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BRIEF NOTES

Some experiments in granular flow

K. WIEGHARDT (HAMBURG)

SOME experimental findings are described about forces in granular flow and the pressure in a bin after vibrations. Unfortunately, there are no models providing explanations a) for the passive earth pressure on a wall of finite breadth and b) for the influence of vibrations on the pressure in gravitating granular material.

1. Forces on a body in granular flow

THE FLOW of a granular medium has been studied in a rotating vessel of 1.40 m diameter filled with sand (about 50 cm deep) into which rods with various cross-sections were dipped (at a radius of 50 cm), the forces necessary to hold the rod were measured at various speeds (Annual Review of Fluid Mechanics, 7, 1975). Further tests were then made with natural and synthetic sand, both either dry or with water surplus. Results for a vertical rod with a rectangular cross-section 20 by 3 mm at various angles of incidence are shown in Fig. 1. The drag D and the lift or cross-force L are made dimensionless as simply as possible, viz. by means of the specific bulk weight γ in dense packing and the cube of the immersion depth h. Since the paths of the undisturbed sand particles are not straight lines but circles, the forces on the rod are somewhat different for a positive or negative angle of incidence. The curves in Fig. 1 show the mean values of drag or lift for $\pm\alpha$ vs α. The full lines are for dry sand and the broken lines for sand and water, both at a speed of $v = 0.8$ m/s at the rod and $h = 12$ cm. Here dry sand and the sand/water mixture give almost the same forces.

However, this holds only for this special velocity. Figure 2 gives the drag at $\alpha = 90^0$ or 0^0 for three immersion depths vs speed. For the plate perpendicular to flow direction ($\alpha = 90^0$) in dry sand, at first the drag decreases with speed; as usual, dry friction at rest is higher than in motion. Only at higher speed does the inertia momentum of displaced sand become noticeable giving an additional quadratic term to resistance. The broken lines are for dry synthetic sand (polystyrol-sand) which has almost the same angle of repose as natural sand, yet only a third of its bulk weight γ. The ratio D/γ is almost the same at a greater depth $h = 9$ or 12 cm. Differences at the smallest depth of 6 cm might be due to different "bow waves" in front of the rod and to different through and furrow making after the rod. With the plate in flow direction, when wall friction becomes more important than inner friction, there is no decrease of drag with low speed. Still, grain size here (nat. sand 0.74 mm, synth. sand 1.43 mm) is no longer small compared with the breadth of the plate.

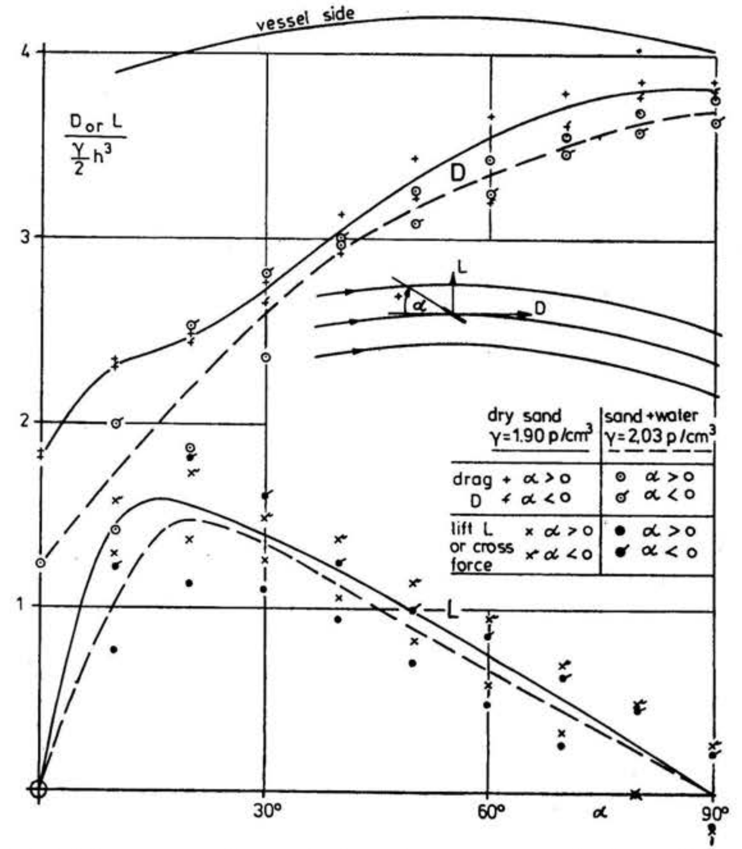

FIG. 1. Forces on a vertical rod with rectangular cross-section 2 cm × 0.3 cm dipped into a rotating sand bed, immersion depth $h = 12$ cm, local sand velocity $v \approx 0.8$ m/s, $v/\sqrt{gh} \approx 0.74$. (Rod at radius 50 cm, vessel radius 70 cm). Dry and wet sand.

