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Resonant gas oscillations in open pipes(*)

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THIS CONTRIBUTION is concerned with oscillations of gas in a pipe in which a piston executes harmonic oscillations at one end, whereas the other end is open to the atmosphere. Emphasis is put on a range of frequencies near resonance when the wavelength is about four times the length of the pipe. Under those circumstances work is done by the piston on the gas. For a quasi-steady flow this work must be dissipated somehow. Dissipative mechanisms are: viscous and thermal dissipation in boundary layers, acoustic radiation from the open end and vortex shedding from the mouth of the pipe. The relative importance of these will be discussed for pipes with sharp edges and for pipes with a smoothly rounded-off mouth. The discussion is based on both experimental and theoretical results obtained in our laboratory.

Praca dotyczy drgań gazu w rurze, w której tłok wykonuje drgania harmoniczne w jednym końcu, podczas gdy drugi koniec rury jest otwarty. Nacisk położono na zakres częstości rezonansowej w przypadku kiedy długość fali równa jest około czterem długościom rury. W procesach quasi-statycznych praca wykonywana w gazie przez tłok musi być w pewien sposób rozpraszana. Jako mechanizmy dysypacyjne przyjęto: lepką i termiczną dysypacje w warstwach przyściennych, promieniowanie akustyczne otwartego końca oraz wirowy wypływ z końca rury. Porównano względny wpływ tych czynników dla rur z ostrymi oraz zaokrąglonymi krawędziami. Zbadano wyniki teoretyczne i doświadczalne otrzymane w laboratorium.

В работе рассматриваются осцилляции газа в трубе с поршнем на конце совершающим гармонические колебания; второй конец трубы открыт. Внимание уделяется диапазону околорезонансовых частот, когда длина волны равна около четырех длин трубы. При этих условиях поршень проделывает работу над газом. При квазистационарном процессе эта работа должна каким-то образом рассеиваться. Основные диссипативные механизмы следующие: вязкая и термическая диссипация в пограничном слое, акустическое излучение и вихревое истечение через открытый конец. Относительный вклад этих механизмов рассматривается для труб с острой кромкой и с закругленными краями. Рассуждения основаны как на теоретических так и экспериментальных результатах полученных в лаборатории авторов.

1. Introduction

WE CONSIDER in this paper oscillations of air in pipes open to the atmosphere at one end. At the other end a piston executes harmonic oscillations of the form

$$x = \delta \cos \Omega t, \tag{1.1}$$

δ being the amplitude and Ω the angular frequency of the motion. For small δ, the amplitude of the oscillating air velocity will be small too, that is to say of the order δ. Near the resonance frequencies, the lowest one of which is given by

$$\Omega_0 = \frac{\pi a_0}{2L}, \tag{1.2}$$

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a_0 being the velocity of sound and L the length of the pipe, the amplitude is of the order δ^N, $N < 1$, for not too small δ. The phenomena are nonlinear. Since the pioneer work by LETTAU (1939) much has been done to determine the resonant motion in the pipe for given a_0, L, R and δ, where in addition to the quantities already mentioned R is the radius of the pipe. This problem is still unsolved. Our understanding of the phenomena under resonance in a closed pipe is, due to the work by CHU and YING (1963), CHESTER (1964) and others, fairly complete. The open pipe case is much more difficult due to the complicated interaction of the flow in the pipe with the outside atmosphere. This interaction involves radiation of acoustic energy and convection of energy and momentum during in- and outflow. Inside the pipe the flow can be accurately described by a one-dimensional, unsteady and inviscid gas flow separated from the walls by thin boundary layers of the Stokes type.

The main problem is to formulate a boundary condition for this one-dimensonal gas flow at the open end. Two proposals have been made. Although motivated in different ways, both SEYMOUR and MORTELL (1973) and JIMENEZ (1973) assumed between the pressure, relative to the atmospheric pressure p_0, and the velocity u_e at the exit, a relation of the kind

$$p_e - p_0 = j u_e. \tag{1.3}$$

The parameter j is zero in the completely linear case and, therefore, in the nonlinear case small. Experiments by STURTEVANT (1974) and VAN WIJNGAARDEN and WORMGOOR (1974) give little or no support to Eq. (1.3) neither as to the local behaviour of p and u, nor as to the predicted peak values of p_e and u_e at the exit.

