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Separation of turbulent boundary layer at a curved wall with normal shock

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THE SHOCK-BOUNDARY layer interference very often causes flow separation. The question at which pressure gradient or at what Mach number separation starts to develop is of interest for both theory and applications. Our report will be about a simple criterium that relates the dependency of a turbulent boundary layer to the Mach number and the Reynolds number. The starting point is an analytical solution of the shock boundary layer problem given by BOHNING and ZIEREP [1]. By using a three layer model we get analytical expressions for the pressure and velocity distributions along a curved wall. Employing these results it is easy to find a criterium for vanishing wall shear stress. In general there exists an expansion immediately behind the shock. This after-expansion is dependent on the wall curvature and on the outer flow. It diminishes the tendency towards separation. This effect is bounded by two limits that are given by the conditions for which the boundary layer may separate and for which the boundary layer must separate.

Interferencja uderzeniowej warstwy przyściennej bardzo często powoduje odrywanie się strug gazu. Pytanie, przy jakim gradiencie ciśnienia lub przy jakiej liczbie Reynoldsa rozpoczyna się proces odrywania strug, jest interesujące zarówno z punktu widzenia teorii jak i zastosowań. W niniejszej pracy podano proste kryterium, które wiąże turbulencyjną warstwę przyścienną z liczbą Macha i liczbą Reynoldsa. Punktem wyjścia jest rozwiązanie analityczne problemu uderzeniowej warstwy przyściennej podane przez BOHNINGA i ZIEREPA [1]. Stosując trójwymiarowy model warstwy, otrzymaliśmy analityczne wyrażenia na rozkłady ciśnienia i prędkości wzdłuż zakrzywionej ścianki. Wykorzystując te wyniki łatwo już znaleźć kryterium znikania naprężenia stycznego na ściance. Na ogół ekspansja następuje bezpośrednio za falą uderzeniową. Zależy ona od krzywizny ścianki, przepływu zewnętrznego oraz zmniejsza tendencję do odrywania się strug. Efekt ten ograniczony jest dwoma wartościami granicznymi określonymi odpowiednio przez warunki, przy których warstwa przyścienna może się odrywać i musi się odrywać.

Интерференция ударного пограничного слоя очень часто вызывает отрыв струй газа. Вопрос при каком градиенте давления или при каком числе Рейнольдса начинается процесс отрыва струй, интересен так с точки зрения теории, как и применений. В настоящей работе дается простый критерий, который связывает турбулентный пограничный слой с числом Маха и числом Рейнольдса. Исходной точкой является аналитическое решение проблемы ударного пограничного слоя, приведенное Бонингом и Зипером [1]. Применяя трехмерную модель слоя, получено аналитическое выражение для распределений давления и скоростей вдоль искривленной стенки. Используя эти результаты, легко уже можно найти критерий исчезновения касательного напряжения на стенке. Вообще расширение наступает непосредственно за ударной волной. Оно зависит от кривизны стенки, внешнего течения и уменьшает стремление к отрыву струй. Этот эффект ограничен двумя предельными значениями определенными соответственно через условия, при которых пограничный слой может оторваться и должен оторваться.

THE INTERACTION between shock waves and boundary layers often causes separation. The question as to what pressure gradient (or Mach number) and given Reynolds number separation starts is important both for theory and for applications. Our final result is a diagram that gives the Mach number and Reynolds number dependence for the separation of a compressible turbulent boundary layer due to a normal shock at a curved wall.

The starting point is an analytical solution due to the authors [1] in the domain of the interaction of the shock wave and the boundary layer. Figure 1 shows the model used. The turbulent boundary layer at a curved wall in transsonic slightly supersonic flow (Fig. 1, left) is disturbed by a weak normal shock (Fig 1, middle). With a three-layer model

