Arch. Mech., 40, 5-6, pp. 795-805, Warszawa 1988

Influence of thermal effects on micro-damage mechanism in dynamic processes

P. PERZYNA and A. DRABIK (WARSZAWA)

THE AIM of this paper is to investigate the influence of thermal effects on micro-damage mechanism of dissipative solids. Taking into account temperature changes, the evolution equation for the void volume fraction parametr ξ is derived. It describes the growth of micro-voids during the dynamical deformation process. Determination of the equilibrium state is the basic result obtained. The second part contains an alternative description of a temperature-dependent micro-damage process. The main role in this model is played by evolution equations for two parameters N, R which characterize the density of microvoids and the average size of microvoid, respectively. The material functions describing viscous effects and threshold stress for nucleation are determined. Finally, the relation between parameters ξ and N, R is discussed.

Celem pracy jest zbadanie wpływu efektów termicznych na mechanizm zniszczenia ciał dysypatywnych. Wyprowadzono, uwzględniając zmiany temperatury, równanie ewolucji parametru ξ opisującego objętościowy udział pustek. Równanie to określa wzrost mikro-uszkodzeń w procesie dynamicznego zniszczenia. Ważnym rezultatem jest określenie zależności naprężenia równowagi od temperatury. Druga część pracy zawiera alternatywną koncepcję zależnego od temperatury opisu procesu zniszczenia. Podstawowymi w tym modelu są równania ewolucji dla dwóch parametrów N i R, które określają odpowiednio liczbę pustek na jednostkę objętości i średni wymiar pustki. Zaproponowano postać funkcji materiałowych opisujących efekty lepkie i naprężenie progowe dla nukleacji. Przedstawiono także zależność pomiędzy parametrami ξ i N, R.

Целью работы является исследование влияния термических эффектов на механизм разрушения диссипативных тел. Было выведено, с учетом влияния температуры, уравнение эволюции параметра ξ, описывающего объемную долю пустот. Этим уравнением определяется рост микро-повреждений в процессе динамического разрушения. Важным результатом является определение зависимости равновесного напряжения от температуры. Вторая часть работы содержит альтернативную концепцию описания процесса разрушения с учетом температурной зависимости. Фундаментальными для этой модели являются уравнения эволюции двух параметров N и R, определяющих соответственно число пустот в единице объема и средний их размер. Был предложен вид материальных функций, описывающих вязкие эффекты и пороговые значения напряжения для зарождения пустот. Представлены также зависимости между параметрами ξ, N и R.

1. Introduction

EXPERIMENTAL results as well as theoretical descriptions show that fracture of metals occurs by a process of void nucleation, growth and coalescence. A particular analysis of the mechanisms responsible for each of these three phenomena suggests that they are very sensitive to the temperature changes. At increasing temperature, the stress required to obtain a given strain rate is reduced in such a way that the growth rate increases but the nucleation rate decreases. At low temperatures the situation is different.

The aim of our paper is to investigate the influence of thermal effects on micro-damage process of dissipative solids. The first part contains an analysis of the microvoids growth mechanism during the dynamical deformation process. It is described by the evolution of internal imperfection parameter ξ, which is interpreted as the void volume fraction. Taking into account temperature changes, the evolution equation for the parameter ξ is derived. It depends not only on the porosity but also on the size of microvoids. Determination of the equilibrium state as a function of temperature is the basic result obtained.

The second part contains a description of micro-damage process composed of two cooperative phenomena: nucleation and growth of microvoids. The nucleation is described by the evolution equation for density of voids N, and the growth of voids is controlled by the evolution equation for the average size of microvoid R. Basing on the available experimental results, the temperature-dependent material functions describing viscous effects and threshold stress for nucleation are determined.

Finally, by introducing a relation between the parameters ξ and N, R, the evolution equation is obtained for parameter ξ which describes the nucleation as well as growth of microvoids.

2. Physical model of porous material

The description of ductile fracture phenomenon by the evolution of porosity parameter ξ, measuring the void volume fraction, was applied by many authors.