The other suggested boundary condition (VAN WIJNGAARDEN 1968) is

$$(4.1) \qquad \begin{aligned} p_e - p_0 &= c\frac{\partial u_e}{\partial t} + \varrho u_e^2, \quad && u_e > 0. \\ p_e - p_0 &= 0, && u_e < 0. \end{aligned}$$

The physical picture behind Eq. (1.4) is that during inflow it is as if a sink were located in the exit of the pipe. Conservation of momentum of the fluid within the surface Σ in Fig. 1 then leads to the first line in Eq. (1.4). Here the term $c\partial u_e/\partial t$ represents the pressure that is needed to increase the kinetic energy within the surface. This pressure is proportional to

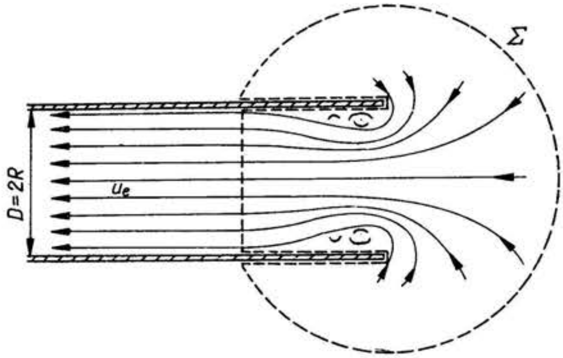

FIG. 1. Inflow in a pipe with a sharp edge. There is separation at the edge. The first line in Eq. (1.4) results from a momentum balance of the fluid within Σ.

$\partial u_e/\partial t$, so c is a constant. During outflow the air is assumed to issue in the form of a jet, at ambient pressure.

Experiments (VAN WIJNGAARDEN (1968)) showed that a theory using Eq. (1.4) fairly well predicts the peak value $\hat{p}_p - p_0$ of the pressure p_p at the piston or $\hat{u}_e$, but that (VAN WIJNGAARDEN and WORMGOOR (1974)) the behaviour of the pressure in the exit is not like that assumed in Eq. (1.4). Very recently KELLER (1977) used the computational and experimental results by STURTEVANT (1974) to compare the relative validity of Eqs. (1.3) and (1.4). His conclusion is that although Eq. (1.4) gives results which agree much better with observation than Eq. (1.3), further improvement is needed because the gap between the theory and observation is still too wide. In the work of VAN WIJNGAARDEN and WORMGOOR (1974) it is suggested that the deviation of the observed flow from Eq. (1.4) is for a large part due to the formation at inflow and shedding at outflow of ring vortices. These result from boundary layer separation at the sharp edge during inflow. It may be expected therefore that Eq. (1.4) holds good for round edges when only jet formation occurs and there is no separation at inflow. An experimental study was therefore carried out on oscillations in pipes with round edges.

2. Resonant oscillations in pipes with a round edge

In order to assess the importance of boundary layer dissipation, it was anticipated that at low enough values of δ/R no jet formation takes place and that boundary layer dissipation together with acoustic radiation completely control the flow. More specifically this means that for low δ/R the work done by the piston (we recall that at resonance the pressure at the piston is in phase with the piston velocity) is radiated away as acoustic energy and dissipated by viscosity and thermal conduction in the boundary layer at the wall. The former can be obtained from the results of LEVINE and SCHWINGER (1948), the latter from a straightforward calculation based on the velocity and temperature distribution in the Stokes' boundary layer separating the wall from a linear acoustic standing-wave in the pipe (see e.g. TEMKIN 1968).

