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Conditional correlations and probabilities of temperature fluctuations in a turbulent boundary layer

R. DUMAS, E. ARZOUMANIAN, L. FULACHIER and A. FAVRE
(MARSEILLE)

An improved procedure to investigate the structure of turbulence consists in using a passive scalar contaminant which describes the turbulent field better than a single velocity component. When this contaminant is heat, the sign of the temperature fluctuations reveals the origin of fluid lumps. This indirect approach has been used to study the turbulent structure of the boundary layer. In this respect, space-time correlations and probabilities of temperature fluctuations self-conditioned by their signs have been measured. The present analysis allows to describe the diffusion of both the hot events coming from the wall region and the cold events due to the inrush of the outward fluid.

Udoskonalona procedura badania struktury turbulencji polega na wykorzystaniu biernej skalarnej kontaminanty, która opisuje pole turbulentne lepiej niż pojedyncza składowa prędkości. Gdy tą kontaminantą jest ciepło, znak fluktuacji temperatury wykazuje powstawanie kropel cieczy. To pośrednie podejście zastosowano do eksperymentalnego skonstruowania struktury warstwy przyściennej. W tym celu zmierzono przestrzenno-czasowe korelacje i prawdopodobieństwa fluktuacji temperatur samo-uwarunkowanych przez ich znaki. Niniejsza analiza pozwala na opis dyfuzji obydwu efektów nagrzewania pochodzącego od obszaru ścianek i oziębienia wynikłego wskutek naporu cieczy z zewnątrz.

Усовершенствованная процедура исследования структуры турбулентности заключается в использовании пассивного скалярного контаминанта, который описывает турбулентное поле лучше чем единственная составляющая скорости. Когда этим контаминантом является тепло, знак флуктуации температуры указывает на возникновение капель жидкости. Этот непосредственный подход применен к экспериментальному построению структуры пограничного слоя. С этой целью измерены пространственно-временные корреляции и вероятности флуктуации температур самообусловленные их знаками. Настоящий анализ позволяет описать диффузии обоих эффектов: нагрева, происходящего от области стенок, и охлаждения, возникающего вследствие напора жидкости извне.

1. Introduction

Although the mean properties of the turbulent boundary layer are well known, its mechanism is not well understood, especially concerning the generation of the turbulence at the edge of the viscous sub-layer by the so-called bursting process. Therefore experimental investigations are still to be undertaken. In this regard the structure of the turbulent field has been examined using heat as a passive contaminant. The wall was only slightly heated in order to avoid gravity effects, at least concerning the turbulent field [1]. The temperature can permanently mark the fluid [2, 3]. The sign of the temperature fluctuations indicates the origin of he tturbulent fluid lumps. This allows the efficient use of the conditional sampling techniques according to the sign of the temperature fluctuations.

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In the first part, two point space-time correlations, conditioned by the sign of the temperature fluctuations, are presented at several distances from the wall. When the upstream point is fixed in the buffer-layer these correlations are significant of the behaviour of the coherent fluid lumps coming from the wall. Special attention has been brought on the celerities of the hot fluid lumps which can be related to the ejected process [4].

In the second part, the diffusion of hot fluid lumps from the wall is examined by using conditional space-time probabilities. Reduced contingencies are defined to characterize the degree of statistical linkage. Different threshold levels are considered to distinguish the behaviour of the eddies according to their energy. Agreement is found with a Gaussian process only for the eddies corresponding to small amplitude fluctuations. In the physical coordinates, contingency lines are compared with the most favoured trajectories of ejected eddies visualized by RUNSTADLER and KLINE [4] and a Lagrangian path obtained experimentally by SHLIEN and CORRSIN [5]. The comparison is also applied to a relation proposed by BATCHELOR [6] for the Lagrangian path.

2. Experimental configuration

The turbulent boundary layer is developed on a slightly heated wall. The characteristic parameters at the upstream point are:

outer velocity, $u_e = 12.2\ \mathrm{ms}^{-1}$,

boundary layer thickness, $\delta = 59$ mm (for $\bar{u} = u_e$),

displacement thickness, $\delta_1 = 7.8$ mm,

friction velocity, $u^* = 46\ \mathrm{cm\ s}^{-1}$,

temperature difference between the wall and the outer flow, θ_p and θ_e respectively: $\theta_p - \theta_e \simeq 22$ K.

In these conditions the effects of heating on the turbulent velocity field is not disclosed. However, an effect appears on the mean normal velocity component in the viscous sub-layer [7].

