H(x)$ are derived from Eqs. (3.11) to (3.14) as

$$E[\alpha H(x)] = \alpha+0(\alpha^2), \tag{3.16}$$

$$\mathrm{Var}[\alpha H(x)] = 2\alpha+0(\alpha^2), \tag{3.17}$$

$$R_{\alpha H}(x_0) = 2\alpha e^{-\beta|x_0|}+0(\alpha^2). \tag{3.18}$$

4. Experimental observations

Steady state creep rates of 23 aluminium specimens at 190°C were measured. With the least-square method a straight mean characteristic was adapted to the data. The scatter of $\log C_L$ was analysed and is presented on normal distribution paper in Fig. 2. The straight line indicates log-normal scatter with the standard deviation $s_L = 0.42$. The tests on aluminium are all performed under rigorous laboratory conditions with material from the same batch and with identical heat treatment. The scatter for 49 stainless steel specimens at 550°C are shown in Fig. 3 for comparison. The steady state creep rates of the stainless steel are from the STAL-LAVAL Company tests [9, 1972] performed over a long period. The material is from different batches and there is no control of the heat-treatment

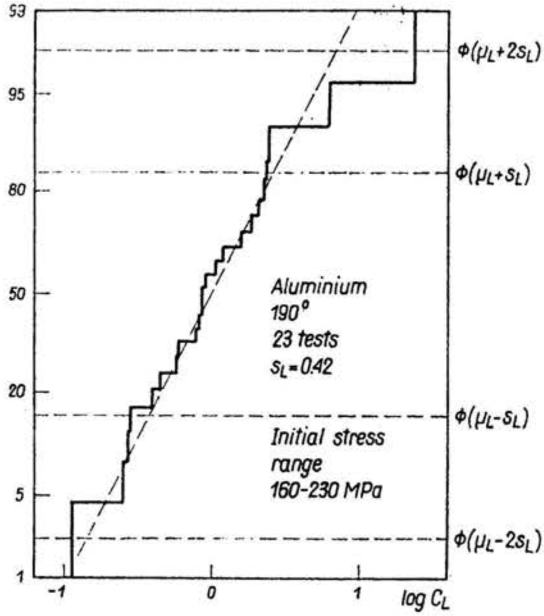

FIG. 2. Normal distribution plot of creep rate in aluminium.

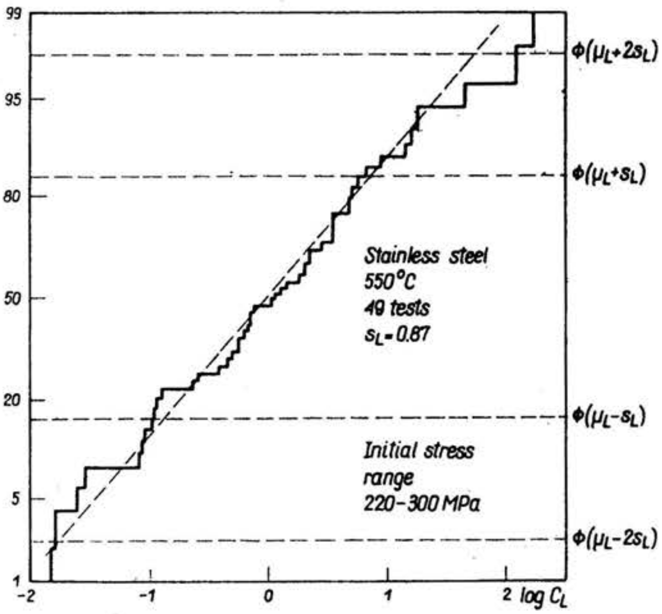

FIG. 3. Normal distribution plot of creep rate in a stainless steel.

of the specimens, thus the scatter is larger with a standard deviation $s_L = 0.87$. Both materials are ductile at the current test temperature, thus they can be adequately described by the constitutive equation (2.1).

Moreover, the scatter along the specimen was studied in one test. The local strain of an aluminium specimen was measured with strain gages (SG) in four points, and compared with the ordinary measurements of the mean strain over the gage length with differential transformers (DT). In Fig. 4 the strain rate time diagram is shown for this test. The tertiary