The derivation and a detailed discussion of static and dynamic void-growth relation for elastic-plastic material was presented by M. M. CARROLL and A. C. HOLT [1]. Extension of this theory can be found in the papers of J. N. JOHNSON [4] and P. PERZYNA [6]. Our aim was to utilize this model in the description of dynamic fracture phenomenon with regard to temperature changes.

The physical model of ductile porous material is such that we consider the rectangular volume element containing a representative distribution of imperfections, as it is shown in Fig. 1. External pressure $\bar{p}$ acts over the surface of this element(¹). It is assumed that matrix material is homogeneous, isotropic and incompressible.

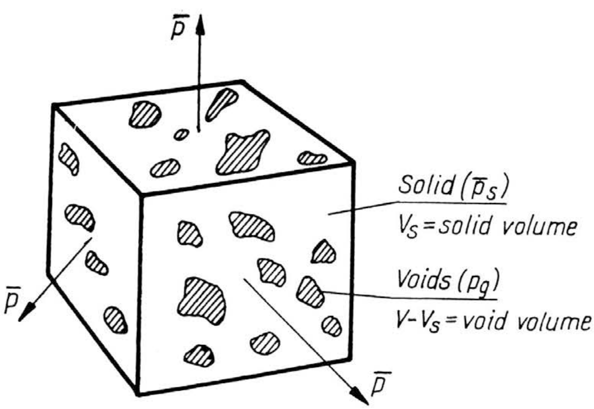

FIG. 1. Material element with representative distribution of imperfections (after J. N. JOHNSON [4]); V — total volume of rectangular element.

(¹) p_s is the mean stress in the solid material, p_g is the internal pressure in the solid.

The porosity parameter ξ is defined as the ratio of imperfections volume to solid volume

$$\xi = \frac{V - V_S}{V}. \tag{2.1}$$

Our calculations are reduced to the simplified model of the porous element. We assume that there is a spherical imperfection of radius a in the material sphere of radius b, subject to internal pressure p_g and external pressure $\bar{p}$ (see Fig. 2).

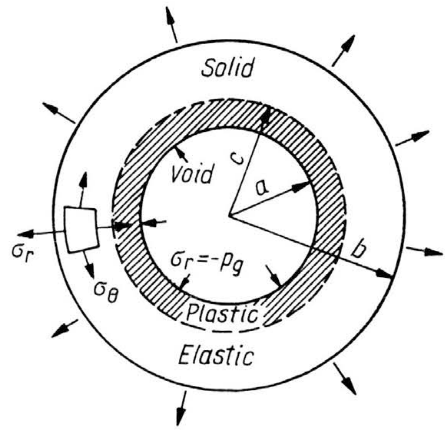

FIG. 2. Simplified model of the porous element (after J. N. JOHNSON [4]).

The definition of porosity parameter is now as follows:

$$\xi = \frac{a^3}{b^3}. \tag{2.2}$$

Additional assumption is that the surfaces $r = a$ and $r = b$ are kept at the temperatures $\bar{\vartheta}_1$, $\bar{\vartheta}_2$, respectively ($\bar{\vartheta}_1 > \bar{\vartheta}_2$), where $\bar{\vartheta}$ is defined as $\dfrac{\vartheta - \vartheta_0}{\vartheta_0}$; ϑ_0 — initial value of temperature.

From the equation of heat conduction we obtain the temperature distribution in the whole sphere [2]

$$\bar{\vartheta}(t) = \frac{a\bar{\vartheta}_1}{r} + \frac{(b\bar{\vartheta}_2 - a\bar{\vartheta}_1)(r-a)}{r(b-a)} + \frac{2}{r\pi}\sum_{n=1}^{\infty} \frac{b\bar{\vartheta}_2 \cos n\pi - a\bar{\vartheta}_1}{n} \sin\frac{n\pi(r-a)}{b-a}$$
$$\times e^{-\frac{kn^2\pi^2 t}{(b-a)^2}} + \frac{2}{r(b-a)}\sum_{n=1}^{\infty} \sin\frac{n\pi(r-a)}{b-a}\, e^{-\frac{kn^2\pi t}{(b-a)^2}} \int_a^b \tau f(\tau) \sin\frac{n\pi(\tau-a)}{b-a}\, d\tau, \tag{2.3}$$

where

$$f(r) = (\bar{\vartheta}_2 - \bar{\vartheta}_1)\frac{r-a}{b-a} + \bar{\vartheta}_1$$

is postulated as the initial temperature of the sphere.