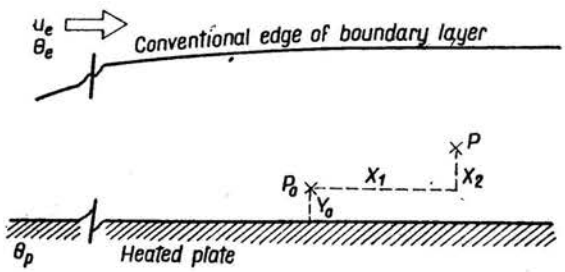

FIG. 1. Experimental configuration.

Figure 1 gives the scheme of the boundary layer and the coordinates concerning the two points considered.

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3. Space-time conditional statistics

The turbulence is considered as a stationary time process. Space-time correlation coefficients of temperature fluctuations $\theta'_0(t)$ and $\theta'(t+\tau)$, corresponding respectively to the upstream fixed point P_0 and to the movable downstream point P, are defined by:

$$r_{\theta_0\theta}(\tau) = \frac{\overline{\theta'_0(t)\theta'(t+\tau)}}{[\overline{\theta'^2_0}\,\overline{\theta'^2}]^{1/2}}, \tag{3.1}$$

where τ is a variable time delay. We also consider the following two point triple coefficient:

$$r_{\theta_0,\theta\theta}(\tau) = \frac{\overline{\theta'_0(t)\,\theta'^2(t+\tau)}}{[\overline{\theta'^2_0}\,\overline{(\theta'^2-\overline{\theta'^2})^2}]^{1/2}}. \tag{3.2}$$

This parameter emphasizes the role played by the large amplitude fluctuations.

In order to identify the origin of the fluid lumps, we introduce the conditional space-time correlations:

$$R_{m,n}(\tau) = \frac{\overline{J(t)\theta'_0(t)\theta'(t+\tau)}}{[\overline{\theta'^2_0}\,\overline{\theta'^2}]^{1/2}}, \tag{3.3}$$

where m, n are the signs + or −. $J(t) = 1$ when $\theta'_0(t)$ and $\theta'(t+\tau)$ have the signs m and n, respectively. $J(t) = 0$ otherwise.

These conditional correlations $R_{m,n}$ are not correlation coefficients but they represent the contribution of different sign events to the total coefficient as it appears in the following relation:

$$r_{\theta_0\theta} = R_{++}+R_{+-}+R_{-+}+R_{--}. \tag{3.4}$$

Similar definitions and relations apply to the triple conditional correlations:

$$R_{m,nn}(\tau) = \frac{\overline{J(t)\,\theta'_0(t)\theta'^2(t+\tau)}}{[\overline{\theta'^2_0}\,\overline{(\theta'^2-\overline{\theta'^2})^2}]^{1/2}}, \tag{3.5}$$

$$r_{\theta_0,\theta\theta}(\tau) = R_{+++}+R_{+--}+R_{-++}+R_{---}. \tag{3.6}$$

Furthermore, in order to determine the statistical linkage we introduce the contingency defined from the conditional probabilities.

For example, in the case of hot fluid events occurring at P_0 and P we consider the contingency φ^{++}:

$$\varphi^{++} = \text{prob}\left(\frac{\theta'_0(t)}{\sigma_0} > h_0 \text{ and } \frac{\theta(t+\tau)}{\sigma} > h\right) - \text{prob}\left(\frac{\theta'_0}{\sigma_0} > h_0\right)\text{prob}\left(\frac{\theta'}{\sigma} > h\right), \tag{3.7}$$

where σ_0, σ are the standard deviations and h_0, h the thresholds.

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More conveniently, in order to obtain a degree of linkage, we defined a reduced contingency:

$$(3.8)\quad \Phi^{++} = \frac{\varphi^{++}}{\left[1-\text{prob}\left(\frac{\theta'}{\sigma} > h\right)\right]\text{prob}\left(\frac{\theta'_0}{\sigma_0} > h_0\right)} = \frac{\text{prob}\left(\frac{\theta'(t+\tau)}{\sigma} > h \text{ if } \frac{\theta'_0(t)}{\sigma_0} > h_0\right) - \text{prob}\left(\frac{\theta'}{\sigma} > h\right)}{1-\text{prob}\left(\frac{\theta'}{\sigma} > h\right)}.$$

For the statistical independence $\varphi^{++} = \Phi^{++} = 0$ and, reciprocally, for complete dependence $\Phi^{++} = 1$.

Similar definitions apply with other signs, but for negative signs we consider for example $\theta'/\sigma < -h$. These probabilities are computed with a numerical data acquisition system. They represent the proportion of the time when the fluctuations $\theta'_0(t)$ and $\theta'(t+\tau)$ fulfill the above conditions. It is obvious that the contingency corresponding to an excess or a lack of temperature is significant of the dispersion of heat by the turbulence.

