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Spectrally filtrated model of a nonlinear visco-elastic medium

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CONSTRUCTION of a filtrated model of a geometrically nonlinear visco-elastic medium is presented. The nature of the filtration procedure and construction of effective operators are shown. To this end displacement u_i and velocity v_i fields are decomposed into the filtered parts $\bar{u}_i, \bar{v}_i$ and fluctuations u_i', v_i', respectively. The effective operator is determined to within an accuracy of the spectral fluctuation fields, which may be evaluated on the basis of the measurement data. If such data are not available, the fluctuation fields are determined with $\bar{u}_i$ and $\bar{v}_i$ by means of a recursive procedure described in the paper.

W pracy przedstawiono konstrukcję odfiltrowanego modelu dla geometrycznie nieliniowego ośrodka lepko-sprężystego. Przedstawiono istotę procedury filtrowania, a następnie konstrukcji operatora efektywnego. W tym celu podzielono pola przemieszczeń u_i i prędkości v_i na część odfiltrowaną $\bar{u}_i, \bar{v}_i$ i część fluktuacyjną u_i', v_i'. Operator efektywny uzyskuje się z dokładnością do pól fluktuacji widmowych. Pola takie można wyznaczyć na podstawie danych pomiarowych. Przy braku takich danych pola fluktuacji można wyznaczyć łącznie z polami $\bar{u}_i, \bar{v}_i$ według przedstawionej w pracy procedury rekursyjnej.

В работе представлено построение отфильтрованной модели для геометрически нелинейной вязкоупругой среды. Представлена сущность процедуры фильтрования, а затем построения эффективного оператора. С этой целью поля перемещений u_i и скоростей v_i разделены на отфильтрованную часть $\bar{u}_i, \bar{v}_i$ и флуктуационную часть u_i', v_i'. Эффективный оператор получается с точностью до полей спектральных флуктуаций. Такие поля можно определить на основе измерительных данных. При отсутствии таких данных поля флуктуаций можно определить совместно с полями $\bar{u}_i, \bar{v}_i$ согласно представленной в работе рекурсивной процедуре.

1. Introduction

THE PRESENT state of development of phenomenological models of material continua is the result of parallel development of analytical methods in mathematics, of the theory of generalized functions and functional spaces; this enabled the description of space-time processes modelling various physical phenomena of arbitrarily irregular character. The irregularities may be due to the external causes influencing the object under investigation, or due to the internal structure of the model and its theory itself. It frequently happens that external actions may be treated as a superposition of causes of arbitrary degree of irregularity. Ability of a rational classification of such causes and of tracing their individual results makes it possible to understand the nature of processes taking place within the material and the properties of the theory applied.

External causes are now frequently modelled by means of measurements made at a finite number of space-time points. Results of such measurements contain errors characteristic for each measurement technique. If such results are to be used as initial or boundary data fields, they should first be properly processed in order to eliminate the random errors, to perform the necessary interpolation and to refer them to the calculation point arrays assumed for numerical analysis. Situations of such kind are encountered in evaluation of ecological, meteorological and hydrological forecasts.

Realisation of both types of tasks (separation of causes and effects and assimilation of measurement results) necessitates the application of space, time or space-time filtration procedures. Such procedures have been developed in the signal theory and are usually applied to filtrate the time processes. Material field filtration processes are at a lower stage of development; it applies also to the assimilation of measurement results containing random errors and obtained by means of filtration procedures. Particularly interesting is the filtration procedure based on the notion of the Kalman–Bucy filter [1], which is of a recursive character and enables filtrated assimilation in the model of a given set of current measurements. As a result, the forecasted process is a resultant of the "work" done by the forecast model and the actual measurements.