3. Description of micro-damage process by porosity parameter

3.1. Constitutive relations

The constitutive relations for viscoplastic material with internal imperfections were proposed by P. PERZYNA [6].

The equation for the rate of the inelastic deformation tensor $\mathbf{E}^p$ has the form

$$\mathbf{E}^p(t) = \frac{\gamma}{\varphi}\left\langle \Phi\left[\frac{f(\cdot)}{\varkappa} - 1\right]\right\rangle \partial_{\mathbf{T}(t)} f, \tag{3.1}$$

where γ denotes the temperature-dependent viscosity coefficient, φ is the control function dependent on $(I_2/I_2^S)-1$, where I_2 is the second invariant of the rate of deformation tensor $\dot{\mathbf{E}}^p$, I_2^S is its static value and Φ denotes the viscoplastic overstress function, $\varkappa$ is a material function describing the work-hardening effects. The symbol $\langle[\ ]\rangle$ is defined as

$$\langle[\,]\rangle = \begin{cases} 0 & \text{if} \quad f \leqslant \varkappa, \\ [\,] & \text{if} \quad f > \varkappa. \end{cases} \tag{3.2}$$

The yield function for damaged solid is postulated in the form

$$f(\cdot) = J_2'\left[1-(n_1+\xi n_2)\frac{J_3'^2}{J_2'^3} + (n_3+\xi n_4)\frac{J_1^2}{J_2'}\right], \tag{3.3}$$

where n_i $(i = 1, 2, 3, 4)$ denote material constants, J_1 is the first invariant of the Cauchy stress tensor; J_2', J_3' — the second and third invariants of the stress deviator.

3.2. Evolution equation

The basis for our calculations will be the relation between parameter ξ and external pressure $\bar{p}$ derived by J. N. JONSON [4]

$$(\bar{p}(t)-p_g)\frac{1}{1-\xi} - 2\int\limits_a^b \frac{\Delta s}{r}\,dr = 0. \tag{3.4}$$

It was obtained from the equation of motion for the material surrounding the void, under the assumption that void expansion takes place in such a way that matrix material does not change volume.

After specification of Δs in the Eq. (3.4), we obtain the equation for parameter ξ describing the evolution of porosity during the dynamical deformation process. For viscoplastic material it is assumed that

$$\Delta s = \sigma_r - \sigma_\theta = Y(\bar{\vartheta}) + H(\bar{\vartheta})\bar{\varepsilon}^p + \eta(\bar{\vartheta})\dot{\bar{\varepsilon}}^p, \tag{3.5}$$

where $Y(\vartheta)$ is the temperature-dependent yield function, $H(\bar{\vartheta})$ is the material function describing work-hardening effects, $\eta(\bar{\vartheta})$ is the material function describing viscous effects, $\bar{\varepsilon}^p$ is the equivalent plastic deformation.

To simplify further calculations we postulate linear forms of these functions

$$\begin{aligned} Y(\bar{\vartheta}) &= Y_0 - Y_1\bar{\vartheta}, \\ H(\vartheta) &= H_0 - H_1\bar{\vartheta}, \\ \eta(\bar{\vartheta}) &= \eta_0 + \eta_1\bar{\vartheta}, \end{aligned} \tag{3.6}$$

where $Y_0, Y_1, H_0, H_1, \eta_0, \eta_1$ are constants.

Very useful is also the following expression for the equivalent plastic deformation [4]

$$\overline{\varepsilon}^p = \left(1+\frac{B(t)}{r^3}\right)^{\frac{1}{3}} + \left(1+\frac{B(t)}{r^3}\right)^{-\frac{2}{3}}, \tag{3.7}$$

where

$$\frac{B(t)}{a^3} = \frac{\xi_0-\xi}{\xi(1-\xi_0)}, \qquad \frac{B(t)}{b^3} = \frac{\xi_0-\xi}{1-\xi_0}.$$