If we assume Gaussian relationships for the variables $\theta'_0(t)$ and $\theta'(t+\tau)$, we can compute the joint probabilities from the correlation coefficient $r_{\theta_0\theta}(\tau)$ or r; for example [10]:

$$(3.9)\quad \text{prob}\left(\frac{\theta'_0(t)}{\sigma_0} > h_0 \text{ and } \frac{\theta'(t+\tau)}{\sigma} > h\right) = \frac{1}{4}\left(1-\text{erf}(h_0)\right)\left((1-\text{erf}(h)\right) + \frac{r}{2\pi}\left[\exp\left(-\frac{h_0^2+h^2}{2}\right)\right]\sum_{p=0}^{\infty}\alpha_p(h_0)\,\alpha_p(h),$$

$$(3.10)\quad \alpha_p(h) = \frac{1}{((p+1)!)^{1/2}}\, r^{P/2} H_p(h),$$

H_p — Hermite polynomial.

In particular, for thresholds equal to zero the following relations apply:

$$(3.11)\quad \text{prob}\,(\theta'_0(t) > 0 \text{ and } \theta'(t+\tau) > 0) = \text{prob}\,(\theta'(t) < 0 \text{ and } \theta'(t+\tau) < 0) = \frac{1}{4} + \frac{1}{2\pi}\arcsin r$$

and

$$(3.12)\quad \Phi^{++} = \Phi^{--} = \frac{2}{\pi}\arcsin r.$$

One can verify also that for statistical independency the conditional correlations R_{++}, R_{--}, tend to $1/2\,\pi$ and R_{+-}, R_{-+} tend to $-1/2\,\pi$.

4. Experimental results

Figure 2 gives the correlation coefficient and the conditional correlations for two points located on the same mean streamline in the intermittent region. τ^* is a non-dimensional

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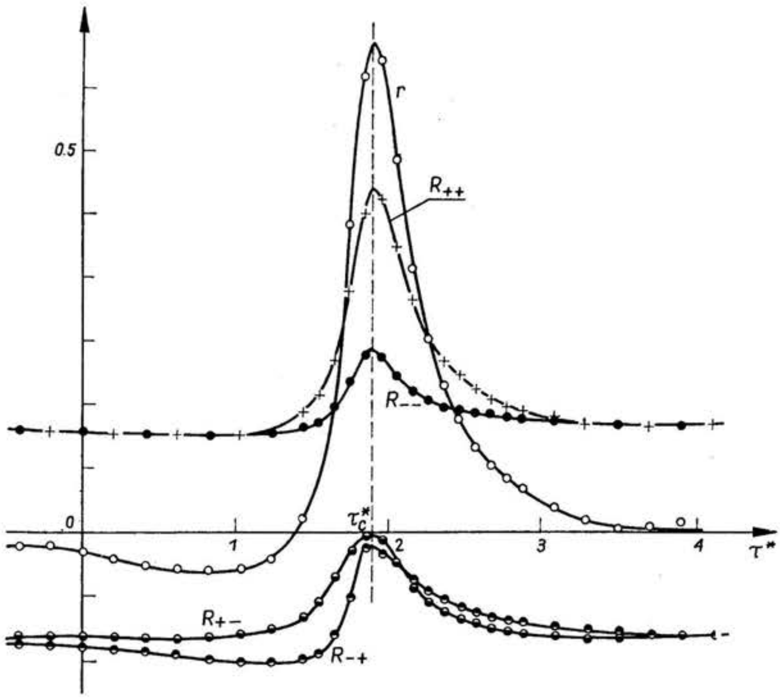

FIG. 2. Conditional space-time correlations in the intermittent region; $y/\delta = 0.814$, $\gamma = 0.45$, $\bar{u}/u_e = 0.991$.

time delay, $\tau^* = \frac{\tau u_e}{\delta}$. τ_c^* is the time corresponding to the local mean velocity $\tau_c^* = (X_1/\delta)(u_e/\bar{u})$. The coefficient r reaches a maximum value at an optimum time delay τ_m which is practically the same as the optimum time of conditional correlations R_{++} and R_{--}. For a time delay very different from the optimum time τ_m, the conditional correlations tend to constant values which are closed to the Gaussian values $\pm 1/2\pi$.

Near the time τ_m the cross sign correlations R_{+-} and R_{-+} are almost null. All these results confirm that the intermittent field pattern is very conservative along the general movement.

Figure 3A shows similar measurements on a mean streamline in the central zone, $y/\delta = 0.339$, where there is no longer intermittency.