The result is presented here for the impedance $(\hat{p}_p - p_0)/\Omega\delta$ at the piston, rendered dimensionless with $(\gamma p_0)/a_0$, γ being the ratio of specific heats,

$$Q = \frac{(\hat{p}_p - p_0)a_0}{\gamma p_0 \Omega\delta} = \left\{\left(\frac{\pi R}{4L}\right)^2 + \left(\frac{L}{R}\right)^{\frac{1}{2}}\left(\frac{\pi\nu}{4a_0 R}\right)^{\frac{1}{2}}\left(1+\frac{\gamma-1}{\mathrm{Pr}^{\frac{1}{2}}}\right)\right\}^{-1}. \tag{2.1}$$

In this expression ν is the kinematic viscosity of the employed gas and Pr its Prandtl number. If the oscillation were an acoustic standing wave, Q would be equal to $\hat{u}_e/\Omega\delta$.

The first term which stems from radiation becomes rapidly small with respect to the second one with increasing L/R. In Fig. 2 Q obtained from experiments is plotted against $\delta/D(D = 2R)$ for various pipe lengths.

The solid lines on the left represent the value for Q given for each length by Eq. (2.1). The difference between the measured values and these solid lines is about 5%.

We shall return to this in Sect. 3.

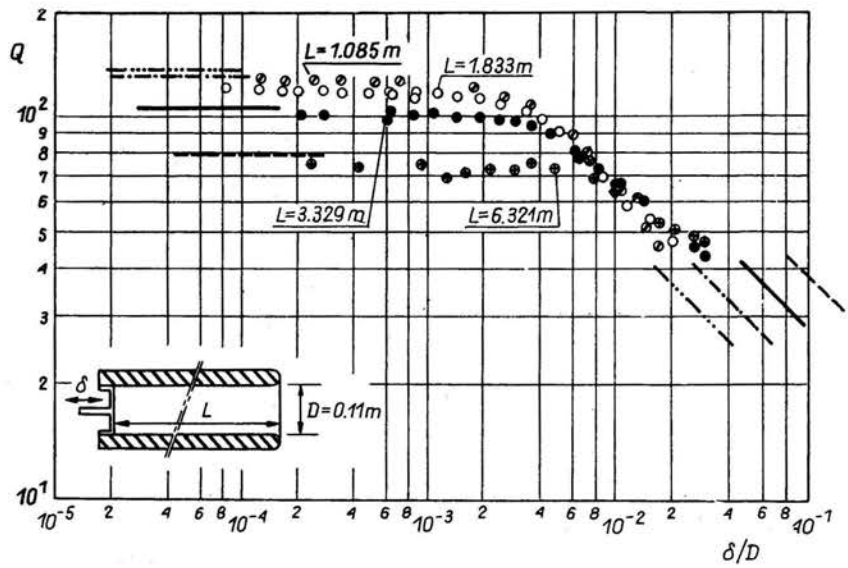

FIG. 2. The quantity $Q = \dfrac{(\hat{p}_p - p_0)a_0}{\gamma p_0 \Omega \delta}$ as a function of δ/D for pipes with a round edge.

Theory	Exp.	
—...—..—	⊘ ⊘ ⊘	$L = 1.085$ m,
—.—.—	○ ○ ○	$L = 1.833$ m,
———	● ● ●	$L = 3.329$ m,
--------	⊕ ⊕ ⊕	$L = 6.321$ m.

Whereas up to $\delta/D \sim 10^{-2}$ the phenomena are linear, for larger values Q depends on δ, as follows from Fig. 2. Asymptotically one expects that the velocity in the pipe becomes so large that boundary layer dissipation is negligible and the magnitude of the signal in the pipe is completely dependent on the conditions at the exit. For round edges jet formation can be expected at outflow and smooth flow without boundary layer separation at inflow.

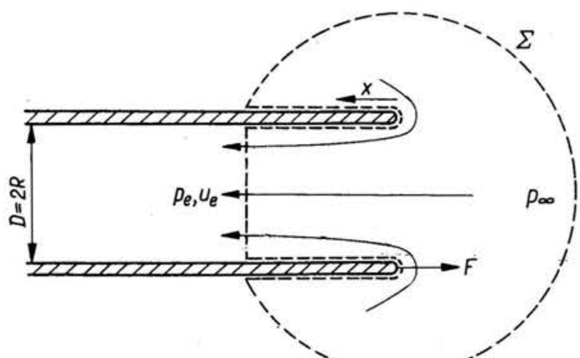

FIG. 3. Inflow in a pipe with a round edge. There is no separation at the edge. A momentum balance for the fluid enclosed in Σ gives the first line in Eq. (2.3).