In the present paper and in papers [2–5] a filtration procedure is used which resembles that applied in the signal theory; it consists in a proper selection of spatial spectrum intervals in spectral images of material fields. Such choice follows partly from cognitive reasons (determination of model reaction to individual spectrum intervals), and partly from practical reasons connected with the numerical method applied, the assimilated measurement data, etc. This procedure accounts also, whenever it is possible, for minimization of the spectral effects coming from without the selected spectrum intervals.

In the space of positions, the filtration operation performed in the spectral space reduces to a convolution; it is applied both to the equations of the theory and to the data fields. This problem of fundamental importance for the filtration procedures has been tackled in [2] and in the present paper (Section 2).

It should be noted that spatial filtration procedures are sometimes unconsciously used by numerous authors. Some examples of such applications are given in [2]. Here let us mention several examples: procedure of averaging based on spatial ergodic hypothesis [6], construction of pseudo-continuum (quasi-continuum) models [7, 8], theories of generalized functions etc. Numerical methods may also be treated as certain filtration procedures.

It was found in [3, 4] that an effective operator may be constructed for linear models. Considerable difficulties are encountered in cases of nonlinear models. In order to construct an effective nonlinear operator, the so-called closure procedures must be applied. Such procedures are used in the turbulence theory (closure K and higher approximations) [9]. An example of the closure procedure based on measurement results was presented in [4].

It was observed in [5] that in formulating the boundary value problems for filtered models, a characteristic boundary layer appears; it is connected with a nonlocal effect and contains body forces dependent on the displacements or stresses at the boundary.

The aim of the present paper is to construct a filtered model for a nonlinear viscoelastic medium and to demonstrate the fundamental difficulties appearing in the construction of an effective operator. It is known that in the nonlinear case spectral intervals are not separated and spectral exchange (of momentum or energy) between the intervals occurs. Thus it becomes necessary to propose a suitable "parametrisation" of the extra-interval effects, what represents a rather difficult task. It will be considerably simplified in the case when the set of measurements data characterizing the process modelled is given. In numerous cases the data may be used to solve the problem and to construct the effective operator.

The filtration procedure is presented in Sect. 2, construction of the filtered model for a nonlinear visco-elastic medium is discussed in Sect. 3, the concept of closure in nonlinear filtered models is explained in Sect. 4; conclusions following from the results obtained are presented in Sect. 5.

2. Filtration procedure

Spectral filtration is equivalent to the following convolution operation:

$$\Phi(k)u(k) \to \int_{\Omega} \Phi(\mathbf{x}-\mathbf{x}')u(\mathbf{x}')d^3x', \tag{2.1}$$

where $\Phi(k)$ is the function of spectral "cutout". In the simplest case of uniform filtration

$$\Phi(k) = \begin{cases} 1, & |k| < k_L, \\ 0, & |k| > k_L. \end{cases} \tag{2.2}$$

The convolution kernel $\Phi(\mathbf{x}-\mathbf{x}')$ may be treated as a test function in the construction of generalized functions (distributions). Hence in the filtered models the boundary value problem is formulated and solved for such generalized functions and, in this manner, the generalized solutions have a well-defined interpretation. Functions Φ are usually assumed to have the following properties:

$$\Phi(\mathbf{x}-\mathbf{x}') = \Phi(\mathbf{x}'-\mathbf{x}), \tag{2.3}$$

$$\Phi(\mathbf{x}-\mathbf{s}') = 0, \qquad \Phi_{,j}(\mathbf{x}-\mathbf{x}') = 0. \tag{2.4}$$

The fundamantal problem in constructing the filtered model consists in the determination of the effective operator A_{ef} defined as follows:

$$\langle \Phi A u\rangle = A_{ef}\langle \Phi u\rangle. \tag{2.5}$$

It was shown in [1] that A_{ef} is easily determined for linear operators A. In the case when A is a nonlinear operator with an additive linear term, then

$$\langle \Phi A u\rangle = A_{ef}\langle \Phi u\rangle + \langle \Phi N(\bar{u}, u')\rangle, \tag{2.6}$$

where

$$\bar{u} = \frac{\langle \Phi u\rangle}{\langle \Phi\rangle}, \qquad u' = u - \bar{u}. \tag{2.7}$$

For the sake of simplicity it is usually assumed that $\langle \Phi\rangle = 1$. It resembles the procedure of division of the field u into the average value $\bar{u}$ and fluctuation u'.