The optimum time delay for R_{++} is higher that for R_{--}. Then, as it was expected, the celerity of coherent lumps coming from the wall is slower than the celerity of fluid lumps coming from the external side of the considered mean streamline. At the same position Fig. 3B gives the triple coefficient and conditional correlations (3.2), (3.5). As it is usually the rule in the fully turbulent zones, the triple correlation coefficient has an antisymmetrical pattern. It appears that the triple coefficient, which has relatively low values, is separated (3.6) into conditional correlations which look very much like double correlations and have relatively high individual values. Moreover, since the celerities of hot and cold lumps are different. i.e. the extrema of R_{+++} and R_{---} are different, this

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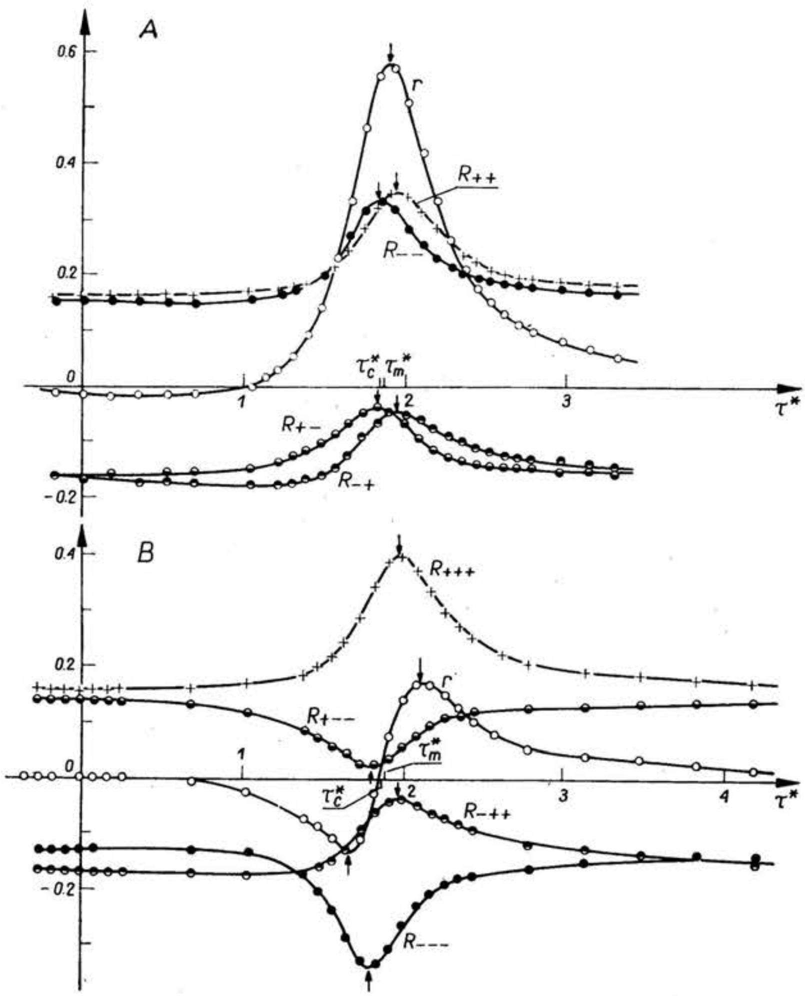

FIG. 3. Conditional space-time correlations in the central zone; $y/\delta = 0.339$, $\gamma = 1-\varepsilon$, $\bar{u}/u_e = 0.884$. A–double; B–triple.

contributes to the antisymmetrical pattern of the coefficient. This underlines that, although the triple coefficients have relatively small values, they are in fact due to opposite sign events which are strongly correlated.

Figure 4 shows the results concerning a mean streamline in the inner region at $y/\delta = 0.034$. The optimum time delay for hot fluid lumps is larger that for cold lumps, but it is noticed that this delay is smaller than the time τ_c^* corresponding to the mean velocity. Therefore, there are highly correlated turbulent lumps, coming from the wall region, which travel faster than the mean local velocity. This result is emphasized much better in Fig. 5 concerning the buffer layer, the mean streamline being at the distance from the wall $y^+ = \dfrac{yu^*}{\nu} = 22$.