Putting the expressions (3.5), (3.6) into the Eq. (3.4) and making use of (3.7), we obtain the final form of the evolution equation for parameter ξ

$$\dot{\xi} = \frac{1}{\dfrac{\eta_0}{F(\xi,\xi_0)} - \dfrac{(1-\xi)D(a)}{\xi}} \left\{\overline{p}(t) - p_g + \frac{2Y_0}{3}(1-\xi)\ln|\xi| \right.$$
$$\left. + \frac{2H_0}{3}(1-\xi)F_1(\xi,\xi_0) + 2(1-\xi)H_1\overline{\vartheta}_1\left(\frac{1-\xi}{1-\xi_0}\right)^{\frac{1}{3}}\left[\left(\frac{\xi_0}{\xi}\right)^{\frac{1}{3}} - \xi^{\frac{1}{3}}\right] + (1-\xi)E(a)\right\}, \tag{3.8}$$

where

$$F_1(\xi,\xi_0) = 3\left(\frac{1-\xi}{1-\xi_0}\right)^{1/3}\left[1-\left(\frac{\xi_0}{\xi}\right)^{1/3}\right],$$

$$F(\xi,\xi_0) = \frac{3\xi}{2}\left(\frac{1-\xi}{1-\xi_0}\right)^{1/3}\left[\xi-\left(\frac{\xi_0}{\xi}\right)^{1/3}\right]^{-1},$$

$$D(a) = \frac{2\eta_1}{3}\frac{\xi_0}{\xi(1-\xi_0)}\sum_{n=1}^{\infty}\mathrm{C}_n\sum_{k=0}^{\infty}\sum_{m=0}^{k}\frac{(-1)^m\sin\left(\dfrac{m\pi}{2}-\dfrac{1}{\xi^{-1/3}-1}\right)}{(2m+1)!}\alpha^{2m+1}$$
$$\times\left[\frac{\xi_0-\xi}{\xi(1-\xi_0)}\right]^{-\frac{5}{3}-k+m}\left[\binom{-\frac{5}{3}}{k-m}\frac{1}{2+3k-m}\left(\xi^{\frac{-(2+3k-m)}{3}}-1\right) + \frac{\xi_0-\xi}{\xi(1-\xi_0)}\left(\frac{a^{-1}}{3k-m+1}\right)\right.$$
$$\left.\times\left(\xi^{\frac{-(3k-m-1)}{3}}-1\right)\right] + \frac{2}{3}\eta_1[\overline{\vartheta}_1 - a(\xi^{-\frac{1}{3}}\overline{\vartheta}_2-\overline{\vartheta}_1)]\frac{\xi}{\xi(1-\xi_0)}\left(\frac{1-\xi}{1-\xi_0}\right)^{-\frac{2}{3}}\left[\left(\frac{\xi_0}{\xi}\right)^{-\frac{2}{3}}-\xi^{\frac{4}{3}}\right]$$
$$+\eta_1 a(\xi^{-\frac{1}{3}}\overline{\vartheta}_2 - \overline{\vartheta}_1F_2(\xi,\xi_0)),$$

$$E(a) = -\frac{6Y_1\overline{\vartheta}_1}{a^2}(\xi-1) + 2Y_1a(\xi^{-\frac{1}{3}}\overline{\vartheta}_2-\overline{\vartheta}_1)\left[\frac{3}{a^2}(\xi-1)-\frac{1}{3}\ln|\xi|\right]$$
$$+2Y_1\sum_{n=1}^{\infty}\mathrm{C}_n\left[\cos\frac{\alpha}{a}\left(\mathrm{si}\left(n\pi+\frac{\alpha}{a}\right)-\mathrm{si}\frac{\alpha}{a}\right)+\sin\frac{\alpha}{a}\left(\mathrm{ci}\frac{\alpha}{a}-\mathrm{ci}\left(n\pi+\frac{\alpha}{a}\right)\right)\right.$$
$$+2H_1\sum_{n=1}^{\infty}\mathrm{C}_n\alpha^2\left(\frac{\xi_0-\xi}{\xi(1-\xi_0)}\right)^{\frac{1}{3}-m}\sum_{k=0}^{\infty}\sum_{m=0}^{k}\binom{\frac{1}{3}}{m}\frac{(-1)^{k-m}\cos\left[\dfrac{(k-m)\pi}{2}-\alpha\right]}{[2(k-m)]!}$$
$$\times a^{2(k-m-1)}\left(\xi^{\frac{-(2k+m-1)}{3}}-1\right)+2H_1a(\xi^{-\frac{1}{3}}\overline{\vartheta}_2-\overline{\vartheta}_1)\left(\frac{1-\xi}{1-\xi_0}\right)^{\frac{1}{3}}(\xi^{-\frac{1}{3}}-1),$$