The conditions (1.4) would then be applicable. However, when the flow around the edge is smooth, there is a net suction force at the edge, indicated with F in Fig. 3. The magnitude of this force is

$$F = -\frac{1}{2}\varrho u_e^2 \pi R^2, \tag{2.2}$$

which follows from making up, e.g. for steady flow, the balance of momentum for the material within the surface Σ in Fig. 3 and comparing the results with the outcome of Bernoulli's Theorem,

$$p_\infty = p_e + \frac{1}{2}\varrho u_e^2.$$

With no separation at inflow we should have instead of Eq. (1.4)

$$\begin{aligned} p_e - p_0 &= c\frac{\partial u_e}{\partial t} + \frac{1}{2}\varrho u_e^2, && u_e > 0, \\ p_e - p_0 &= 0, && u_e < 0. \end{aligned} \tag{2.3}$$

In VAN WIJNGAARDEN (1968) it was calculated that under resonance, and with Eq. (1.4), the pressure p_p at the piston is

$$\frac{p_p - p_0}{p_0} = \gamma\left(\frac{\delta}{L}\right)^{\frac{1}{2}}\pi|\sin\Omega_0 t|^{\frac{1}{2}}\operatorname{sgn}\Omega_0 t, \tag{2.4}$$

neglecting boundary layer dissipation. For round edges we should have Eq. (2.3) instead of Eq. (1.4). (the lack of a factor 1/2 in Eq. (1.4) was noticed also by KELLER (1977)). The result is that the right hand side of Eq. (2.4) becomes larger by a factor $2^{1/2}$ giving for the peak value

$$\frac{\hat{p}_p - p_0}{p_0} = \gamma\pi\left(\frac{2\delta}{L}\right)^{\frac{1}{2}}, \tag{2.5}$$

or

$$Q^2 = \frac{4a_0}{\Omega D}\frac{D}{\delta}. \tag{2.6}$$

Equation (2.6) is plotted for resonant frequencies Ω_0 in the various pipes in Fig. 2. In all cases, but for the longest pipe ($L = 6.32$ m), the observed values approach asymptotically the lines representing Eq. (2.6). In the case of the pipe of the length $L = 6.32$ m the measurements should extend to larger values of δ/D than given in the figure so as to judge whether eventually the line representing Eq. (2.6) is approached.

3. Dissipation at round edge

To account for the small but systematic difference between the calculated and measured values for low values of δ/D, we studied the additional dissipation produced in the flow around the edge. While a detailed calculation of this dissipation will be given in the junior author's forthcoming thesis, an outline for a fairly accurate approach will be given here. The calculations was made for the case of zero wall thickness, but it can be expected that it gives a good approximation in the case the wall thickness is small as compared with the tube diameter. Additional dissipation occurs because of the high velocities in the flow near the edge.

The basis for the calculation are the unsteady two-dimensional boundary layer equations with the convective terms neglected. This is justified by noting that δ/D is a small quantity. This means that the convective acceleration term $u\dfrac{\partial u}{\partial x}$ in the equation of motion is neglibly small with respect to $\partial u/\partial t$.

The best way to calculate the vorticity (the square of which is the viscous dissipation) is to employ parabolic coordinates ξ and η, see Fig. 4.

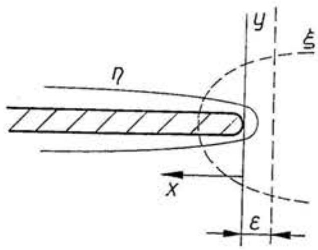

FIG. 4. Parabolic coordinates in a boundary layer along the edge. The domain of integration for the calculation of the dissipation is bounded by the vertical broken line.