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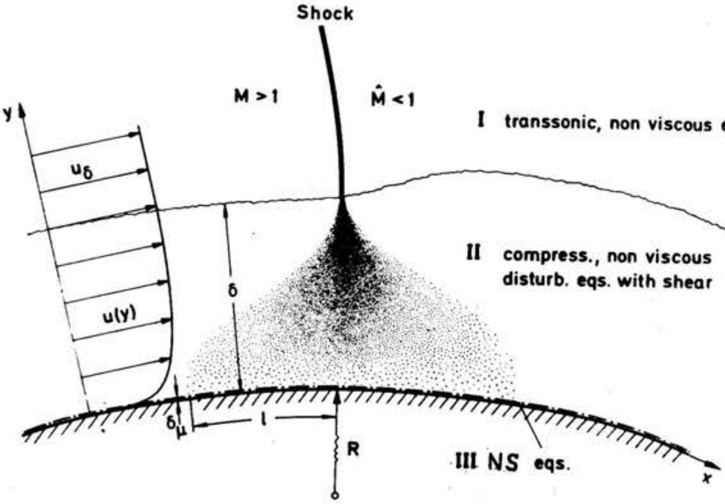

FIG. 1. Three-layer model used for calculation of the normal shock boundary layer interaction.

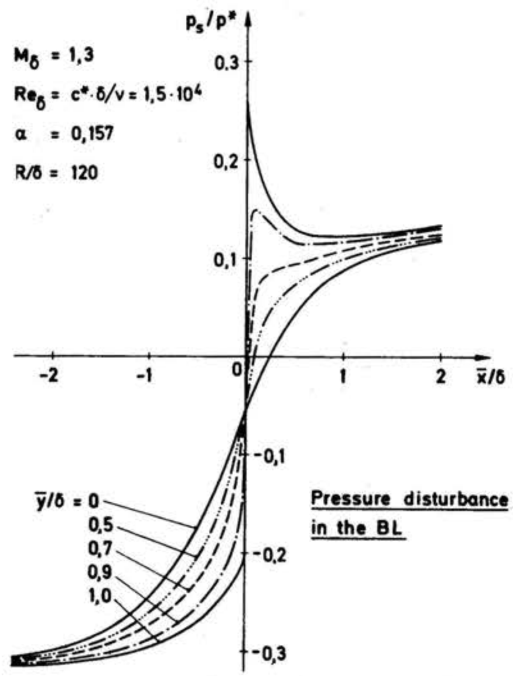

FIG. 2. Pressure distributions in the boundary layer for different wall distances. α is the exponent of the undisturbed velocity profile in the boundary layer.

(Fig. 1, right) — originally due to Lighthill — we get expressions in a closed form for pressure and velocity in the boundary layer. Figure 2 shows the pressure distribution for different wall distances. In front of the shock we get an upstream influence, at the outer edge of the boundary layer we have, in this general case as well, the singular after-expansion found by OSWATITSCH and ZIEREP [2] in the inviscous case. This behaviour was first discussed in the experiments of ACKERET, FELDMANN and ROTT [3].

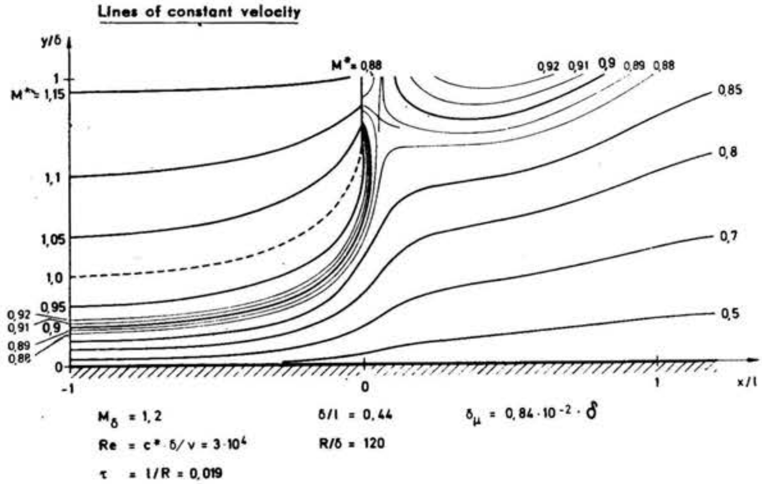

FIG. 3. Lines of constant Mach number in the boundary layer.