$$F_2(\xi, \xi_0) = \frac{2}{3(1-\xi_0)}\left(\frac{1-\xi}{1-\xi_0}\right)^{-\frac{2}{3}}\left[\xi - \left(\frac{\xi_0}{\xi}\right)^{\frac{1}{3}}\right], \quad \alpha = \frac{n\pi}{\xi^{-\frac{1}{3}}-1},$$

$$\mathsf{C}_n = \int\limits_a^b \tau f(\tau)\sin\frac{n\pi(\tau-a)}{b-a}\, d\tau.$$

Eq. (3.8) describes the temperature-dependent microvoids growth mechanism during the dynamical deformation process. The nucleation part is neglected, so the existence of the initial porosity ξ_0 is assumed. Practical application of this relation is very difficult. There are the terms $D(a)$ and $E(a)$ which depend on the size of microvoid a. They can not be approximated by the expressions which depend only on porosity ξ and its initial value ξ_0.

3.3. Discussion of the equilibrium state

The equilibrium state is reached in the case when there are no changes of porosity ($\dot{\xi} = 0$), so with this assumption from the Eq. (3.8) is obtained

$$\tag{3.9} \bar{p} = p_{\mathrm{eqn}}(\xi) = (1-\xi)\left\{-\frac{2}{3}Y_0 \ln|\xi| - \frac{2}{3}H_0 F_1(\xi, \xi_0) - 2H_1\bar{\vartheta}_1\left(\frac{1-\xi}{1-\xi_0}\right)^{1/3}\left[\left(\frac{\xi_0}{\xi}\right)^{1/3} - \xi^{1/3}\right] - E(a)\right\}.$$

The expression (3.9) shows direct temperature influence on the value of equilibrium pressure p_{eqn} for porosity ξ. We have met here the same difficulty as that mentioned at the end of the previous chapter. The value of p_{eqn} depends not only on porosity but also on the size of microvoid a.

4. Alternative description of micro-damage process

4.1. Evolution equations

To overcome the difficulties concerned with the description of micro-damage process by the evolution of porosity parameter presented in the previous chapters, the second method is taken into account. We define two internal state variables N, R which characterize the density of microvoids (the number of microvoids per unit volume) and the average size of microvoid, respectively. It is worth to mention that such description is suggested not only by the theoretical considerations but also by a large number of experimental results (see Refs. [10, 9, 11, 12, 13]). The damage of the material is described by statistical distribution of imperfections, not by individual one. We assume that fracture phenomenon is caused by the nucleation of new voids and by growth of the existing ones. It is also assumed that there exists an initial porosity of the material, so that nucleation produces additional voids. The voids continue to increase in size until the void fraction exceeds a critical value and separation of the material occurs. Evolution of the parameters N and R which are responsible for the nucleation and growth of imperfection is postulated by the following relations [7]

$$\dot{N} = \dot{N}_0(\bar{\vartheta})\left\{\exp\frac{m[I_n - T_N(\bar{\vartheta})]}{k\bar{\vartheta}} - 1\right\}, \tag{4.1}$$

$$\dot{R} = \frac{1}{\eta(\bar{\vartheta})}[I_g - T_G(\bar{\vartheta})]R, \tag{4.2}$$

where

$$I_n = a_1 J_1 + a_2\sqrt{J_2'} + a_3\sqrt{J_3'},$$
$$I_g = b_1 J_1 + b_2\sqrt{J_2'} + b_3\sqrt[3]{J_3'}$$

are the stress intensity invariants for nucleation and growth respectively, a_i and b_i ($i = 1, 2, 3, 4$) are material constants, J_1 denotes the first invariant of the Piola–Kirchhoff stress tensor T; J_2' and J_3' are the second and third invariants of the stress deviator, $\dot{N}_0$ is a temperature dependent material function, m is material constant, k is the Boltzmann constant, η is temperature-dependent viscosity coefficient; T_N and T_G are the threshold stresses for nucleation and growth, respectively. Eqs. (4.1) and (4.2) show that for dynamical processes the thermally-activated mechanism is most important for the nucleation of voids and that growth process depends on the viscoplastic properties of material.

