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On plastic flow mechanisms in perfectly plastic-slackened structures

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The work concerns the plastic flow mechanisms in so-called "slackened structures". Structure slackening means the presence of clearances at structural connections. It has been proved that the presence of clearances does not affect the ultimate limit load as well as the corresponding plastic flow mechanism if the clearance region is convex and closed. The ultimate load state is usually preceded by a sequence of so-called "sublimit states" associated with different flow mechanisms and smaller values of the load multiplier. Two examples illustrate the theory.

Praca dotyczy mechanizmów plastycznego płynięcia tzw. "konstrukcji poluźnionych". Poluźnienie konstrukcji polega na występowaniu luzów w węzłach układu. Wykazano, że obecność luzów nie wpływa na ostateczną nośność graniczną, o ile obszar luzów jest zamknięty i wypukły. Osiągnięcie ostatecznej nośności granicznej poprzedzone jest zazwyczaj sekwencją tzw. stanów „podgranicznych" stowarzyszonych z odmiennymi mechanizmami płynięcia i mniejszymi mnożnikami obciążenia. Załączone przykłady ilustrują rezultaty rozważań teoretycznych.

Рабрта касается механизмов пластического течения т. наз. "разрыхленных конструкций". Разрыхление конструкции заключается в выступании зазоров в узлах системы. Показано, что присутствие зазоров не влияет на конечную предельную несущую способность, если только область зазоров замкнута и выпукла. Достижение конечной предельной несущей способности обычно предшествует последовательности т. наз. "допредельных" состояний ассоциированных с другими механизмами течения и меньшими множителями нагружения. Приведенные примеры иллюстрируют результаты теоретических рассуждений.

1. Introduction

Structure "slackening" means the presence of clearances at joints. Thus, in so-called "slackened structures", the relative generalized displacements between members and connecting elements (joints) are not fully constrained. In other words, one can introduce the idea of generalized clearance hinge where a constrained motion of the member face and of the corresponding element is permitted. The problem of slackened structure is part of the mechanics of systems with unilateral constraints and, independently of the mechanical properties of the structure material, is essentially a nonlinear one. It is known that the presence of clearances strongly influences the elastic strength of structures, [1]. Therefore a fundamental question arises: "does the connection slackening affect an ultimate collapse load?" The present work deals with this problem assuming:

a perfectly plastic material,
frictionless reactions at connections,
small clearances and displacements,
a quasi-static structure behaviour,
a geometric stability of a reference "ideal" structure without clearances.

Thus, the limit load problem generalized in such a way will be considered here within the frame of the geometrically linear theory.

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2. Connection slackening

According to the Finite Element Method formulation, a given structure is assumed to be an assembly of deformable rigid-plastic elements and undeformable ideal rigid "connecting elements" (joints) of very small dimensions. In the interior of each connecting element, a certain point called "node" is distinguished, and the external loads can be applied only at the nodes.

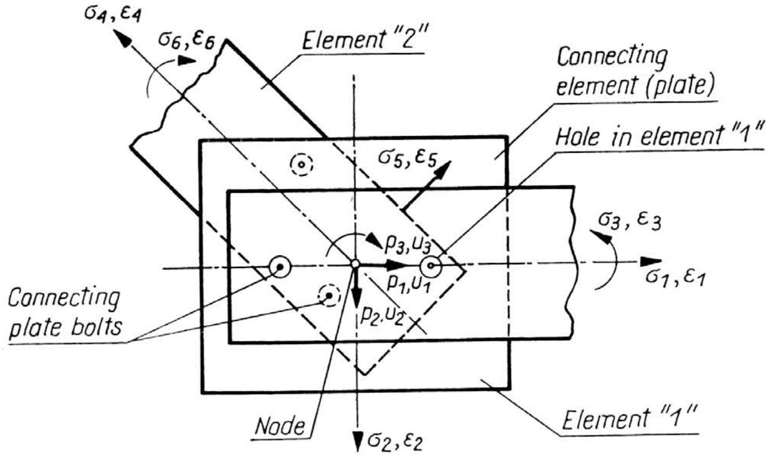

FIG. 1. Model of slackened connection.