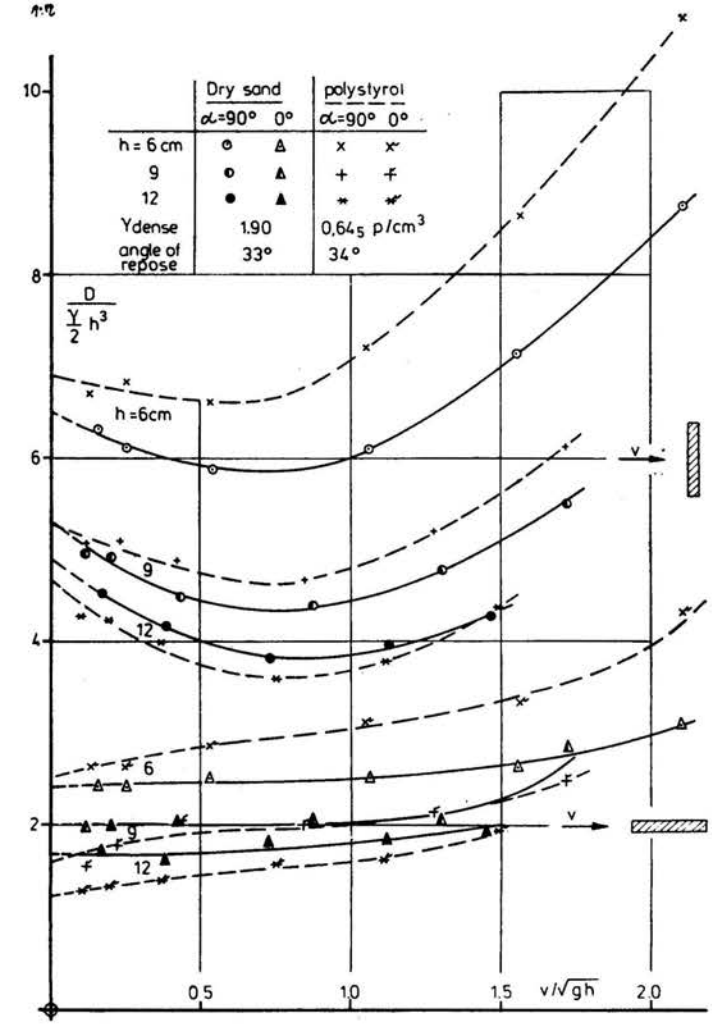

FIG. 2. Natural and synthetic dry sand. Drag at $\alpha = 90°$ and 0° vs. Froude number.

For sand with water surplus, as in Fig. 3, the drag at zero speed is obviously finite again, yet here it increases with speed v or the Froude number $v/\sqrt{gh}$ at first. Surprisingly, at higher speeds the drag decreases. But this is due to a disadvantage of the facility. Since the sand grains are 2.6 times denser than the displaced water, centrifugal forces yield the densest packing at the vessel side and a dilution of the mixture elsewhere, i.e. at the rod the mixture becomes fluidized.

On the other hand, the polystyrol grains are almost "swimming" in water at any speed since their specific weight is only 6% higher than that of water. In this mixture the drag is