The problem arises how to postulate the temperature-dependent functions $\dot{N}_0$, η, T_N and T_G. Our suggestions deduced from the available experimental results will be presented in the next chapters.

4.2. Relation between porosity parameter and N, R

Before specifying all material functions from the Eqs. (4.1), (4.2), let us put our attention to the important dependence between parameter ξ and parameters N, R. They are connected by the following relation:

$$\xi = \Xi N R^3, \quad \xi \in [0, 1], \quad \xi(0) = \xi_0, \tag{4.3}$$

where Ξ is a material constant.

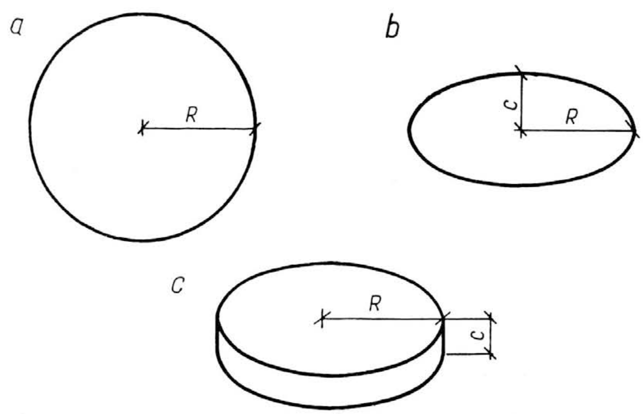

FIG. 3.

FIG. 3. Schematic presentation of the shapes of imperfections (a — spherical, b — ellipsoidal, c — pennyshaped).

The constant Ξ plays a significant role in the description of micro-damage process. Taking into account different values of Ξ it is possible to describe various shapes of micro-voids.

Here are given the examples of several values of Ξ.

1. The shape of void is spherical (Fig. 3a);

$$\xi = \frac{4}{3}\pi N R^3 \Rightarrow \Xi = \frac{4}{3}\pi. \tag{4.4}$$

2. The shape of void is ellipsoidal (Fig. 3b);

$$\xi = \frac{4}{3}\pi N R^3 c \quad \text{for } c = \mu R \quad \text{and} \quad \mu \in (0, 1) \Rightarrow \Xi = \frac{4\mu}{3}\pi. \tag{4.5}$$

3. For penny-shaped voids (Fig. 3c)

$$\xi = \pi R^2 c, \quad c = 0.1 R \Rightarrow \Xi = 0.1\pi. \tag{4.6}$$

Evolution of the porosity parameter ξ can be determined by differentiation of Eq. (4.3), and then by replacing the values N and $\dot{R}$ with Eqs. (4.1), (4.2). This procedure leads to the following result:

$$\dot{\xi} = h(\xi, \bar{\vartheta})\left\{\exp\frac{m[I_n - T_N(\bar{\vartheta})]}{k\bar{\vartheta}} - 1\right\} + g(\xi, \bar{\vartheta})[I_g - T_G(\bar{\vartheta})], \tag{4.7}$$

where

$$h(\xi, \bar{\vartheta}) = \xi\frac{\dot{N}_0}{N_0}, \quad g(\xi, \bar{\vartheta}) = \frac{3\xi}{\eta}.$$

Relation (4.7) shows the equivalence between the description of micro-damage process by porosity parameter ξ and by the parameters N, R.

4.3. Modification of yield function

The formula (3.3) gives a definition of yield function for the viscoplastic material with internal imperfections, which depends on porosity parameter ξ. For the description of fracture phenomenon by parameters N and R we have to modify this form.