As an example, consider a model of plane connection with clearances shown in Fig. 1. Two bars are joined by four bolts attached to the undeformable connecting plate. Because of the presence of gaps between the bolts (treated here as points) and holes drilled in the end part of the bars, a relative motion of the bar elements and connecting plate is permitted. It is known [3, 4] that in the discrete formulation of plastic structures the displacement discontinuities (i.e. longitudinal and transversal relative displacements, and a relative angle of rotation) are regarded as the generalized plastic strains at the generalized plastic hinge. Similarly, one can introduce the idea of "a generalized clearance hinge" where the relative displacements due to clearances are treated as "generalized clearance strains". Moreover, one can construct a certain region in the generalized clearance strain space bounded by the so-called "clearance surface" that plays the same role as the locking surface in the theory of locking materials [5] or structures [6]. The stressless states correspond to a "clearance region" interior, whereas nonvanishing generalized stress (i.e. normal and shear forces, and bending moment) states can occur only if the respective generalized strain point lies on the clearance surface. Since frictionless motions and contacts are assumed, the generalized stress vector appears to be orthogonal to the clearance surface at the corresponding strain point [7]. Turning now to the example considered above, the clearance surface related to the end of element *1* is presented in Fig. 2.

It is worth noticing that an initial unloaded position of the system may be geometrically unstable due to connection slackening and therefore one has to establish a certain "ideal configuration" that should be chosen from all the kinematically admissible configurations.

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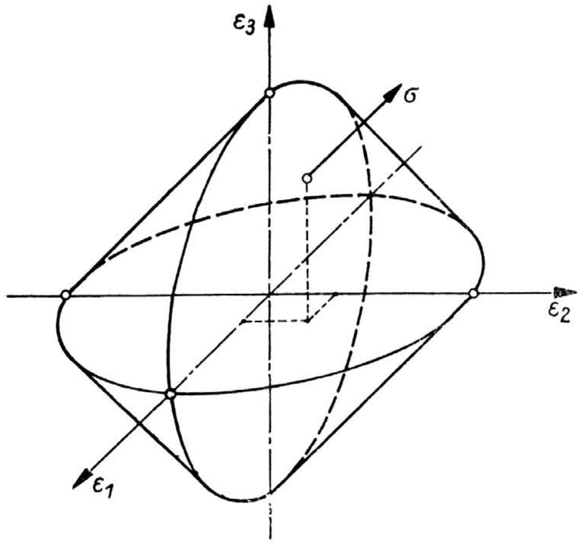

FIG. 2. Clearance surface of clearance hinge of element *1*.

Thus the clearance strains as well as connecting element displacements are related to this ideal position of the system. Otherwise, the kinematical quantities would not be uniquely determined. Note that the connecting plate and bar elements shown in Fig. 1 are situated at their ideal positions and the clearance strains in Fig. 2 are related to this ideal configuration.

Passing on to the mathematical model of connection slackening, assume that the clearance region is described by a "clearance function" $g(\boldsymbol{\epsilon}_L)$:

$$g(\boldsymbol{\epsilon}_L) \leqslant 0, \tag{2.1}$$

where $\boldsymbol{\epsilon}_L$ denotes the generalized clearance vector and the clearance surface equation is

$$g(\boldsymbol{\epsilon}_L) = 0. \tag{2.2}$$

The orthogonality of the generalized stress vector $\boldsymbol{\sigma}$ to the clearance surface can be expressed in the form

$$\boldsymbol{\sigma} = \psi \mathbf{g}_{,\boldsymbol{\epsilon}_L}. \tag{2.3}$$

Here $\mathbf{g}_{,\boldsymbol{\epsilon}_L}$ indicates the clearance surface gradient vector and ψ is a non-negative (due to unilateral constraints) stress multiplier that satisfies the conditions

$$\begin{aligned} &\psi \geq 0 \quad \text{if} \quad g = 0 \quad \text{and} \quad \dot{g} = 0,\\ &\psi = 0 \quad \text{if} \quad g < 0 \quad \text{and also if} \quad g = 0 \quad \text{and} \quad \dot{g} < 0, \end{aligned} \tag{2.4}$$

where the dot denotes differentiation with respect to time. By virtue of the conditions (2.4), one can state that the following relations hold:

$$\begin{aligned} \psi g &= 0,\\ \psi \dot{g} &= 0. \end{aligned} \tag{2.5}$$

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It is pointed out that the strain state uniqueness occurs only if the clearance region is strictly convex. Then, for the given stress vector, the corresponding clearance strain vector can be uniquely determined. The clearance region convexity essentially depends on the shapes and dimensions of the element holes.