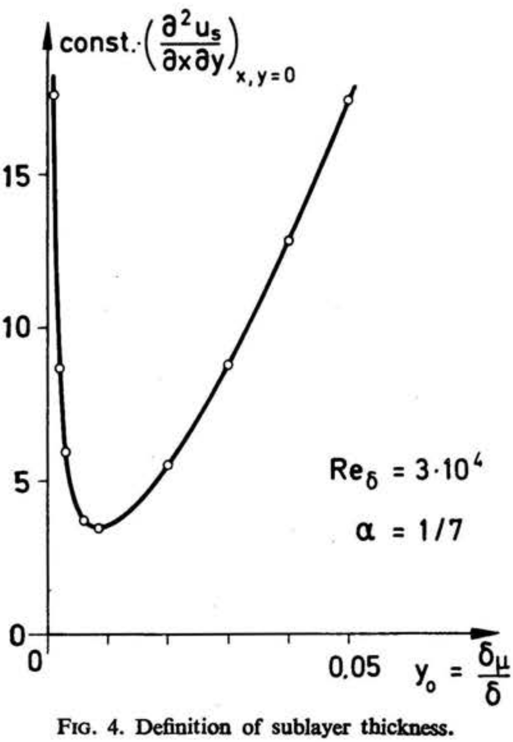

FIG. 4. Definition of sublayer thickness.

The structure of the flow field is given in Fig. 3 by drawing the lines of a constant Mach number. In front of the shock we have the upstream influence ranging about some boundary layer thicknesses. Behind the shock in the neighbourhood of the wall the shear stress diminishes, the boundary layer thickness increases. At the outer edge of the boundary layer we have the after-expansion. Both regimes are separated from one another by a saddle point.

Figure 4 illustrates the definition of the sublayer thickness $\delta_\mu/\delta = y_0$. We calculate a characteristic quantity — for instance $\partial\tau_w/\partial x$ — for different sublayer thicknesses. The abscissa y_0 where $\partial\tau_w/\partial x$ has an extremum is preferred because in the neighbourhood of this value we have no dependence of $\partial\tau_w/\partial x$ from y_0. An increasing Reynolds number leads to a decreasing sublayer thickness (Fig. 5).

With the explicit solution of the authors it is possible to calculate the wall shear stress. Figure 6 shows the local friction coefficient at a constant Mach number and a constant wall curvature for different Reynolds numbers with (— — —) and without (———) after

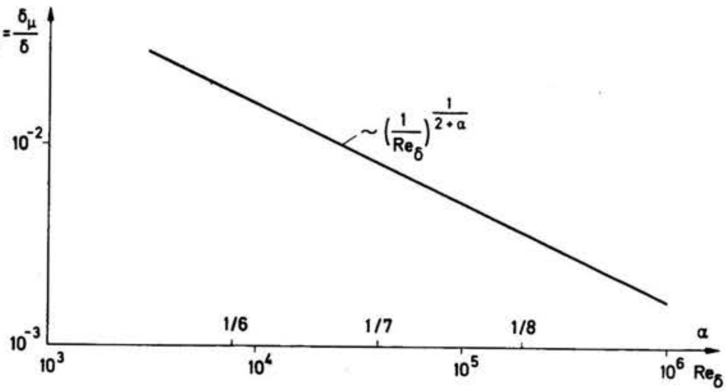

FIG. 5. Reynolds number dependence of sublayer thickness.