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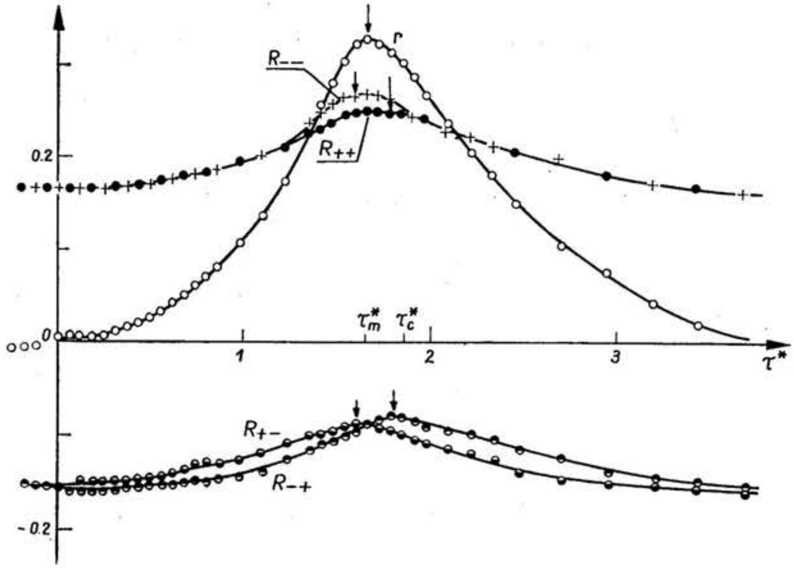

FIG. 4. Conditional space-time correlations in the inner zone; $y/\delta = 0.034$, $y^+ = 64$, $\bar{u}/u_e = 0.594$.

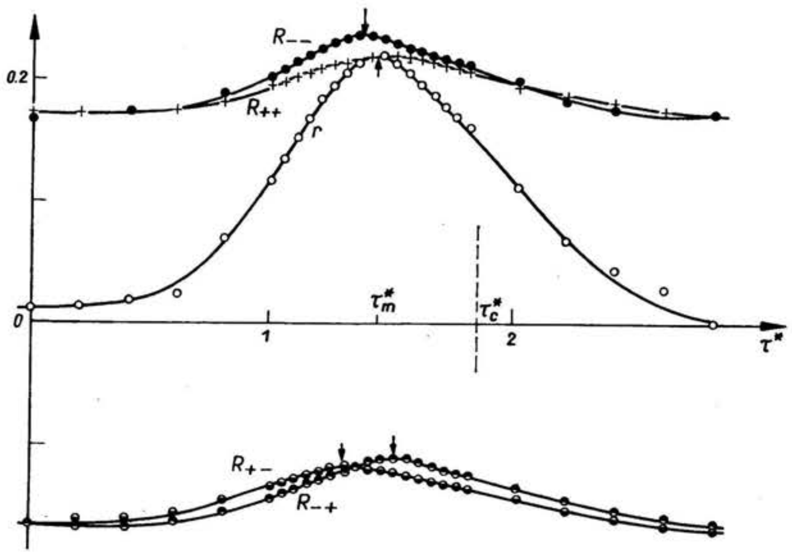

FIG. 5. Conditional space-time correlations in the buffer layer; $y/\delta = 0.012$, $y^+ = 22$, $\bar{u}/u_e = 0.490$.

The optimum time delay for R_{++} is 14.8 10^{-3} s while the time τ_c is 18.4 10^{-3} s. This difference is very large compared to the experimental errors of the order of 0.1 10^{-3} s.

Velocity measurements were also performed by considering the longitudinal component u'_1 at the upstream point and the transverse u'_3 at the downstream point as Reynolds stresses. However, the size of X wires used did not allow the wall to be approached closer than $y^+ = 117$.

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Even at that distance to the wall, it can be noticed in Fig. 6 that the optimum time delay for R_{-+}, corresponding to lumps coming from the wall region, is slightly shorter than the time τ_c. This is in agreement with the results obtained by the temperature correlations.

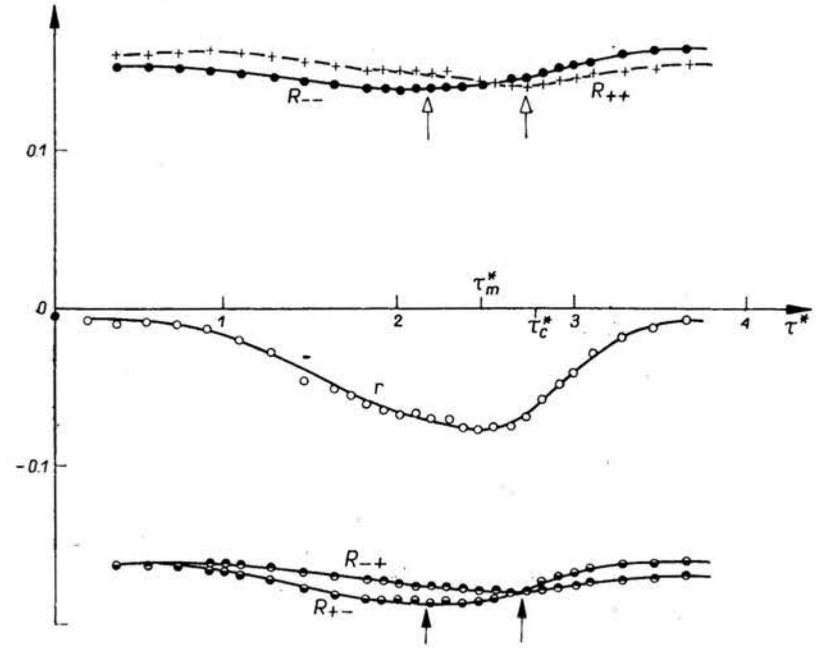

FIG. 6. Conditional space-time correlations in the inner region for velocities $(u_1')_0$ and u_3'; $y/\delta = 0.060$, $y^+ = 117$, $\bar{u}/u_e = 0.620$.