Further considerations will be referred to the whole structure. First of all, in order to utilize the linear programming method, we assume that the clearance surface can be approximated by a hyperpolyhedron. Hence the clearance region is described by a linear matrix inequality:

$$\mathbf{g} = \mathbf{M}^T \boldsymbol{\epsilon}_L - \mathbf{l} \leq \mathbf{0}, \tag{2.6}$$

where T indicates the matrix transposition, $\mathbf{M}$ is a rectangular matrix that collects all the external normals of the clearance hyperpolyhedrons of all the clearance hinges; and $\mathbf{l}$ is the vector that determines the distance of the individual side to the origin.

In view of the inequality (2.6), it is clearly seen that the clearance region is weakly convex and therefore we should be aware of the possibilities of strain state non-uniqueness.

The normality rule yields

$$\boldsymbol{\sigma} = \mathbf{M}\boldsymbol{\psi}, \quad \boldsymbol{\psi} \geq \mathbf{0}, \tag{2.7}$$

where $\boldsymbol{\psi}$ is the vector of the stress multipliers. Taking into account the relation (2.5), the following orthogonality condition holds:

$$\boldsymbol{\psi}^T \mathbf{g} = \boldsymbol{\psi}^T(\mathbf{M}^T \boldsymbol{\epsilon}_L - \mathbf{l}) = 0, \tag{2.8}$$

that is equivalent to

$$U_L = \boldsymbol{\sigma}^T \boldsymbol{\epsilon}_L = \boldsymbol{\psi}^T \mathbf{l}. \tag{2.9}$$

Here U_L represents so-called "clearance work". After differentiating the condition (2.8) with respect to time and using the relations (2.5), we arrive at

$$\dot{\boldsymbol{\psi}}^T \mathbf{g} = \boldsymbol{\psi}^T \dot{\mathbf{g}} = 0 \tag{2.10}$$

or

$$D_L = \boldsymbol{\sigma}^T \dot{\boldsymbol{\epsilon}}_L = 0, \tag{2.11}$$

$$\dot{\boldsymbol{\sigma}}^T \boldsymbol{\epsilon}_L = \dot{\boldsymbol{\psi}}^T \mathbf{l}, \tag{2.12}$$

what results from the relations (2.6) and (2.7).

In view of Eqs. (2.11) and (2.12), we can conclude that a "clearance dissipation" D_L always equals zero and the stress variations can occur only if a contact condition is satisfied. These conclusions are of real value for the structure analysis.

3. Problem of original structure

Suppose that a given discrete system is geometrically unstable due to slackening of connections. Now, a non-trivial problem arises: "find a non-zero stiffness structure that can carry prescribed external loads $\mathbf{p}_0$" [9]. The solution of this problem corresponds to the conversion of a mechanism into so-called "original structure" depending on the given load vector $\mathbf{p}_0$. The original structure is completely determined by the generalized displace-

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ment vector $\mathbf{u}_0$ that describes the positions of all the connecting elements. Usually the original structure is statically determinate and, generally, does not depend on material properties.

To formulate the original structure problem, we recall the well-known relations for the discrete structures. The fundamental equations can be presented in matrix notations as follows:

equilibrium equations

$$\mathbf{C}^T\boldsymbol{\sigma}_0 = \mathbf{p}_0, \tag{3.1}$$

geometric equations

$$\mathbf{C}\mathbf{u} = \boldsymbol{\epsilon}_{L_0}, \tag{3.2}$$

where $\boldsymbol{\sigma}_0$ and $\boldsymbol{\epsilon}_{L_0}$ are supervectors that collect all the generalized stresses and strains, respectively, of all the structural elements, and $\mathbf{C}$ is a rectangular matrix that depends on the geometry of the "ideal structure" (i.e. a structure without clearances). Note that

$$\det[\mathbf{C}^T\mathbf{C}] \neq 0, \tag{3.3}$$

what follows from the assumption of geometric stability of the ideal structure.