Problem of closure of the theory consists in finding such an operator G, linear or nonlinear, which allows for expressing the nonlinear operator N in the form

$$\langle \Phi N(\bar{u}, u')\rangle = G\langle \Phi u\rangle = G\bar{u}. \tag{2.8}$$

A good example of such procedure is found in the theory of turbulence (K-approximation). In the general case the problem is very complicated.

3. Construction of filtered models for nonlinear visco-elastic medium

Equations of motion of the medium have the form

$$-\varrho\frac{\partial^2 u_i}{\partial t^2}+t_{ij,j}+\varrho f_i=0, \tag{3.1}$$

where $t_{ij}={}_st_{ij}+{}_Dt_{ij}$, ${}_st_{ij}$ — elastic stresses, ${}_Dt_{ij}$ — dissipative stresses. Taking into account geometric nonlinearity only, the following constitutive equations of the medium are assumed:

$$ {}_st_{ij}=c_{ijkl}e_{kl}=c_{ijkl}\left(\varepsilon_{kl}-\frac{1}{2}u_{m,k}u_{m,l}\right), \tag{3.2}$$

$$ {}_Dt_{ij}=D_{ijkl}\dot{e}_{kl}=D_{ijkl}[d_{kl}-(e_{kn}v_{n,l}+e_{ln}v_{n,k})], \tag{3.3}$$

where

$$\varepsilon_{kl}=u_{(k,l)},\qquad d_{kl}=v_{(k,l)}. \tag{3.4}$$

Under the assumption of constant density, $\varrho=\text{const}$, convolution with the kernel $\Phi(\mathbf{x}-\mathbf{x}')$ yields the equation

$$-\varrho\frac{\partial^2}{\partial t^2}\langle\Phi u_i\rangle+\langle\Phi t_{ij,j}\rangle+\varrho\langle\Phi f_i\rangle=0, \tag{3.5}$$

where

$$\langle\Phi u_i\rangle=\int_\Omega\Phi(\mathbf{x}-\mathbf{x}')u_i(\mathbf{x}')d^3x'. \tag{3.6}$$

In order to determine the effective operator "acting" on $\langle\Phi u\rangle$, the term containing stress t_{ij} is integrated by parts to lead to

$$\langle\Phi t_{ij,j}\rangle=\langle(\Phi t_{ij})_{,j}\rangle-\langle\Phi_{,j}t_{ij}\rangle=\langle\Phi(\mathbf{x}-\mathbf{s})t_{ij}n_j(\mathbf{s})\rangle+\langle\Phi t_{ij}\rangle_{,j}. \tag{3.7}$$

According to Eq. (2.4), the first (surface) term vanishes, so that the operation of partial differentiation commutes with the operation of convolution. This rule is, however, violated in the boundary layer (close to the boundary), its dimenssion being dependent of the support of function $\Phi(\mathbf{x}-\mathbf{x}')$ where, in general,

$$\langle\Phi(\mathbf{x}-\mathbf{s}')t_{ij}(\mathbf{s}')n_j(\mathbf{s}')\rangle\neq 0. \tag{3.8}$$

In an elastic homogeneous medium, elastic stresses ${}_st_{ij}$ satisfy, acccording to Eqs. (3.2), (3.3), the equations

$$\langle\Phi_s t_{ij}\rangle_{,j}=C_{ijkl}\left\langle\Phi\left(\varepsilon_{kl}-\frac{1}{2}u_{m,k}u_{m,l}\right)\right\rangle_{,j}. \tag{3.9}$$