The inviscid flow at the outer edge of the boundary layer has the form

$$u = u_e c\left(\frac{R}{x}\right)^{\frac{1}{2}} \cos\Omega t, \tag{3.1}$$

where c is a constant. The problem is to determine from the three-dimensional flow as in Fig. 3 the constant c in the effectively two-dimensional flow (3.1) around the edge.

We determined c as follows: according to the potential flow theory the suction force exerted in the velocity field whose potential is

$$\begin{aligned} w &= 2u_e c(Rz)^{\frac{1}{2}} \cos\Omega t, \\ z &= x+iy, \end{aligned} \tag{3.2}$$

and which gives the flow (3.1) along the edges is, at $t = 0$,

$$F = -2\pi R^2 c^2 u_e^2.$$

On the other hand we know that F is also given by Eq. (2.2), from which it follows that $c = (4\pi)^{-\frac{1}{2}}$, whence

$$u = \frac{1}{2} u_e \left(\frac{R}{\pi x}\right)^{\frac{1}{2}} \cos\Omega t. \tag{3.3}$$

In calculating the dissipation in a boundary layer having Eq. (3.3) as the inviscid outer flow, an improper integral can be avoided by extending the integration a little bit in the right half of the x, y plane or, in other words, by integrating ξ from 0 to ∞ and η from 0 to $(\xi^2+\varepsilon^2)^{\frac{1}{2}}$ (see Fig. 4) in the ξ, η plane.

The result of the viscous dissipation at the edge, ϕ_{edge}, relative to the dissipation in the wall boundary layer, ϕ_{wall}, is found to be

$$\frac{\phi_{\text{edge}}}{\phi_{\text{wall}}} = \frac{R\log\dfrac{a_0 R^2}{8\pi\nu L}}{2L\pi\left(1+\dfrac{\gamma-1}{\mathrm{Pr}^{\frac{1}{2}}}\right)}. \tag{3.4}$$

For resonant air oscillations with $R = 0.055$ m, as used in the experiments, this is 0.04 at a length of 1 m and 0.02 for a pipe of a 2 m length. This "edge" dissipation therefore partly explains the small difference between calculated and observed values in Fig. 2.

4. Resonant oscillations in open pipes with a sharp edge

In Fig. 5 corresponding results for a sharp edge are given. For comparison the results for a round edge and almost the same value of L are given as well.

It is evident that for a sharp edge there is no linear regime where Q does not depend

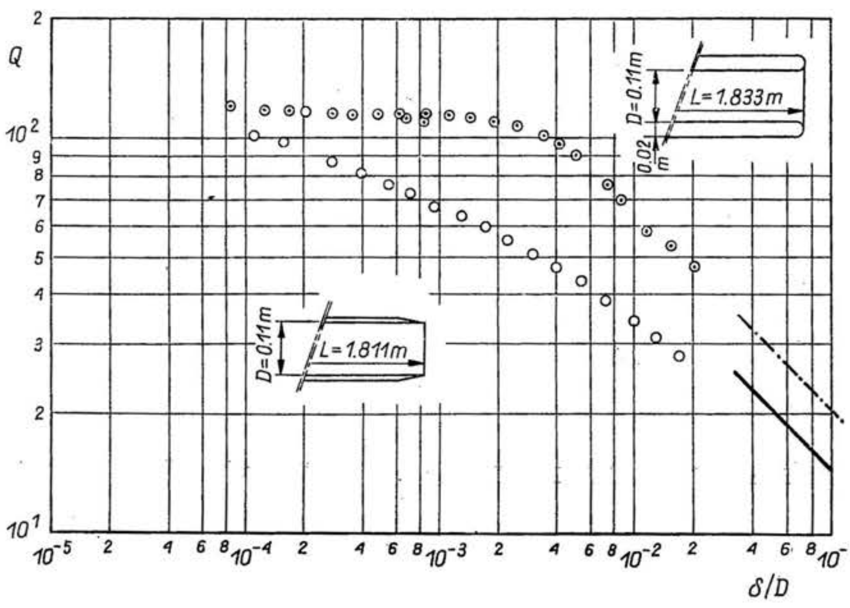

FIG. 5. The quantity $Q = \dfrac{(\hat{p}_p - p_0)a_0}{\gamma p_0 \Omega\delta}$ as a function of δ/D for a sharp edged pipes. For comparison the results for a pipe with a smooth edge are also shown.