The complete system of relations describing the original structure problem can be presented in the form

$$\begin{array}{llll} 1) & \mathbf{C}\mathbf{u}_0 = \boldsymbol{\epsilon}_{L_0}, & 4) & \mathbf{g}_0 = \mathbf{M}^T\boldsymbol{\epsilon}_{L_0} - \mathbf{1} \leq \mathbf{0}, \\ 2) & \mathbf{C}^T\boldsymbol{\sigma}_0 = \mathbf{p}_0, & 5) & \boldsymbol{\psi}_0 \geq \mathbf{0}, \\ 3) & \boldsymbol{\sigma}_0 = \mathbf{M}\boldsymbol{\psi}_0, & 6) & \boldsymbol{\psi}_0^T\mathbf{g} = 0. \end{array} \tag{3.4}$$

Treating these relations as Kuhn–Tucker's conditions and basing on the exellent book by A. BORKOWSKI [4], we can formulate the proper dual extremum principles in the frame of the linear programming method, namely

$$[F' = \mathbf{u}_0^T\mathbf{p}_0] \to \max|\mathbf{M}^T\mathbf{C}\mathbf{u}_0 - \mathbf{1} \leq \mathbf{0}, \tag{3.5}$$

$$[F'' = \boldsymbol{\psi}_0^T\mathbf{1}] \to \min|\mathbf{C}^T\mathbf{M}\boldsymbol{\psi}_0 - \mathbf{p}_0 = \mathbf{0}, \quad \boldsymbol{\psi}_0 \geq \mathbf{0}. \tag{3.6}$$

One has to be aware of the following possibilities which can arise when the linear programming method is applied:

a) the solution is unique and corresponds to the finite value of the object function,

b) the solution is non-unique but the corresponding object function appears to be unique and finite,

c) the form of constraints does not allow to reach the finite value of the object function,

d) the solution does not exist due to the constraint contradiction.

Cases c) and d) cannot occur, but the appearance of case b) is quite possible. It corresponds to the situation when the clearance strain point lies on an individual side of the clearance hyperpolyhedron.

After solving the original structure problem, one can divide $\mathbf{g}_0$, $\boldsymbol{\psi}_0$ and $\mathbf{M}$ into active and passive parts:

$$\begin{aligned} \mathbf{g}_0 &= \{\mathbf{g}_a, \mathbf{g}_p\}; \quad \mathbf{g}_a = \mathbf{0}, \ \mathbf{g}_p < \mathbf{0}, \\ \boldsymbol{\psi}_0 &= \{\boldsymbol{\psi}_a, \boldsymbol{\psi}_p\} \\ \mathbf{M} &= [\mathbf{M}_a, \mathbf{M}_p]. \end{aligned} \tag{3.7}$$

This observation will be utilized in further considerations.

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4. Yield load problem for perfectly plastic-slackened structures

The common limit load problem for the prescribed reference load vector $\mathbf{p}_0$ consists in determining a scalar load multiplier μ and plastic displacement rate vector $\dot{\mathbf{u}}$ describing the corresponding plastic flow mechanism. Assuming a piece-wise linear approximation of the yield condition, namely

$$\mathbf{f} = \mathbf{N}^T\boldsymbol{\sigma} - \mathbf{k} \leq \mathbf{0}, \tag{4.1}$$

the generalized plastic strain rates $\dot{\boldsymbol{\epsilon}}_P$ can be expressed as

$$\dot{\boldsymbol{\epsilon}}_P = \mathbf{N}\dot{\boldsymbol{\lambda}}, \quad \dot{\boldsymbol{\lambda}} \geq \mathbf{0} \tag{4.2}$$

together with the orthogonality condition

$$\dot{\boldsymbol{\lambda}}^T\mathbf{f} = 0 \tag{4.3}$$

or

$$D_P = \boldsymbol{\sigma}^T\dot{\boldsymbol{\epsilon}}_P = \dot{\boldsymbol{\lambda}}^T\mathbf{k} \geq 0. \tag{4.4}$$

Here $\mathbf{N}$ is a rectangular matrix that collects all the external normals of the yield hyper-polyhedrons of all the elements, the vector $\mathbf{k}$ determines the distances of the separate sides to the origin, $\dot{\boldsymbol{\lambda}}$ is a vector of the plastic strain intensity rates and D_P denotes the non-negative plastic dissipation.