Using the relation (4.3) we obtain from (3.3) that

$$f(\cdot) = J_2'\left[1 - (n_1 + n_2' N R^3)\frac{J_3'^2}{J_2'^3} + (n_3 + n_4' N R^3)\frac{J_1^2}{J_2'}\right], \tag{4.8}$$

where n_2' and n_4' are constants dependent on Ξ,

$$\text{i.e. } n_2' = n_2\Xi, \quad n_4' = n_4\Xi.$$

4.4. Determination of material functions

Let us return to the postulated evolution equations for parameters N and R (Eqs. (4.1), (4.2)). The basis for determination of the temperature-dependent functions describing threshold stresses for nucleation and growth T_N, T_G, viscosity η and $\dot{N}_0$ will be the experimental results.

Experiments performed for a various types of materials show that temperature has significant influence on the mechanisms of fracture. We observe different mechanism responsible for nucleation and growth phenomena when fracture occurs at elevated temperature, in comparison with the same process running at room temperature.

As an example, consider a copper polycrystal with a grain size of 10 μm, strained at a constant strain-rate of 10^{-4}/s. At high temperatures, the polycrystal will flow predominantly by power-law creep, at low temperatures its flow occurs predominantly by the glide motion of dislocations. The dependence between stress and temperature calculated for this experiment is shown in Fig. 4 (see also [8]).

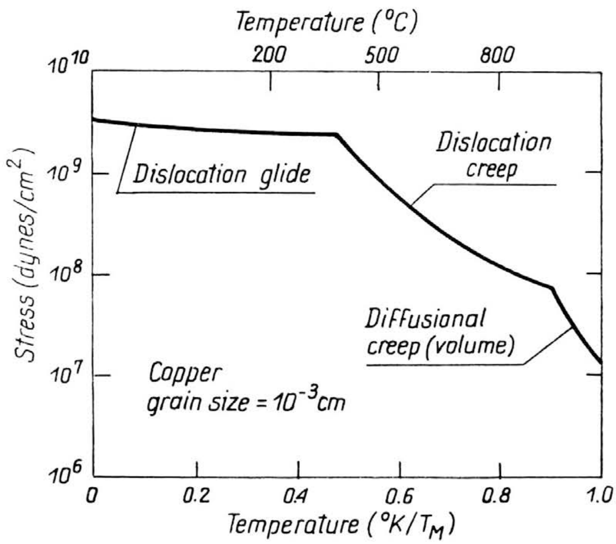

FIG. 4. Variation of stress with temperature when a polycrystal of copper of grain size 10μ m is strained at 10^{-4}/s (after R. RAJ and M. F. ASHBY [8]).

Temperature influences the mechanisms of nucleation and growth of voids in such a way that for increasing temperature the growth rate increases but the nucleation rate decreases. At low temperatures nucleation occurs very fast but growth is slow. Taking into account this facts we suggest the functions T_N, T_G and $\dot{N}_0$ in the forms

(4.9)
$$
\begin{aligned}
T_N(\bar{\vartheta}) &= \alpha_1 - \alpha_2 \bar{\vartheta}^{\alpha_3},\\
T_G(\bar{\vartheta}) &= \beta_1 - \beta_2 \bar{\vartheta}^{\beta_3},\\
\dot{N}_0(\vartheta) &= \delta_1 \bar{\vartheta}^{-\delta_2},
\end{aligned}
$$

where $\alpha_i, \beta_i, \delta_i$ $(i = 1, 2)$ are constants.

Next, we utilize the linear dependence between viscosity coefficient and temperature (Refs. [3] and [5]).

(4.10)
$$\eta(\bar{\vartheta}) = \gamma_1 + \gamma_2 \bar{\vartheta},$$

where γ_1, γ_2 are constants.

Putting the formulas (4.9) and (4.10) into the Eqs. (4.1), (4.2) we obtain the final form of the evolution equations for imperfections parameters N and R

$$\dot{N} = \delta_1 \bar{\vartheta}^{-\delta_2} \left\{ \exp \frac{m[I_n - \alpha_1 + \alpha_2 \bar{\vartheta}^{\alpha_3}]}{k\bar{\vartheta}} - 1 \right\}, \tag{4.11}$$

$$\dot{R} = \frac{1}{\gamma_1 + \gamma_2 \bar{\vartheta}} [I_g - \beta_1 + \beta_2 \bar{\vartheta}^{\beta_3}] R. \tag{4.12}$$

The values of constants α_i, β_i, γ_i, δ_i ($i = 1, 2$) must also be determined from experimental results. At temperature above $0.7 T_M$ the rate of nucleation is near to zero.