Taking into account the fact that the operations of differentiation and convolution commute, we obtain at $\langle\Phi\rangle=1$ the relation

$$\bar{u}_{mk}=\frac{\langle\Phi u_{m,k}\rangle}{\langle\Phi\rangle}=\langle\Phi u_m\rangle_{,k}=\bar{u}_{m,k}. \tag{3.10}$$

what makes it possible to decompose the displacement gradient field

$$u_{m,k}=\bar{u}_{m,k}+u'_{m,k}. \tag{3.11}$$

Similar decomposition is made in the case of velocity gradient fields. Definition $u'_{m,k} = u_{m,k} - \bar{u}_{m,k}$ yielding the result

$$\langle \Phi u'_{m,k} \rangle = 0. \tag{3.12}$$

Taking this into account, $\langle \Phi_s t_{ij} \rangle$ is written in the form

$$\langle \Phi_s t_{ij} \rangle = C_{ijkl} \left\langle \Phi \left[(\bar{\varepsilon}_{kl} + \varepsilon'_{kl}) - \frac{1}{2} (\bar{u}_{m,k} + u'_{m,k}) (\bar{u}_{m,l} + u'_{m,l}) \right] \right\rangle \tag{3.13}$$

or, after transformations and using formula (3.12), the form

$$\langle \Phi_s t_{ij} \rangle = C_{ijkl} \left[\bar{\varepsilon}_{kl} - \frac{1}{2} (\bar{u}_{m,k} \bar{u}_{m,l}) - \frac{1}{2} \langle \Phi u'_{m,k} \bar{u}'_{m,l} \rangle \right]. \tag{3.14}$$

The first two right-hand terms of this equation contain derivatives of the filtered displacement field $\bar{u}_i$. The third term is of a different structure; it is responsible for "interaction" between the large-scale and small-scale phenomena, the interaction being characteristic for nonlinear theories. The problem of closure reduces in that case to expressing it in terms of $\bar{u}_{m,k}$. It should be stressed that the problem of closure in nonlinear models remains an open question as yet, positive answers to the question being rare.

Expression for the dissipative stresses ${}_D t_{ij}$ has the form

$$\begin{aligned} {}_D t_{ij} = D_{ijkl} \left[d_{kl} - (\varepsilon_{kn} v_{n,l} + \varepsilon_{ln} v_{n,k}) + \frac{1}{2} (u_{m,k} u_{m,n} v_{n,l} + u_{m,l} u_{m,n} v_{n,k}) \right] \\ = {}_D t^1_{ij} + {}_D t^2_{ij} + {}_D t^3_{ij} \end{aligned} \tag{3.15}$$

decomposition of ${}_D t_{ij}$ being made in anticipation of further transformations of the formula. Making use of gradient field decomposition, cf. Eq. (3.11), we obtain after transformations

$$\langle \Phi_D t^1_{ij} \rangle = D_{ijkl} \bar{d}_{kl}, \tag{3.16}$$

$$\langle \Phi_D t^2_{ij} \rangle = -D_{ijkl} (\bar{\varepsilon}_{kn} \bar{v}_{n,l} + \bar{\varepsilon}_{ln} \bar{v}_{n,k}) - D_{ijkl} (\langle \Phi \varepsilon'_{kn} v'_{n,l} \rangle + \langle \Phi \varepsilon'_{ln} v'_{n,k} \rangle), \tag{3.17}$$