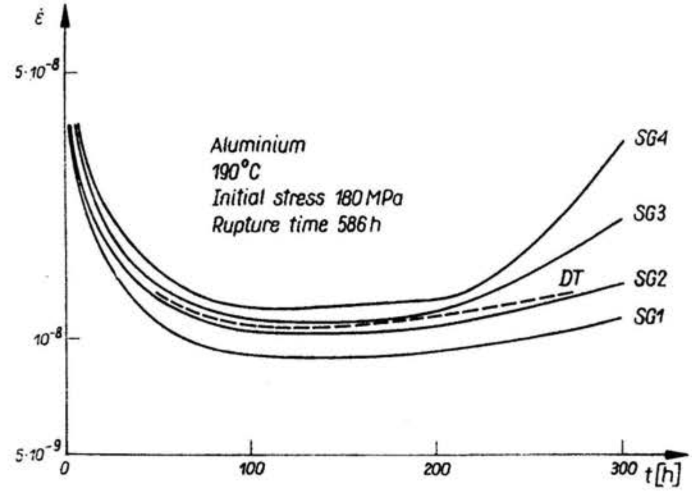

FIG. 4. Creep rate at different locations along one aluminium specimen.

phase is seen to be well begun. The strain gages, numbered from 1 to 4 from below, with the gage length of 8 mm, were placed with equal spacing along the total gage length of 100 mm. Rupture took place after 586 h immediately below the upper clamping edge. Note that the difference in $\log C$ between strain gage 4 and 1 is 0.35 which should be compared with the standard deviation $s_L = 0.42$.

5. Creep under random temperature

DORN [4, 1954] has shown that the temperature dependence of the creep rate can be described as a variation of only $\dot{\varepsilon}_0$ in Norton's law, Eq. (2.2). Other representations of the temperature dependence of the creep law are presented by PENNY and MARRIOTT [8, 1971].

Norton's creep law for an ideal scatter-free material can be written in the form

$$\dot{\varepsilon}(x) = \dot{\varepsilon}_0 \exp\left\{-\frac{Q}{R}\left[\frac{1}{T(x)} - \frac{1}{T_0}\right]\right\}\left(\frac{\sigma}{\sigma_n}\right)^n. \tag{5.1}$$

Here $\dot{\varepsilon}_0$ and n are material constants, Q is the activation energy, R is Boltzmann's constant, $T(x)$ is the absolute temperature along the specimen and T_0 is a constant reference temperature. Thus the creep rate may be expressed as in Eq. (3.2) with

$$C(x) = \exp\left\{-\frac{Q}{R}\left[\frac{1}{T(x)} - \frac{1}{T_0}\right]\right\}. \tag{5.2}$$

If the temperature can be described as a small random variation superimposed on the constant reference temperature,

$$T(x) = T_0+\alpha T_1(x), \tag{5.3}$$

where α is a constant with the property $|\alpha T_1(x)| \ll T_0$, then Eq. (5.2) may be written as

$$C(x) = \exp\left\{\frac{Q}{RT_0}\left[\alpha\frac{T_1(x)}{T_0}+0(\alpha^2)\right]\right\}. \tag{5.4}$$

Hence $C(x)$ may be written as in Eq. (3.15) with

$$H(x) = \frac{Q}{RT_0}\frac{T_1(x)}{T_0}. \tag{5.5}$$

If random temperature variations, with normal distribution, are considered as the origin of scatter in creep rate, it follows from Eq. (5.4) that $C(x)$ is log-normal distributed.

6. Analysis of a creep test specimen

A creep test specimen under constant tensile load is considered. The total strain is expressed as an elastic part and a creep part

$$\varepsilon(x) = \sigma/E+\varepsilon_c(x). \tag{6.1}$$

The first order theory is considered, i.e. the stress is constant not depending on the strain. The equilibrium condition may then be simply stated as

$$\sigma = P/A_0, \tag{6.2}$$

where P is the load and A_0 is the initial area of the specimen. The mean strain rate is given by Eq. (2.1). The constitutive equation (3.2) yields together with Eqs. (3.15), (6.1) and (6.2)

$$\dot{\varepsilon}(x) = \dot{\varepsilon}_0[1+\alpha H(x)]\left(\frac{\sigma}{\sigma_n}\right)^n+0(\alpha^2). \tag{6.3}$$

Insertion of Eq. (6.3) in Eq. (2.1) gives

$$\dot{\varepsilon}_L = \frac{\dot{\Delta}}{L} = \dot{\varepsilon}_0(1+\alpha H_L)\left(\frac{\sigma}{\sigma_n}\right)^n+0(\alpha^2), \tag{6.4}$$

where

$$H_L = \frac{1}{L}\int_0^L H(x)dx. \tag{6.5}$$

The statistical properties of the mean strain rate then follow as

$$E[\dot{\varepsilon}_L] = E[\dot{\varepsilon}(x)] = (1+\alpha)\dot{\varepsilon}_0\left(\frac{\sigma}{\sigma_n}\right)^n+0(\alpha^2), \tag{6.6}$$

$$\mathrm{Var}[\dot{\varepsilon}_L] = \frac{4\alpha}{(\beta L)^2}(\beta L-1+e^{-\beta L})\dot{\varepsilon}_0^2\left(\frac{\sigma}{\sigma_n}\right)^{2n}+0(\alpha^2). \tag{6.7}$$