In the above results, the two points were located on a mean streamline. Since the turbulence is principally generated in the buffer layer by the bursting process, the study of the structure has been carried out by locating the upstream hot wire near the wall at $y^+ = 23$, and moving the downstream point longitudinally and transversely. Moreover, we have used the statistical criterion of linkage Φ defined above (3.8).

As an example, Figs. 7A, B, C present, for different threshold levels, the reduced contingencies Φ^{++} and Φ^{--} as a function of the transverse distance to the wall, the longitudinal separation being $X_1 = 0.93\ \delta$. The time delay is constant and equal to the optimum delay for the maximum maximorum value of the correlation coefficient in this section X_1. It is practically the optimum time delay for the maximum maximorum values of Φ^{++} or Φ^{--} with $h_0 = h = 0$ in the same section. It appears that as the thresholds become larger and larger, the behaviour of contingencies are more and more different from the Gaussian values computed from the correlation coefficients (3.9).

Figure 7D shows in the same conditions the behaviour of the conditional correlations R_{++} and R_{--} which looks like the behaviour of the contingencies Φ^{++}, Φ^{--} for $h_0 = h = 1$ or 1.5. We can infer that correlations are significant principally for the diffusion of the eddies containing a relatively important turbulent energy. This is obvious if we recall that the correlation is a second-order statistical momentum.

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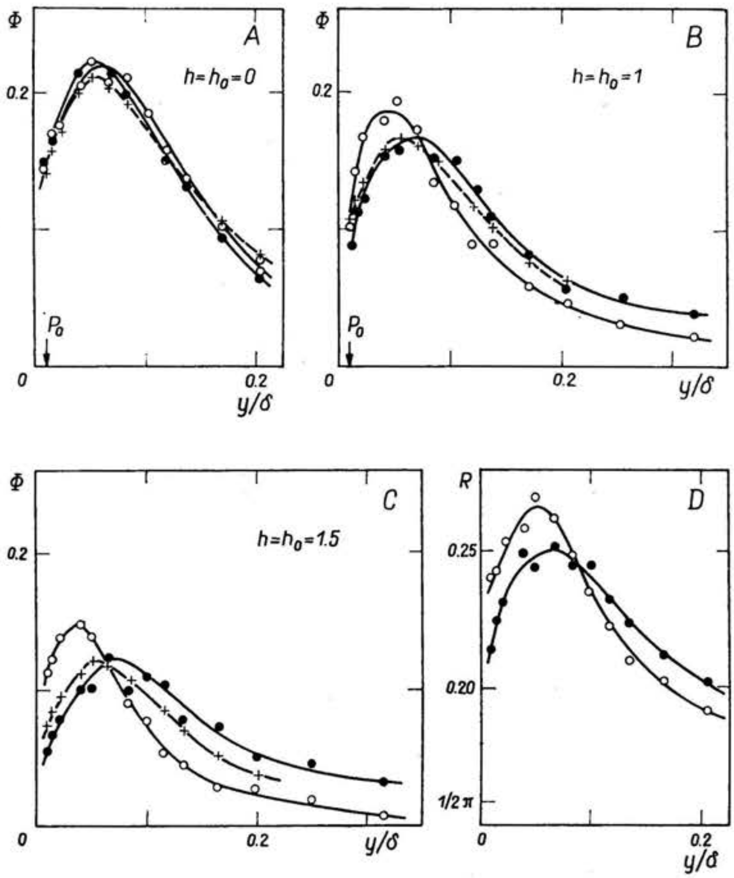

FIG 7. A.B.C. Reduced contingencies for different thresholds (● Φ^{++}; oΦ^{--}; + — gauss). D. Conditional correlations (● R_{++}; oR_{--}. $X_1/\delta = 0.93, y_0/\delta = 0.013, y_0^+ = 23, \tau^* = 1.24$).

In order to have a picture of the diffusion, we used in Fig. 8 a Lagrangian representation for the contingencies Φ^{++} with $h_0 = h = 0$ corresponding to markedly hot fluid lumps coming from the wall region. The line corresponds to constant values of contingencies, or isocontingency curves, equal to half the maximum maximorum value of Φ^{++} at the same optimum time delay τ^*. Besides, the dashed line represents the maximum maximorum contingency.