$$\begin{aligned} \langle \Phi_D t^3_{ij} \rangle = \frac{1}{2} D_{ijkl} [\bar{u}_{m,k} \bar{u}_{m,n} \bar{v}_{n,l} + \bar{u}_{m,l} \bar{u}_{m,n} \bar{v}_{n,k}] + \frac{1}{2} D_{ijkl} [\bar{u}_{m,k} \langle \Phi u'_{m,n} v'_{n,l} \rangle \\ + \bar{u}_{m,l} \langle \Phi u'_{m,n} v'_{n,k} \rangle] + \frac{1}{2} D_{ijkl} [(\langle \Phi u'_{m,k} v'_{n,l} \rangle + \langle \Phi u'_{m,l} v'_{n,k} \rangle) \bar{u}_{m,n}] \\ + \frac{1}{2} D_{ijkl} [\langle \Phi u'_{m,k} u'_{m,n} \rangle \bar{v}_{n,l} + \langle \Phi u'_{m,l} u'_{m,n} \rangle \bar{v}_{n,k}] \\ + \frac{1}{2} D_{ijkl} [\langle \Phi u'_{m,k} u'_{m,n} v'_{n,l} \rangle + \langle \Phi u'_{m,l} u'_{m,n} v'_{n,k} \rangle]. \end{aligned} \tag{3.18}$$

Effective operator will be determined if

$$\langle \Phi \ldots \rangle = 0 \tag{3.19}$$

or if all "primed" values are known. Vanishing of all terms $\langle \Phi u'_m u'_k \rangle$ is possible if the "mixing hypothesis" is satisfied,

$$\langle \Phi u'_m u'_k \rangle = \langle \Phi_1 u'_m \rangle \langle \Phi_1 u'_k \rangle = 0. \tag{3.20}$$

Relation (3.20) is fulfilled if one of the fields appearing in the product varies much faster than the other one, or if both of them are subject to rapid changes in the space (fluctuations). The first case may take place in considering the factorization of the mass flux $\langle \Phi \varrho' v_i' \rangle$. Since the space fluctuations of ϱ' are usually much smaller than those of v_i', the relation holds true

$$\langle \Phi \varrho' v_i' \rangle = \langle \Phi_1 \varrho' \rangle \langle \Phi_1 v_i' \rangle = 0. \tag{3.21}$$

If u_i', v_i' are known, convolutions $\langle \Phi \ldots \rangle$ play the role of coefficients in the equations. In such cases ${}_D t_{ij}^3$ may, for example, be written as

$$\begin{aligned} {}_D t_{ij}^3 = \frac{1}{2} D_{ijkl} [\bar{u}_{m,k} \bar{u}_{m,n} \bar{v}_{n,l} + \bar{u}_{m,l} \bar{u}_{m,n} \bar{v}_{n,k}] + D_{ijkm}^1 \bar{u}_{m,k} + D_{ijml}^2 \bar{u}_{m,l} + D_{ijmn}^3 \bar{u}_{m,n} \\ + D_{ijnl}^4 \bar{v}_{n,l} + D_{ijkn}^5 \bar{v}_{n,k} + D_{ij}^6, \end{aligned} \tag{3.22}$$

where

$$D_{ijkm}^1 = \frac{1}{2} D_{ijkl} \langle \Phi u_{m,n}' v_{n,l}' \rangle, \tag{3.23}$$

$$D_{ijml}^2 = \frac{1}{2} D_{ijkl} \langle \Phi u_{m,n}' v_{n,k}' \rangle, \tag{3.24}$$

$$D_{ijmn}^3 = \frac{1}{2} D_{ijkl} [\langle \Phi u_{m,k}' v_{n,l}' \rangle + \langle \Phi u_{m,l}' v_{n,k}' \rangle], \tag{3.25}$$

$$D_{ijnl}^4 = \frac{1}{2} D_{ijkl} \langle \Phi u_{m,k}' u_{m,n}' \rangle, \tag{3.26}$$

$$D_{ijkn}^5 = \frac{1}{2} D_{ijkl} \langle \Phi u_{m,l}' u_{m,n}' \rangle, \tag{3.27}$$

$$D_{ij}^6 = \frac{1}{2} D_{ijkl} [\langle \Phi u_{m,k}' u_{m,n}' v_{n,l}' \rangle + \langle \Phi u_{m,l}' u_{m,n}' v_{n,k}' \rangle]. \tag{3.28}$$

Thus the linear constitutive equations are obtained, involving material tensors dependent on u_i', v_i'; they may represent the "spectral fluctuations" of the measurements performed.