The local variance follows from Eqs. (3.17) and (6.3) as

$$\mathrm{Var}[\dot{\varepsilon}(x)] = 2\alpha\dot{\varepsilon}_0^2\left(\frac{\sigma}{\sigma_n}\right)^{2n} + 0(\alpha^2). \tag{6.8}$$

The relation between the global and local variances is

$$\frac{\mathrm{Var}[\dot{\varepsilon}_L]}{\mathrm{Var}[\dot{\varepsilon}(x)]} = V(\beta L) = \frac{2}{(\beta L)^2}(\beta L - 1 + e^{-\beta L}) + 0(\alpha^2). \tag{6.9}$$

The function V, cf. Fig. 5, is a volume factor describing the influence of specimen size on creep scatter. When βL is large the measured scatter is small, and when βL is small the

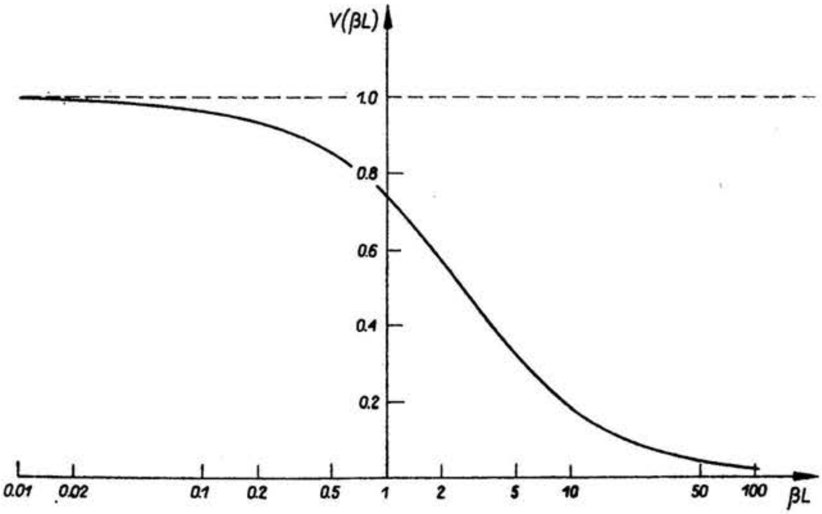

FIG. 5. Ratio of measured scatter and material scatter versus specimen length.

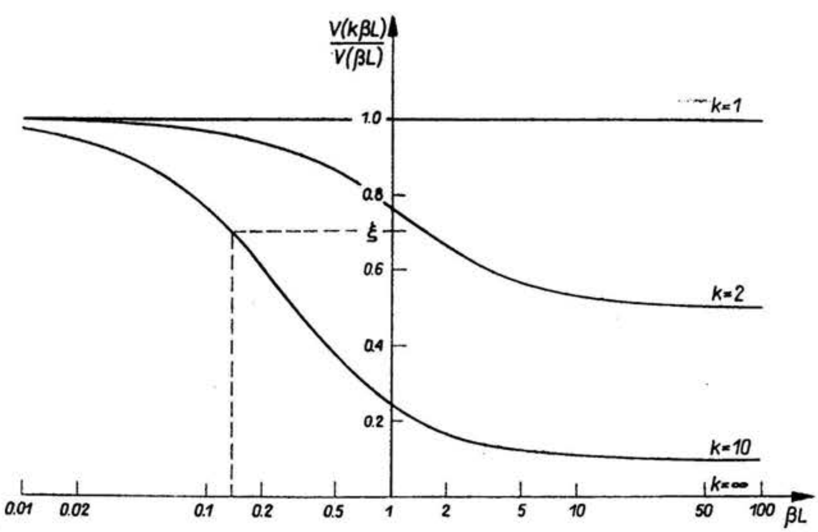

FIG. 6. Ratio of measured scatter at two different specimen lengths versus specimen length.

measured scatter is large. The material constant β determines the absolute value of the gage length necessary to yield a certain measured variance. It is possible to determine β from two series of tests at constant stress level but at two different gage lengths L_1 and kL_1. The value of βL_1 is read from Fig. 6 as the value of βL that satisfies the relation

$$\frac{V(k\beta L)}{V(\beta L)} = \xi, \tag{6.10}$$

where ξ is the ratio of the measured variances. When the value of β is known it can be used to determine the gage length necessary to keep the measured scatter below a desired level.