It is generally known [4, 8] that the limit load problem for perfectly plastic structures can be formulated by means of the dual linear programming method as

$$[F' = \dot{\boldsymbol{\lambda}}^T\mathbf{k}] \to \min|\dot{\boldsymbol{\lambda}} \geq \mathbf{0}, \quad \dot{\mathbf{u}}^T\mathbf{p}_0 = 1, \tag{4.5}$$

$$[F'' = \mu] \to \max|-\mathbf{N}^T\boldsymbol{\sigma}+\mathbf{k} \geq 0, \quad \mathbf{C}^T\boldsymbol{\sigma}-\mu\mathbf{p}_0 = \mathbf{0}, \tag{4.6}$$

where (4.5) and (4.6) correspond to the kinematical and statical theorems, respectively.

In the presence of connection slackening, the formulation described above should be somewhat modified. The mathematical model in this case may be written in the following form:

$$\begin{array}{ll} 1)\ \mathbf{C}\dot{\mathbf{u}} = \dot{\boldsymbol{\epsilon}}, & 7)\ \dot{\mathbf{g}}_a = \mathbf{M}_a^T\dot{\boldsymbol{\epsilon}}_L \leq \mathbf{0}, \\ 2)\ \mathbf{C}^T\boldsymbol{\sigma} = \mu\mathbf{p}_0, & 8)\ \boldsymbol{\psi}_a^T\mathbf{g}_a = 0, \\ 3)\ \dot{\boldsymbol{\epsilon}} = \dot{\boldsymbol{\epsilon}}_L + \dot{\boldsymbol{\epsilon}}_P, & 9)\ \dot{\boldsymbol{\epsilon}}_P = \mathbf{N}\dot{\boldsymbol{\lambda}}, \\ 4)\ \dot{\mathbf{u}}^T\mathbf{p}_0 = 1, & 10)\ \dot{\boldsymbol{\lambda}} \geq \mathbf{0}, \\ 5)\ \boldsymbol{\sigma} = \mathbf{M}_a\boldsymbol{\psi}_a, & 11)\ \mathbf{f} = \mathbf{N}^T\boldsymbol{\sigma} - \mathbf{k} \leqq \mathbf{0}, \\ 6)\ \boldsymbol{\psi}_a \geq \mathbf{0}, & 12)\ \dot{\boldsymbol{\lambda}}^T\mathbf{f} = 0, \end{array} \tag{4.7}$$

where the subscript a indicates the active submatrices determined on the basis of the original structure solution problem.

It can be shown [7] that the solution of the system (4.7) is equivalent to the solutions of the following dual linear programming problems, namely

$$\begin{array}{l} [F' = \dot{\boldsymbol{\lambda}}^T\mathbf{k}] \to \min|\mathbf{M}_a^T(\mathbf{C}\dot{\mathbf{u}} - \mathbf{N}\dot{\boldsymbol{\lambda}}) \leqslant \mathbf{0}, \quad \dot{\mathbf{u}}^T\mathbf{p}_0 = 1, \quad \dot{\boldsymbol{\lambda}} \geq \mathbf{0} \\ [F'' = \mu] \to \max|-\mathbf{N}^T\mathbf{M}_a^T\boldsymbol{\psi}_a + \mathbf{k} \geq \mathbf{0}, \quad \mathbf{C}^T\mathbf{M}_a\boldsymbol{\psi}_a - \mu\mathbf{p}_0 = \mathbf{0}, \end{array} \tag{4.8}$$

$$\boldsymbol{\psi}_a \geqq \mathbf{0}. \tag{4.9}$$

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The solution of both problems allows us to determine kinematical quantities ($\dot{\boldsymbol{\epsilon}}_L$, $\dot{\boldsymbol{\epsilon}}_P$, $\dot{\mathbf{u}}$) as well as statical ones ($\boldsymbol{\sigma}$, μ). In general, these solutions are different from those obtained for the respective structure without clearances, namely the load multiplier usually (always?) appears to be less than the one for the ideal structure. Thus, so-called "sublimit plastic flow mechanisms" can occur.