Once the fields u_i', v_i' are known, the filtered "measurement" model is closed, since now only the filtered fields $\bar{u}_m$, $\bar{v}_m$ (or their derivatives) appear in the equations. Material tensors (3.23)–(3.28) and similar ones appearing in ${}_s t_{ij}$ and ${}_D t_{ij}^1$, ${}_D t_{ij}^2$ determine the properties of visco-elastic medium after filtration; its spectral anisotropy and nonhomogeneity appears, what is an evident effect of nonlinearity of the unfiltered model: in a nonlinear model spectral separation does not occur and the phenomena from without the spectral scale (fluctuations) affect the phenomena observed in the spectrum interval considered. This property may be used to test the models and evaluate the nonlinearity effects. To this end, process simulation in different spectrum intervals and comparative analysis of the results should be performed. This makes it possible to determine the spectral ranges of various phenomena what is of great importance for evaluating the levels of dissipation in material media.

4. Concept of closure in nonlinear filtered models

Several concepts of various validity ranges may be proposed. A frequently applied procedure consists in constructing parametrizations of processes running outside the spectral band considered but affecting the processes occurring in the investigated interval. To this end the knowledge of microphysical phenomena is used, as well as the experience and the asymptotic approximations characterizing certain classes of processes. Such approach is usually applied in gas dynamics and meteorology, characterized by a wide spectral scale of physical phenomena.

According to the most general procedure, a model for the primed values is constructed. This method enables the simultaneous recursive search for both $\bar{u}_m$ and u'_m. In addition, the models for u'_m may considerably be simplified, depending on the class of the processes considered.

Making use of the definition of u'_m, we obtain the equation

$$(4.1)\qquad -\varrho\frac{\partial^2 u'_i}{\partial t^2}+({}_s t_{ij}+{}_D t_{ij}-\langle\Phi_s t_{ij}\rangle-\langle\Phi_D t_{ij}\rangle)_{,j}+\varrho f'_i=0.$$

Stress differences ${}_s t_{ij}-\langle\Phi_s t_{ij}\rangle$, ${}_D t_{ij}-\langle\Phi_D t_{ij}\rangle$ appearing in this equation may be represented in the form (cf. Sect. 3)

$$(4.2)\qquad {}_s t_{ij}-\langle\Phi_s t_{ij}\rangle = C_{ijkl}\,\varepsilon'_{kl}-C^1_{ijml}u'_{m,l}-C^2_{ijkm}u'_{m,k}-\frac{1}{2}C_{ijkl}u'_{m,k}u'_{m,l}+\frac{1}{2}C_{ijkl}\langle\Phi u'_{m,k}u'_{m,l}\rangle,$$

where

$$(4.3)\qquad C^1_{ijml}=\frac{1}{2}C_{ijkl}\bar{u}_{m,k},$$

$$(4.4)\qquad C^2_{ijkm}=\frac{1}{2}C_{ijkl}\bar{u}_{m,l},$$

$$(4.5)\qquad {}_D t^1_{ij}-\langle\Phi_D t^1_{ij}\rangle = D_{ijkl}\,d'_{kl},$$

$$(4.6)\qquad {}_D t^2_{ij}-\langle\Phi_D t^2_{ij}\rangle = -D_{ijkl}[\varepsilon'_{kn}v'_{n,l}+\varepsilon'_{ln}v'_{n,k}+\bar{\varepsilon}_{kn}v'_{n,l}+\bar{\varepsilon}_{ln}v'_{n,k}+\varepsilon'_{kn}\bar{v}_{n,l}+\varepsilon'_{ln}\bar{v}_{n,k}]+D_{ijkl}[\langle\Phi\varepsilon'_{kn}v'_{n,l}\rangle+\langle\Phi\,\varepsilon'_{ln}v'_{n,k}\rangle]$$