However, we cannot consider the upstream point as a source point since the correlated particles passing through the downstream point can come only from the neighbourhood of the upstream point, where the temperature fluctuations are strongly correlated to the fluctuations occurring at the upstream point itself.

We attempted to draw an initial source region at time $\tau^* = 0+\varepsilon$ by taking half a virtual maximum value obtained for τ^* tending to zero. Indeed, Fig. 9 shows the decrease of maximum maximorum space-time correlation and contingencies as a function of optimum time delays.

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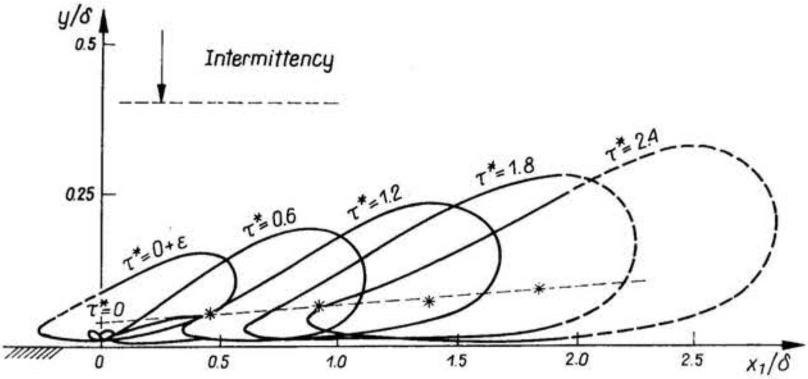

FIG. 8. Relative isocontingencies-hot fluid lumps ($\Phi^{++}/\Phi^{++}_{max} = 0.5$, $h = h_0 = 1$, $y_0/\delta = 0.013$, $y_0^+ = 23$, $*\Phi^{++}/\Phi^{--}_{max} = 1$).

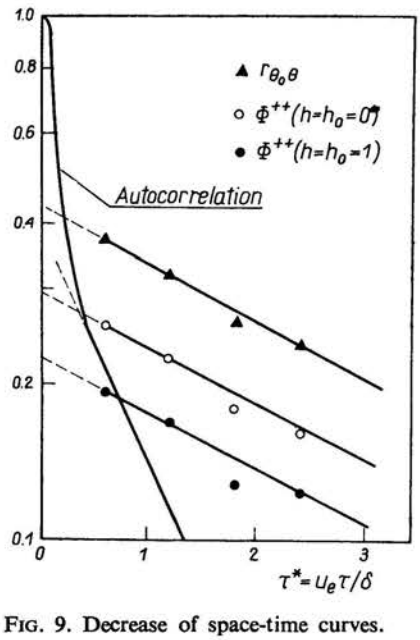

FIG. 9. Decrease of space-time curves.

These decreases appear to be exponential except for the shorter time delays, where the decrease is very fast, perhaps of the order of the auto-correlation decrease [11]. In the case of longer time delays, only the effect of large scale eddies is relevant. In this respect we assume that the initial contingency, to be taken into account, is the value 0.22 on the extrapolated straight line corresponding to Φ^{++}, $h_0 = h = 1$, at $\tau^* = 0$. It appears in Fig. 8 that the initial contour of the source region has the same shape as the others. Besides, the contours enlarge regularly with time. This result is in agreement with the hypothesis of self-similarity proposed by BATCHELOR [6] for the Lagrangian dispersion in the constant stress layer.

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5. Discussion

The results from conditional correlations can be related to the bursting process which generates the turbulence near the wall. From this viewpoint the main feature concerns the convection velocity of the hot fluid lumps, obviously associated with the ejected eddies. That velocity appears to be higher than the mean velocity. This confirms [12] the fact that during the bursting process inrushes of high velocity fluid interact with low velocity streaks ejected from the wall region, and carry them away all across the boundary layer. As a matter of experimental evidence, we compare in Fig. 10 the lines of optimum contin-