Assume now that kinematical quantities can be uniquely determined (it does not occur always!) and the uniform motion of plastic flow is observed. Then, after integrating with respect to time, one obtains

$$\begin{aligned} \dot{\mathbf{u}} &= \mathbf{u}_0 + t\dot{\mathbf{u}}, \\ \boldsymbol{\epsilon}_L &= \boldsymbol{\epsilon}_{L0} + t\dot{\boldsymbol{\epsilon}}_L, \\ \boldsymbol{\epsilon}_P &= \boldsymbol{\epsilon}_{P0} + t\dot{\boldsymbol{\epsilon}}_P, \end{aligned} \tag{4.10}$$

where $\mathbf{u}_0$ and $\boldsymbol{\epsilon}_{L_0}$ are related to the ideal structure position and t denotes the "time" measured from the moment when the plastic flow just begins. A further problem is the determination of $t = t^*$ when the flow mechanism stops due to the appearance of a new contact at a slackened connection. To this end, we utilize the equations of contact at the passive side of the clearance hyperpolyhedron

$$\mathbf{M}_p^T(\boldsymbol{\epsilon}_{L_0} + t\dot{\boldsymbol{\epsilon}}_L) - \mathbf{l}_p = \mathbf{0} \tag{4.11}$$

and the requirement

$$t > 0. \tag{4.12}$$

The problem (4.11) and (4.12) is extremely simple and consists in determining the smallest non-negative root of the linear equation system (4.11) with one unknown, t. For a given value of $t = t^*$, one can obtain the position of a new original structure which is described by

$$\mathbf{u}_0' = \mathbf{u}_0 + t^*\dot{\mathbf{u}}, \qquad \boldsymbol{\epsilon}_{L_0} = \boldsymbol{\epsilon}_{L_0}' + t^*\dot{\boldsymbol{\epsilon}}_L.$$

The next step is to perform a new matrix division into the active and passive parts. Then the above procedure should be repeated again.

Now the last most important question arises: "when will the ultimate limit load be reached?". Since the clearance region represents the closed set, the disappearance of $\dot{\boldsymbol{\epsilon}}_L$ must be at last observed. This means that the final plastic flow mechanism is attained. After substituting $\dot{\boldsymbol{\epsilon}}_L = \mathbf{0}$ into the relations (4.7), we arrive at the principles (4.5) and (4.6) which are valid for the ideal structure. Thus we can conclude that in the case of the convex and closed clearance region the ultimate load and corresponding plastic flow mechanism appear to be identical with those obtained for the respective ideal structure (without clearances).

It is worth observing, however, that this conclusion was derived on the basis of the geometrically linear theory. A more realistic approach should be based on the nonlinear theory of post-yield behaviour where geometry changes as well as dynamic effects are taken into account. This remark relates first of all to optimal plastic structures where the allowance for geometry change effects leads sometimes to the statement that the true initial plastic flow mechanism is quite different from the one predicted by the geometrically linear theory [10].

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5. Examples

Consider a portal frame loaded by horizontal and vertical forces P_x and P_y, respectively (Fig. 3). For the purpose of simplicity, assume that all the bars are of the same uniform cross-section and a plastic flow can be induced only by a bending moment action. Clearances

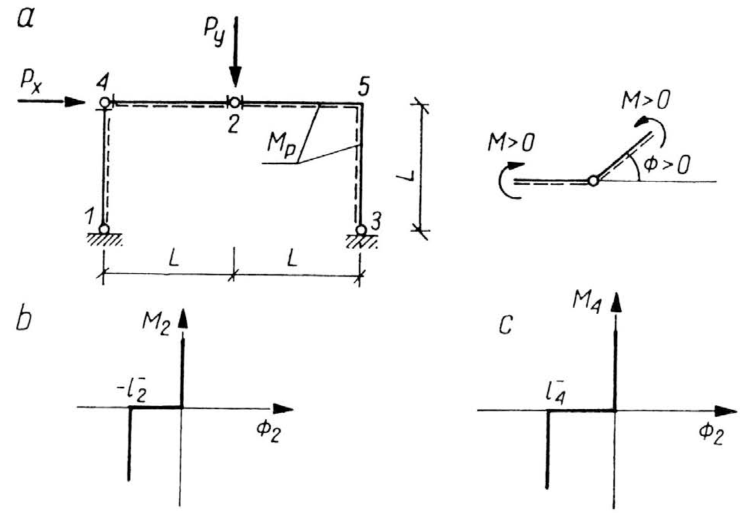

FIG. 3. Slackened portal frame; a) frame geometry and loads; b) mechanical characteristics of hinge *2*, c) mechanical characteristics of hinge *4*.

appear at points 2 and 4 where the common hinges with rotation constraints are introduced. The corresponding mechanical characteristics of these hinges are shown in Figs. 3b and 3c. The initial unloaded position of the frame is geometrically stable and corresponds to the ideal configuration whereas the original structure depends on the signs of P_x and P_y.