on δ/D as it the case of the round edge at low values of δ/D. The solid lines represent Eq. (2.6) for the round edge, and

$$Q^2 = \frac{2a_0}{\Omega D}\frac{D}{\delta} \tag{4.1}$$

for the sharp edge. As explained in Sect. 2 this is based on the result (2.4) obtained by VAN WIJNGAARDEN (1968) using of the boundary condition (2.3). Previous experiments (VAN WIJNGAARDEN 1968, VAN WIJNGAARDEN and WORMGOOR 1974) showed good agreement with Eq. (4.1) but these experiments where for fairly large values of δ/D. It appears now that for smaller δ/D than in those experiments Eq. (4.1) is not satisfied. The reason for this is most likely to be found in the dynamics of vortices in and near the mouth of the pipe. With this in mind we tried to visualize the flow in the mouth during a cycle in a two-dimensional model of the flow around the sharp edge.

This was achieved by heating the airflow, using the density differences which are produced by temperature differences for making the flow visible with the Schlieren method (see Fig. 6a). Vortices were observed very clearly as exemplified in the series of pictures in Fig. 6. This series gives an idea of the formation and shedding of vortices during a full cycle.

To account for the behaviour of Q in Fig. 5 as a function of parameters such as δ/D, the following analytic model is being developed, see Fig. 7.

A two-dimensional flow consists of a uniform flow u_e far down the pipe and two vortices of strength $\Gamma(s, t)$ located in the mouth, s being the distance from the centre of a vortex to the nearest edge. For given u_e, Γ and s the pressure difference between the flow down in the tube and far away from the mouth can be calculated. This provides a boundary condition for the flow in the tube, provided we can express Γ and s in terms of u_e and other overall variables such as diameter D, frequency Ω_0 etc. One relation between Γ, u_e and s is given by the Kutta condition on the sharp edges. The next problem then is to calculate the path $s(t)$ of the vortices in the presence of the plates. The velocity of a votrex ds/dt is given by the velocity induced in its centre. Integration of this relation gives the path $s(t)$ as a function of Γ and u_e. An example of such a calculation has been given recently by SHEFFIELD (1977), but without mean flow u_e and without application of a Kutta condition. Nevertheless, such calculations can be carried out as a rule for cases like those described above.

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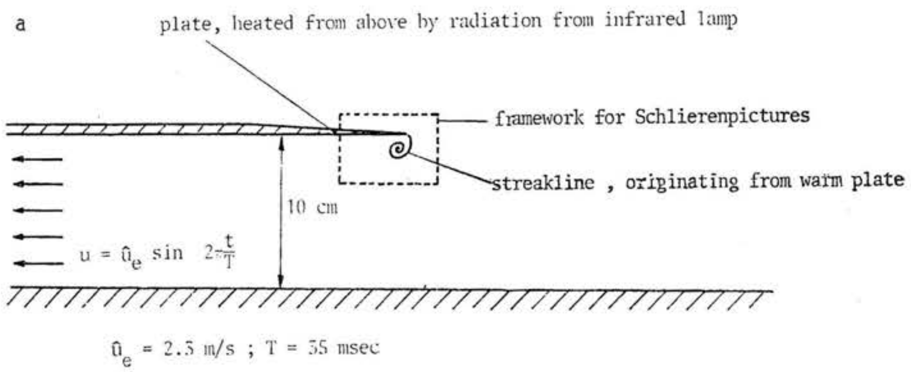