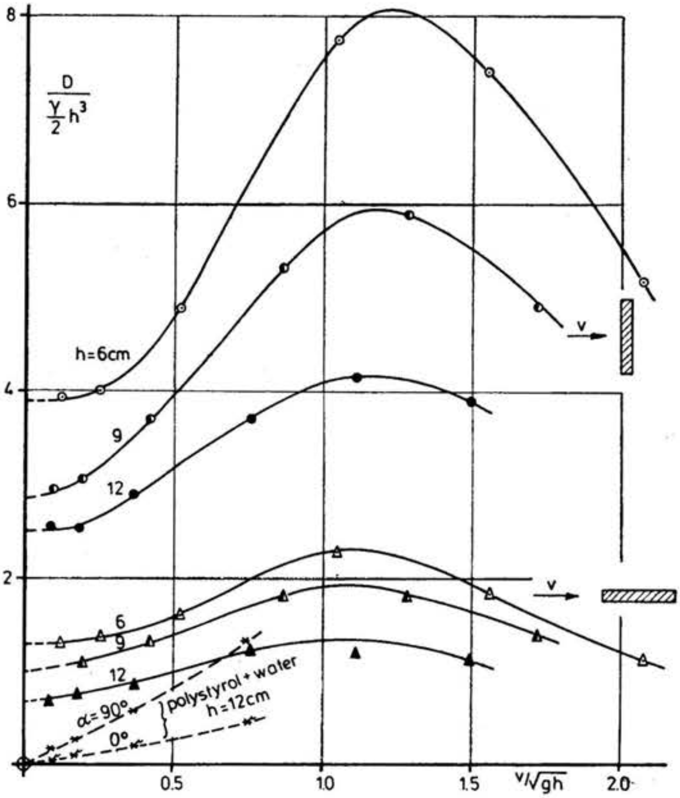

FIG. 3. Sand with water surplus ($\gamma = 2.03$ p/cm³) and polystyrol with water ($\gamma = 1.03$ p/cm³): drag of a vertical rod 2×0.3 cm² with $\alpha = 90°$ or $0°$ (Sand-water mixture at high rotational speed fluidized by centrifugation of sand).

almost proportional to speed (and to immersion depth) as in creeping flow of a Newtonian fluid.

The "glide" ratio, cross-force over drag, varies with the angle of incidence almost in the same manner for all four cases shown in Fig. 4. In particular, for natural sand with or without water, no systematic change is discernible.

To simplify the problem one may extrapolate forces to zero speed $v \to 0$, even if this is — because of the still finite bow wave — not the same limiting case of soil mechanics. Figure 5 shows the dimensionless resultant force ($R = \sqrt{D^2+L^2}$) extrapolated to zero speed vs the ratio of effective breadth over immersion depth for four angles of incidence. Roughly, the points form two single streets, one for the two dry sands and the other one for the sand /water mixture. For the light synthetic sand in water, forces vanish practically with zero speed. On the other hand, natural sand in water still yields finite friction at rest and the force is about half of that in dry sand. Professor Bent Hansen, Lyngby, kindly remarked that in the sand/water mixture only the difference of specific weight for sand and water can give a static earth pressure. That is, instead of the bulk specific weight $(1-\beta)\gamma_s+\beta\gamma_w =$

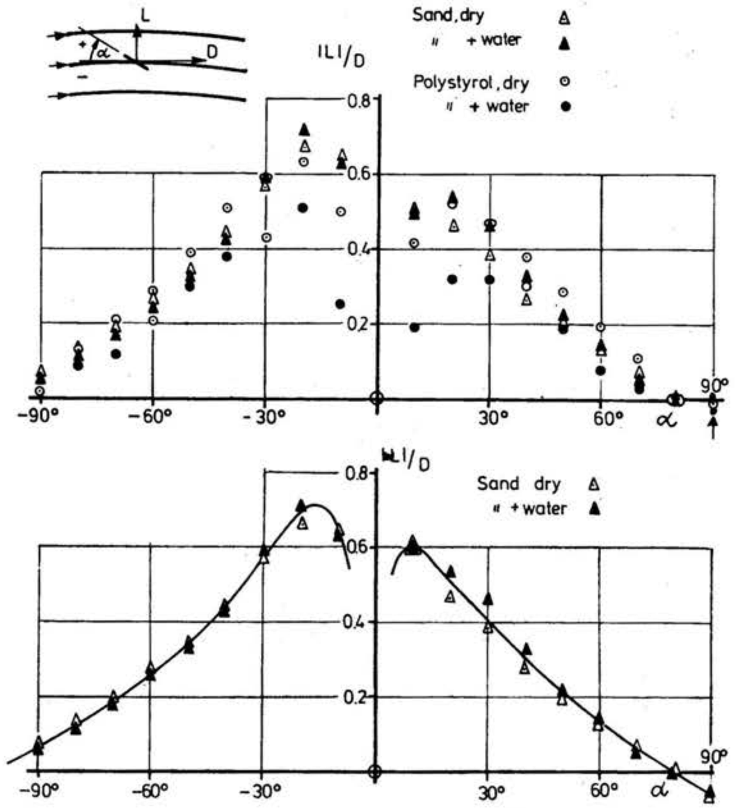

FIG. 4. Ratio of lift or cross force L over drag D for a vertical rod with rectangular cross section 20×3 mm² at immersion depth $h = 12$ cm, mean values at speed $v = 0.2, 0.4, 0.8, 1.6$ m/s.

$= 2.03$ p/cm³, with $\beta =$ porosity, only the difference $(1-\beta)(\gamma_s-\gamma_w) = 2.03-1 = 1.03$ p/cm³ is contributing. In fact, $1.03/2.03 = 0.51$ corresponds well with the factor $6/11 =$ $= 0.55$ in Fig. 5. Just as in triaxial shear tests the inner friction angle ϕ is the same for dry or wet sand, in dense packing for the natural sand it amounts to about 44° and for the synthetic sand to about 39°. Hence, water does not act as a lubricant, apparently here not even for wall friction.