Let us determine the "sublimit" yield load polygon which is valid as long as the rotation constraints are passive. The consecutive elementary plastic flow mechanisms which correspond to the slackened and ideal frame are illustrated in Figs. 4a and 4b, respectively.

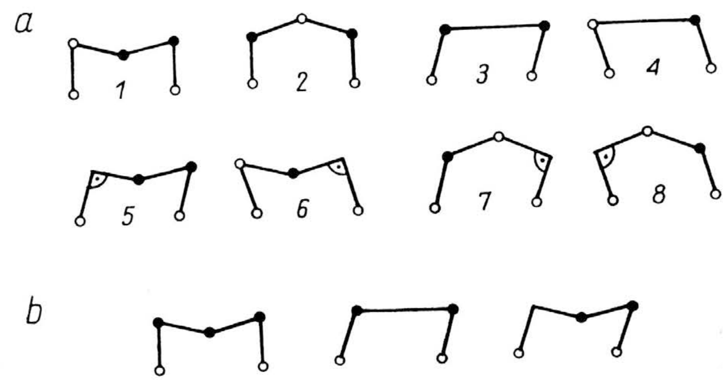

FIG. 4. Separate mechanisms in portal frames; a) slackened frame, b) ideal frame.

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Making use of the virtual work equation, one obtains the yield load polygons for the slackened and ideal frames. Both polygons are shown in Fig. 5a.

Consider now a load path that corresponds to the given reference load vector $\mathbf{p}_0 = \{1, 6\}$, that is $P_x = \mu$, $P_y = 6\mu$. At point A the sublimit state is reached where a combined yield mechanism develops and the following virtual work equation holds:

$$-P_x L\dot{\phi}_1 + P_y L\dot{\phi}_1 = 2M_P\dot{\phi}_1,$$

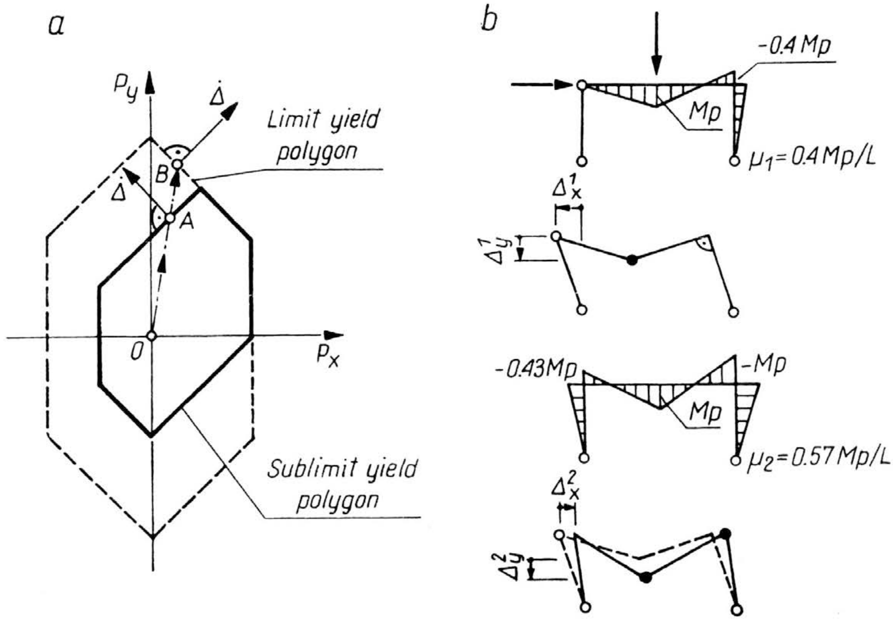

FIG. 5. Sublimit and limit states of the slackened portal frame; a) yield polygons, b) stress states and flow mechanisms.