7. Discussion

A stochastic process was formulated in order to describe creep with inhomogeneous material properties. The same formulation has been shown to be valid for random temperature distributions. The variation of material properties was supposed to be present in one direction only, in this case along the specimen. This can be justified from the manufacturing process of long cylindrical bars. For other geometries, such as thick cylindrical tubes, a variation of the material properties in radial direction could have larger influence on the scatter than axial variations.

This stochastic process was applied to a specimen under steady state creep. A volume effect was shown to exist. The influence of local variation of material parameters on the behaviour of the specimen decreases when the size of the specimen is increased.

The results are now being used to study the influence of variation of material parameters on stresses and deformations in hyperstatic structures. Moreover, the probability of ductile creep rupture, with the aid of extremum value analysis of the stochastic process, can be obtained with these results as a starting point.

Appendix

The stochastic process $\log C(x)$ is normal distributed, and assumed to be stationary, with the density function

$$f(\eta) = \frac{1}{\sqrt{2\pi}\, s} \exp(-\eta^2/2s^2). \tag{A1}$$

Hence the expected value and variance are

$$E[\log C(x)] = \int_{-\infty}^{\infty} \log \eta f(n) d\eta = \mu = 0, \tag{A2}$$

$$\mathrm{Var}[\log C(x)] = E[(\log C - \mu)^2] = E[\log^2 C] - \mu^2 = s^2. \tag{A3}$$

If $\log C(x)$ is of Markov type then the autocorrelation is

$$R_{\log C}(x_0) = E[\log C(x) \log C(x+x_0)] = \iint \log \eta_1 \log \eta_2 f(\eta_1, \eta_2, x_0) d\eta_1 d\eta_2 = s^2 e^{-\beta|x_0|}. \tag{A4}$$

For a normal process the joint density function can be deduced from the autocorrelation as

(A5) $$f(\eta_1, \eta_2, x_0) = \frac{1}{2\pi\sqrt{R^2(0)-R^2(x_0)}} \exp\left\{-\frac{R(0)\eta_1^2-2R(x_0)\eta_1\eta_2+R(0)\eta_2^2}{2[R^2(0)-R^2(x_0)]}\right\}.$$

Insertion of Eq. (A4) yields

(A6) $$f(\eta_1, \eta_2, x_0) = \frac{1}{2\pi s^2\sqrt{1-e^{-2\beta|x_0|}}} \exp\left[-\frac{\eta_1^2-2e^{-\beta|x_0|}\eta_1\eta_2+\eta_2^2}{2s^2(1-e^{-2\beta|x_0|})}\right].$$

The statistical properties of $C(x)$ can be deduced from the statistical properties of $\log C(x)$.

The density function of $C(x)$ is

(A7) $$f_C(\eta) = \frac{f_{\log C}(\log\eta)}{\eta} = \frac{1}{\sqrt{2\pi}\, s}\frac{1}{\eta}\exp[-(\log\eta)^2/2s^2].$$

The joint density function of $C(x_1)$ and $C(x_2)$ is

(A8) $$f_C(\eta_1, \eta_2, x_0) = \frac{f_{\log C}(\log\eta_1, \log\eta_2, x_0)}{\eta_1\eta_2} = \frac{1}{2\pi s^2\sqrt{1-e^{-2\beta|x_0|}}}\frac{1}{\eta_1}\frac{1}{\eta_2} \cdot\exp\left[-\frac{(\log\eta_1)^2-2e^{-\beta|x_0|}\log\eta_1\log\eta_2+(\log\eta_2)^2}{2s^2(1-e^{-2\beta|x_0|})}\right].$$

The expected value and variance of $C(x)$ can be calculated from

(A9) $$I_n = E[C^n(x)] = \int_0^\infty \eta^n f_C(\eta)\,d\eta.$$

Insertion of Eq. (A7) and the variable substitution

(A10) $$y = \frac{\log\eta}{s}$$

yield

(A11) $$I_n = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\exp[-(y^2/2-nsy)]\,dy.$$

The autocorrelation can be calculated from

(A12) $$I_{nm} = E[C^n(x)C^m(x+x_0)] = \int_0^\infty\int_0^\infty \eta_1^n\eta_2^m f_C(\eta_1, \eta_2, x_0)\,d\eta_1\,d\eta_2.$$