Disregarding the points for $\alpha = 0$ where $b_{eff} = 3$ mm is no longer great compared with grain size, one can approximate test results for dry natural and synthetic sand by

$$R(v\to 0) = 13\frac{\gamma}{2}h^2 b_{eff}+1.1\gamma h^3.$$

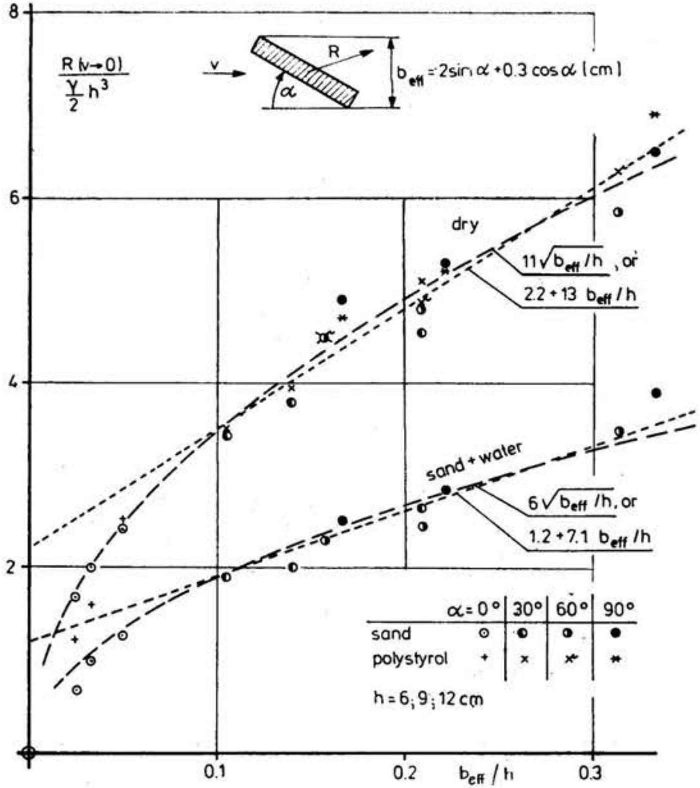

FIG. 5. Resultant force R extrapolated to zero velocity vs. ratio of effective breadth over immersion depth. In sand with water surplus ($\gamma = 2.03$ p/cm³) the passive earth pressure is about 6/11 of that in dry sand ($\gamma = 1.90$ p/cm³).

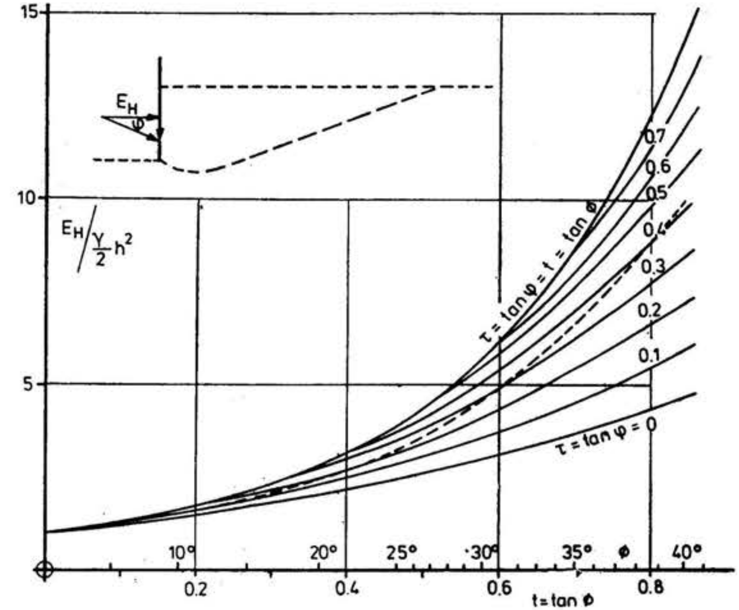

FIG. 6. Horizontal passive soil resistance against a vertical wall: $E_H \Big/ \frac{\gamma}{2} h^2$, γ spec. weight, h depth of wall; $t = \tan \phi$ inner friction, $\tau = \tan \varphi$ wall friction, $\tau = \frac{1}{2} t \ldots$; (After V. V. SOKOLOVSKI, Statics of granular media, 1965).