hence

$$\mu = \mu_1 = 0.40 M_P/L,$$

where M_P is the fully plastic moment of the cross-section. Note that the horizontal displacement is incompatible with the sense of vector P_x. This sublimit plastic flow mechanism stops when the displacement Δ_x^1 (or Δ_y^1) reaches the value of $Ll_4^-/2$. Then the lower limit rotation constraint at hinge 4 (i.e. l_4^-) becomes active and the mechanism converts again into the structure. Now the load can increase up to the fully plastic moment at point 5 (point B in Fig. 5a). Further behaviour of the frame corresponds to the combined plastic flow mechanism that is compatible with the sense of vector P_x. The virtual work equation corresponding to this flow mechanism takes the form

$$P_x L\dot{\phi}_2 + P_y L\dot{\phi}_2 = 4M_P\dot{\phi}_2,$$

hence

$$\mu = \mu_2 = 0.57 M_P/L > \mu_1 = 0.40 M_P/L.$$

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It is noted that the load multiplier as well as the plastic flow mechanism of the slackened frame are identical with those observed for the ideal frame without clearances. The bending moment distributions and the corresponding plastic flow mechanisms are presented in Fig. 5b, whereas the relationship between the load multiplier μ and the displacement components of point 2 are shown in Fig. 6. It is seen that the ultimate value of the limit

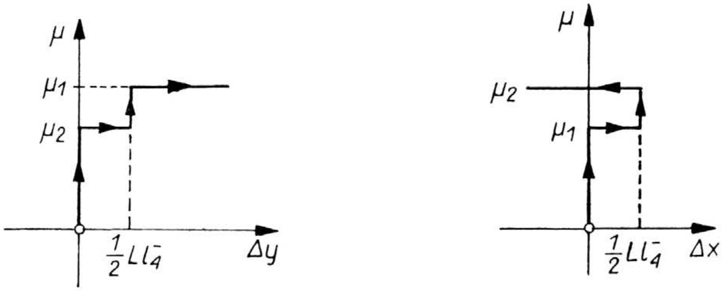

FIG. 6. $\mu - \Delta$ diagrams related to point 2.

load is attained on a step-wise way. This observation appears to be a significant feature of the ideal plastic-slackened structure behaviour.

It should be mentioned that, in general, at the ultimate yield point load all clearance contacts do not have to be active. For this load, according to the results of Sect. 4, only the clearance strain rates have to vanish, as it will be shown in the next example where a portal frame is loaded only by the vertical force P (see Fig. 7). The clearance hinges are

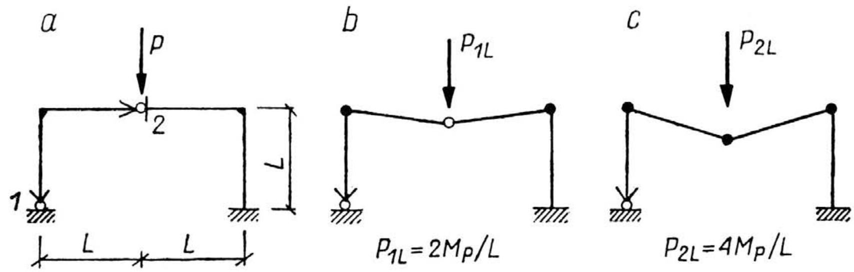

FIG. 7. Portal frame with two clearance hinges, a) frame and load; b) sublimit state, c) limit state.

introduced at points 1 and 2. The initial position of the structure is geometrically stable and corresponds to the original configuration of the two- hinge portal frame. The sublimit (Fig. 7b) and limit (Fig. 7c) plastic flow mechanisms develop according to the beam mechanisms of different plastic dissipations as well as load multipliers. It is clearly seen that for the limit plastic flow mechanism the clearance constraint is active only at hinge 2, whereas at hinge 1 there is an absence of contact and the zero-value of clearance strain rate occurs.

6. Conclusions

1. The problem of plastic flow of perfectly plastic-slackened structures can be resolved into a set of consecutive linear programming problems.

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2. If the clearance region is convex and bounded by a closed clearance surface, then the ultimate collapse load as well as the corresponding plastic flow mechanism appear to be identical with those obtained for the reference structure without clearances.

3. The ultimate collapse load of slackened structures is usually attained on a step-wise way where "sublimit" plastic flow mechanisms can occur before reaching the ultimate one.

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