Insertion of Eq. (A8) and the variable substitutions

(A13) $$y_1 = \frac{\log\eta_1}{s}, \quad y_2 = \frac{\log\eta_2}{s}$$

yield

(A14) $$I_{nm} = \frac{1}{2\pi\sqrt{1-e^{-2\beta|x_0|}}} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\exp\left[-\frac{y_1^2-2e^{-\beta|x_0|}y_1y_2+y_2^2}{2(1-e^{-2\beta|x_0|})}+nsy_1+msy_2\right]dy_1\,dy_2.$$

The solution of the integral, c.f. GRÖBNER and HOFREITER [6, 1949]

(A15) $$\int_{-\infty}^{\infty} \exp[-(ay^2+2by+c)]dy = \sqrt{\frac{\pi}{a}} \exp[(b^2-ac)/a], \quad a>0$$

yields, for Eqs. (A11) and (A14),

(A16) $$I_n = \exp(s^2n^2/2),$$

(A17) $$I_{nm} = \exp[(n^2+2nme^{-\beta|x_0|}+m^2)s^2/2].$$

Hence

(A18) $$E[C] = I_1 = e^{s^2/2},$$

(A19) $$\mathrm{Var}[C] = I_2-I_1^2 = e^{s^2}(e^{s^2}-1),$$

(A20) $$R_C(x_0) = I_{11} = \exp[(1+e^{-\beta|x_0|})s^2].$$

If the scatter is small, the exponential functions may be expanded in a Taylor series. Hence Eqs. (A18) to (A20) yield

(A21) $$E[C] = 1+s^2/2+0(s^4),$$

(A22) $$\mathrm{Var}[C] = [1+s^2+0(s^4)][s^2+0(s^4)] = s^2+0(s^4),$$

(A23) $$R_C(x_0) = 1+s^2(1+e^{-\beta|x_0|})+0(s^4).$$

Moreover, $C(x)$ can be expressed as

(A24) $$C(x) = 1+\alpha H(x)+0(\alpha^2),$$

where α is chosen as half the variance, i.e.

(A25) $$\alpha = s^2/2.$$

Thus

(A26) $$E[C] = 1+E[\alpha H],$$

(A27) $$\mathrm{Var}[C] = \mathrm{Var}[\alpha H],$$

(A28) $$R_C(x_0) = 1+2E[\alpha H]+R_{\alpha H}(x_0)$$

and

(A29) $$E[\alpha H] = \alpha+0(\alpha^2),$$

(A30) $$\mathrm{Var}[\alpha H] = 2\alpha+0(\alpha^2),$$

(A31) $$R_{\alpha H}(x_0) = 2\alpha e^{-\beta|x_0|}+0(\alpha^2).$$

The statistical properties of the mean of αH

(A32) $$\alpha H_L = \frac{1}{L}\int_0^L \alpha H(x)dx$$

can be expressed in terms of the statistical properties of αH.

The expected value of αH_L is

(A33) $$E[\alpha H_L] = \frac{1}{L}\int_0^L E[\alpha H(x)]dx = \alpha+0(\alpha^2).$$

The variance of αH_L is

$$\text{(A34)} \qquad \mathrm{Var}[\alpha H_L] = E[(\alpha H_L)^2] - \{E[\alpha H_L]\}^2,$$

where

$$\text{(A35)} \qquad E[(\alpha H_L)^2] = E\left[\frac{1}{L}\int_0^L \alpha H(x_1)\,dx_1 \cdot \frac{1}{L}\int_0^L \alpha H(x_2)\,dx_2\right] = \frac{1}{L^2}\int_0^L\int_0^L E[\alpha H(x_1)\alpha H(x_2)]\,dx_1\,dx_2.$$

By definition

$$\text{(A36)} \qquad E[\alpha H(x_1)\alpha H(x_2)] = R_{\alpha H}(x_2 - x_1).$$

Since $R_{\alpha H}$ depends only on the difference $x_0 = x_2 - x_1$, Eq. (A35) can be reduced to

$$\text{(A37)} \qquad E[(\alpha H_L)^2] = \frac{2}{L}\int_0^L \left(1 - \frac{x_0}{L}\right) R_{\alpha H}(x_0)\,dx_0.$$

Insertion of Eq. (A31) yields

$$\text{(A38)} \qquad E[(\alpha H_L)^2] = \frac{4\alpha}{(\beta L)^2}(\beta L - 1 + e^{-\beta L}) + 0(\alpha^2).$$

From Eqs. (A33), (A34) and (A38) it follows that

$$\text{(A39)} \qquad \mathrm{Var}[\alpha H_L] = \frac{4\alpha}{(\beta L)^2}(\beta L - 1 + e^{-\beta L}) + 0(\alpha^2).$$

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