Including this fact into Eq. (4.11) we obtain the following relation between material constants

$$\alpha_1 - \alpha_2 \bar{\vartheta}^{\alpha_3} = I_n. \tag{4.13}$$

Specification of the functions T_N, T_G, $\dot{N}_0$ and η in Eq. (4.7) gives also the final form of the evolution equation for the porosity parameter

$$\dot{\xi} = h(\xi, \bar{\vartheta}) \left\{ \exp \frac{m[I_n - \alpha_1 + \alpha_2 \bar{\vartheta}^{\alpha_3}]}{k\bar{\vartheta}} - 1 \right\} + g(\xi, \bar{\vartheta}) [I_g - \beta_1 + \beta_2 \bar{\vartheta}^{\beta_3}],$$

where

$$\begin{aligned} h(\xi, \bar{\vartheta}) &= \frac{\xi \delta_1}{N \bar{\vartheta}^{\delta_2}}, \\ g(\xi, \bar{\vartheta}) &= \frac{3\xi}{\gamma_1 + \gamma_2 \vartheta}. \end{aligned} \tag{4.14}$$

5. Final comments

The aim of our paper was to analyse the temperature-dependent mechanisms of fracture of dissipative solids during dynamical deformation process and to find their theoretical description. The evolution equation for porosity parameter ξ derived in Sect. 3 has some disadvantages. The first one is that it describes only the growth of microvoids and the nucleation part is neglected. The next one consists in the fact that it depends not only on the parameter ξ, but also on the size of microvoid.

A detailed analysis of nucleation and of growth phenomenon shows that it is better to describe them by the evolution of two imperfection parameters N and R. There are several reasons which show that the results obtained from this model are more satisfactory.

1. Using this model we can consider different shapes of microvoids, e.g. spherical, ellipsoidal, penny-shaped;

2. The evolution equations are not complicated and they are convinient in practical applications;

3. Basing on the experimental results it is possible to postulate all material functions in the equations proposed;

4. Relation $\xi = \Xi N R^3$ introduces equivalence between two descriptions of micro-

damage process: first — using internal state variables (N, R) and the second — dealing with porosity parameter ξ.

5. The evolution equations for internal state variables (N, R) describe the temperature-dependent nucleation mechanism as well as the growth process.

References

1. M. M. CARROLL, A. C. HOLT, J. Appl. Phys., **43**, 4, 1972.
2. H. S. CARSLAW, J. C. JAEGER, *Conduction of heat in solids*, Oxford University Press 1959.
3. J. A. GORMAN, D. S. WOOD, T. VREELAND, jr., *Mobility of dislocation in aluminium*, J. Appl. Phys., **40**, 833–841, 1969.
4. J. N. JOHNSON, J. Appl. Phys., **52**, 4, 1981.
5. P. PERZYNA, *Thermodynamics of inelastic materials* [in Polish], PWN, Warszawa 1978.
6. P. PERZYNA, Int. J. Solids Structures, **22**, 7, pp. 797–818, 1986.
7. P. PERZYNA, *Thermomechanical couplings in solids*, Elsevier Science Publishers, IUTAM 1987.
8. R. RAJ, M. F. ASHBY, Acta Metall., **23**, 1975.
9. L. SEAMAN, D. R. CURRAN, W. J. MURRI, J. Appl. Mech., **107**, 593–600, 1985.
10. L. SEAMAN, D. R. CURRAN, D. A. SHOCKEY, J. Appl. Phys., **47**, 4814–4826, 1976.
11. P. J. WRAY, J. Appl. Phys., **40**, 4018–4029, 1969.
12. P. J. WRAY, Metal. Trans., 6A, 1379–1391, 1975.
13. P. J. WRAY, Metal. Trans., 7A, 1621–1627, 1976.

POLISH ACADEMY OF SCIENCES
INSTITUTE OF FUNDAMENTAL TECHNOLOGICAL RESEARCH.

Received April 22, 1988.