or, using the notations

$$(4.7)\qquad D^1_{ijnl}=D_{ijkl}\,\bar{\varepsilon}_{kn},$$

$$(4.8)\qquad D^2_{ijkn}=D_{ijkl}\,\bar{v}_{n,l},$$

$$(4.9)\qquad D^3_{ijkn}=D_{ijkl}\,\bar{\varepsilon}_{ln},$$

$$(4.10)\qquad D^4_{ijnl}=D_{ijkl}\,\bar{v}_{n,k}$$

in the form

$$(4.11)\qquad {}_D t^2_{ij}-\langle\Phi_D t^2_{ij}\rangle = -D_{ijkl}[\varepsilon'_{kn}v'_{n,l}+\varepsilon'_{ln}v'_{n,k}]-D^1_{ijnl}v'_{n,l}-D^2_{ijkn}\,\varepsilon'_{kn}-D^3_{ijkn}v'_{n,k}-D^4_{ijnl}\,\varepsilon'_{nl}+D_{ijkl}[\langle\Phi\,\varepsilon'_{kn}v'_{n,l}\rangle+\langle\Phi\,\varepsilon'_{ln}v'_{n,k}\rangle].$$

The complex relationship for the difference ${}_D t^3_{ij}-\langle\Phi_D t^3_{ij}\rangle$ involving 11 terms is not written here.

Thus we have obtained integro-differential equations for u'_i and v'_i, with material tensors being this time functions of the filtered values $\bar{u}_i$, $\bar{v}_i$. In the most general case, the equations should be solved jointly for $\bar{u}_i$ and u'_i according to the following recursive algorithm:

1) values of $\bar{u}_m(0)$, $u'_m(0)$ are determined from the initial data;
2) convolutions of $\langle \Phi \ldots \rangle$ with magnitudes u'_i, v'_i are calculated;
3) evaluated are: the values of $\bar{u}_i(\Delta t)$,
4) convolutions of $\langle \Phi \ldots \rangle$ with $\bar{u}_n$ and $\bar{v}_i$,
5) and the field $u'_i(\Delta t)$
6) calculations should be continued.

It should be stressed that the problem posed above is very complicated in spite of the sequential linearisation of $\bar{u}_i$ introduced, so that various possible simplifications would be of primary importance.

First, if a weak spectral "exchange" between the intervals takes place, values of u'_i vary but little in time and may be considered as being almost constant in sufficiently large time intervals; it will then be possible to evaluate them only on the basis of the initial data.

Another simplification follows from assuming u'_i to be negligibly small in comparison with $v'_i = \partial_t u'_i$ (fast variation of fluctuations in time). In such a case equation for v'_i takes the comparatively simple form

$$(4.12)\qquad -\varrho \frac{\partial v'_i}{\partial t} + D_{ijkl} d'_{kl,j} - D_{ijkl}[\varepsilon'_{kn}\bar{v}_{n,l} + \bar{\varepsilon}_{ln} v'_{n,k}]_{,j} + \frac{1}{2} D_{ijkl}[\bar{u}_{m,k}\bar{u}_{m,n} v'_{n,l} + \bar{u}_{m,l}\bar{u}_{m,n} v'_{n,k}]_{,j} + \varrho f'_i = 0.$$

This is a linear equation in v'_i with "material constants" depending on $\bar{u}_i$. Introduction of additional simplifications may be the subject of separate considerations.

5. Conclusions

The following general conclusions may be drawn from the results obtained thus far:

1) The filtration procedure has a strict interpretation in the linear case only, when spectral separation occurs, enabling the determination of effective operators.

2) In case of nonlinear theories spectral separation is, in general, impossible. However, this property may be used in the analysis of mutual interaction of effects corresponding to different spectral intervals.

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