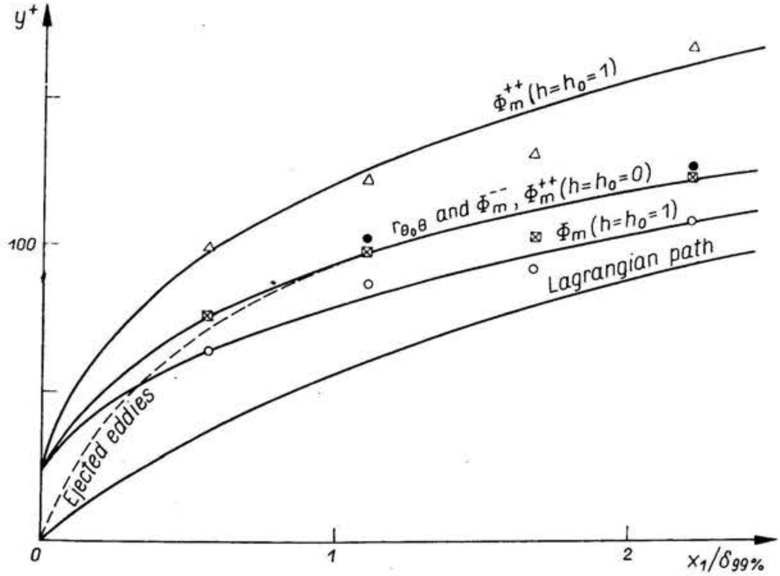

FIG. 10. Dispersion curves; ● $r_{\theta_0\theta}$, □ Φ^{++} $(h = h_0 = 0)$, × Φ^{--} $(h = h_0 = 0)$.

gency Φ^{++}, for $h_0 = h = 1$, with the most favoured trajectory of ejected eddies visualized by RUNSTADLER, KLINE and REYNOLDS [4].

We have also given the line of optimum contingency Φ^{--}, for $h_0 = h = 1$, which can be related to the so-called sweep events. Furthermore, keeping in mind the Lagrangian dispersion, the line of the centroid obtained by SHLIEN and CORRSIN [5] and the line of maximum contingency Φ^{++} or Φ^{--} for $h_0 = h = 0$ are also reported. We emphasize that the lines of maximum contingencies of Φ^{++}, Φ^{--} for $h_0 = h = 0$ and maximum correlations of $r_{\theta_0\theta}$ are practically identical. The line of centroid is the locus of the mean ordinates using the mean temperature profiles as probability distributions in the wake of a heated wire located on the wall. The authors assume this line to be the mean Lagrangian path behind a source point which would be located on the wall. Taking into account the fact that the upstream point is not really a source point, and is not on the wall, we can consider that (except for the initial part of contingency lines) these lines are in agreement with themselves.

However, it is possible to determine a virtual source point located on the wall by adjusting the maximum contingency lines of Φ^{++}, Φ^{--} for $h = h_0 = 0$ with a formula, accord-

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ing to scaling considerations, proposed by Batchelor. This formula gives the coordinates $\overline{X}$ and $\overline{y}$ of the Lagrangian path:

$$\overline{x} = \frac{1}{bK}\left(y\ln\left(\frac{c\overline{y}}{Y_0}\right) - \overline{y} + A\right),$$

where A is the constant of integration which depends on the particle release position. In our case the virtual source point being assumed on the wall $A = 0$. K is the Kármán constant equal to 0.4.; c is a constant taken equal to the unity. Y_0 is computed from the logarithmic part of the mean velocity profiles, in our case $Y_0 = 0.123$ mm. We have adjusted the value of b according to the slope of contingencies for $\tau^* = 2.4$. The experimental values of b are generally rather scattered from 0.1 to 0.4 but Batchelor estimates this value to be less than 0.2 [6]. Our experimental data, adjusted by the Batchelor formula in Fig. 11,

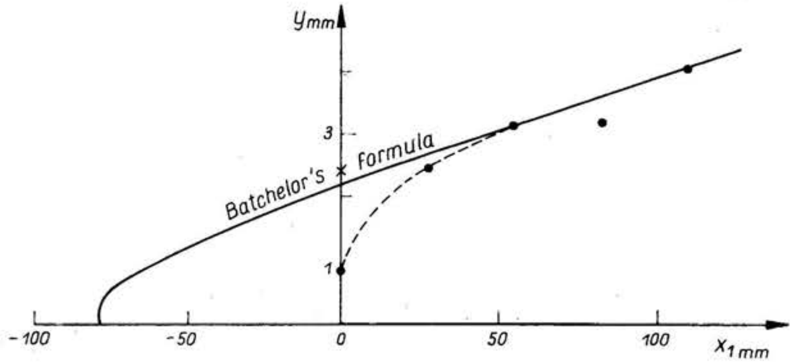

FIG. 11. Adjustment with Batchelor's formula; X point extrapolated on a straight line decrease at $\tau^* = 0$, see Fig. 9.; ● Φ^{++} ($h = h_0 = 0$),

yield $b = 0.13$. This value is consistent with the fact that the above contingency relative o the temperature, can be considered significant of the dispersion, except for the shortest time delay.

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INSTITUT DE MÉCANIQUE STATISTIQUE DE LA TURBULENCE,
MARSEILLE, FRANCE.

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