This suggests we interpret the $13\frac{\gamma}{2}h^2$ in the first term as passive earth pressure for the plane stress state and the second term as a correlation for finite breadth. A theoretical solution seems to exist only for the wall with infinite breadth as in Fig. 6. If one assumes the wall friction ($\tan\varphi$) to be at least half of the inner friction, the above interpretation seems to be roughly confirmed. For the correction term no solution could be found. Obviously, the main difficulty here is to find a kinematically possible, three-dimensional form for the sidewards gliding surfaces.

2. Pressure in a bin after vibrations

For the plane stress state between two parallel vertical walls an exact solution has been given by SAVAGE and YONG in Int. J. Mech. Sciences, 12, 675, 1970. It is reassuring that in

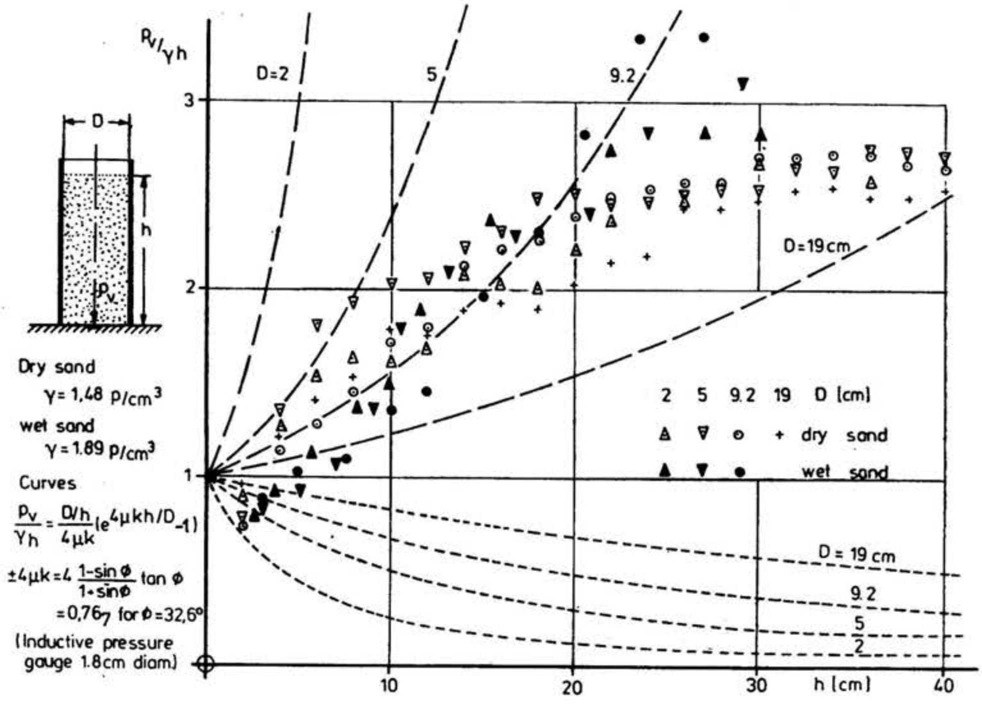

FIG. 7. Maximal vertical pressure p_v in the centre of a cylindrical bin (diam D) filled with sand: after vibrations.

the two examples calculated there the mean vertical bottom pressure deviates only by a few percent from that of H. A. JANSSEN (1895) who assumed constant pressure over the cross-section.

However, vibrations can change the static pressure distribution completely. In a plexiglass cylinder filled with dry or wet sand the bottom pressure was measured by an inductive pressure gauge. When vibrations were introduced into the sand by tapping the cylinder, then the pressure reading jumped up a little and remained at this higher value. The maximal pressure produced by tapping is shown in Fig. 7: p_{vmax} divided by the specific weight γ and the sand height h vs height in various cylinders with the diameter D. Full points are for saturated sand/water mixture.

The lower curves are calculated by the usual assumption that the greatest possible wall shear is directed upwards, as if the sand were just ready to flow out through an aperture in the bottom. However, as long as the sand is completely at rest, one might just as well expect a demon trying to elevate the bottom of the bin, and then a downwards directed shear would be mobilized at the wall, in this case the upper curves would hold. Most of the test points are between these lower and upper boundaries, yet, for the broadest cylinder ($D = 19$ cm) some points are even higher than the corresponding upper curve. Furtheron, it seems as if this maximal pressure — after vibrations had been introduced — depended on sand height only and not on the cylinder diameter; possibly, the grain diameter is a hidden characteristic length.

Of course, large silos without vibrations are still useful because the corn is not crushed near the bottom. Only when the corn is flowing out high pressures have been noticed, perhaps because of the vibrations excited then.

INSTITUT FÜR SCHIFFBAU
HAMBURG, BRD.

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