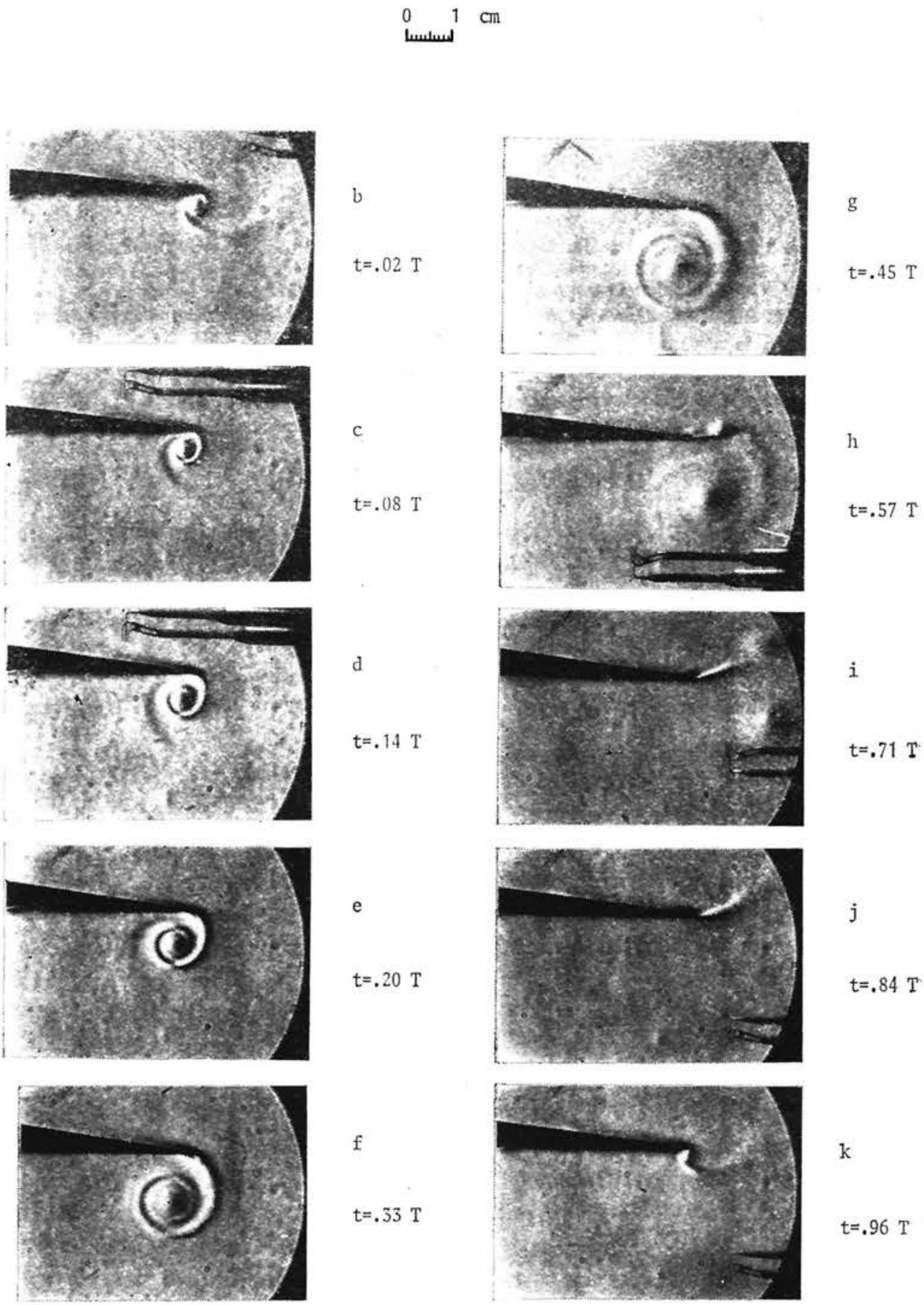

FIG. 6. Schlieren pictures of the formation and shedding of vortices during one cycle. (The hot wire probes that can be seen in some of the pictures are not important for this experiment).

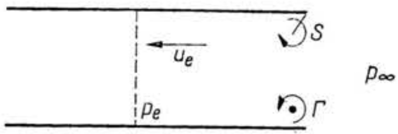

FIG. 7. Flow in the mouth of a pipe with a sharp edge. At the mouth vortices are formed during inflow and shedded during outflow.

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