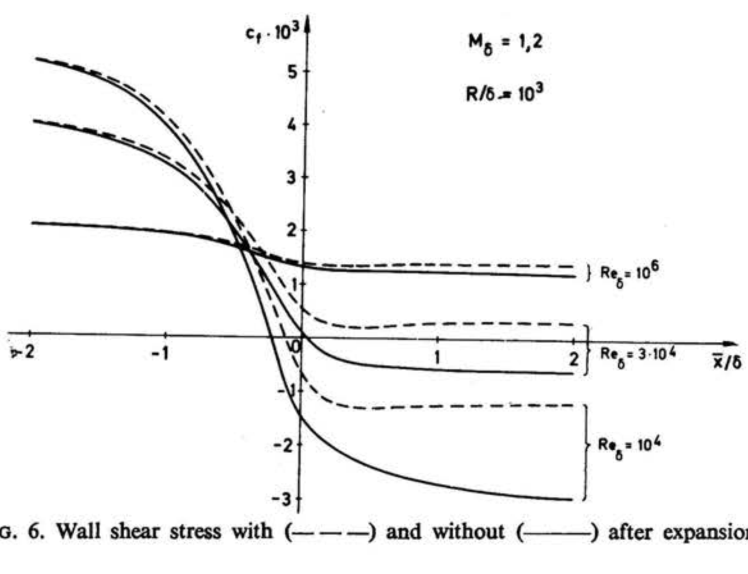

FIG. 6. Wall shear stress with (— — —) and without (———) after expansion.

expansion. We immediately recognize that the after-expansion reduces the tendency for separation. The effect is such as if to reduce the shock strength. The difference may be considerable and can be in the magnitude of 20—30%. Figure 7 gives the final result. For

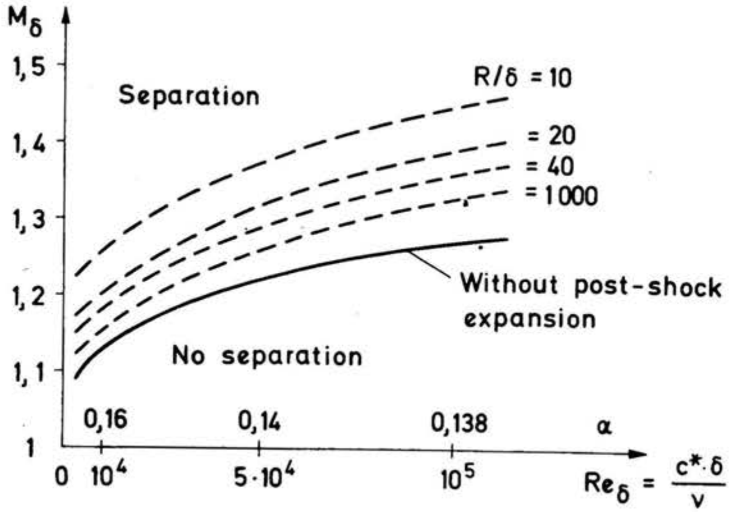

FIG. 7. Mach number versus Reynolds number that lead to separation. Influence of wall curvature.

a given radius of wall curvature R/δ we get the dependency of the Mach number and Reynolds number that leads to separation. Again we get different curves with after-expansion (— — —) and without (———). The interpretation is as follows. Below the solid curve we have no separation at all, above the interrupted curves — belonging to different wall curvatures — the fluid flow must separate. The influence of the wall curvature is such that increasing curvature lifts the interrupted curves, this means that increasing curvature reduces the tendency for separation. We immediately see that the transsonic fluid flow ($M_\delta \leqslant 1.3$) will separate only in the domain of a relatively low Reynolds number and low curvature.

The consequence and others which issue from our results are in good agreement with available measurements.

Two additional remarks are of interest. If we calculate with our explicit solution the boundary layer parameter H_{32} (= ratio of energy displacement thickness to that of momentum) we get in the case of separation 1.65 instead of the value 1.58 known from integral methods of the boundary layer theory. The difference may be looked upon as an error estimation of the integral methods.

The solid curve in Fig. 7 which belongs to the vanishing wall shear stress $\tau_w = 0$ fulfills a simple similarity law. If we apply the Prandtl-Glauert rule to the external flow field, we have

$$\frac{\delta}{l} \cdot \sqrt{M_\delta^2 - 1} = \text{const},$$

δ/l stands for the dimensionless vertical coordinate in the flow field, that means for the thickness parameter of the boundary layer. If we express δ/l by the Reynolds number, we get a simple relation $M_\delta = f(\mathrm{Re}_\delta)$ which approximates extremely well the law for the solid curve in Fig. 7.

References

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