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Theoretical estimation of the applicability range of the differential pressure type flowmeters in presence of pulsation of the mass flux

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IN THE PAPER one- and two-dimensional mathematical models of the pulsating flow of an incompressible viscous fluid through a pipe orifice were formulated. The one-dimensional model was formulated on the grounds of the Cauchy-Lagrange integral, and the two-dimensional model was derived with the use of Reynolds equations and the k-ε turbulence model equations. The Reynolds equations and the equations of turbulence model were solved by the finite difference method. The obtained variable in time, velocity and pressure fields were used for the analysis of flow through a pipe orifice and for the estimation of simplyfying assumptions made in the one-dimensional model. The one-dimensional flow description was found to be limited, what is due to dependence on time of the coefficients in the equations. The range of application of this description was assumed to be dependent on the mass flow pulsation amplitude and on the Strouhal number. The one-dimensional model was used to estimate the metrological properties of orifice meters in presence of mass flow pulsations. The additional measurement error caused by pulsation was related to the frequency, amplitude and shape of pulsation. For the assumed value of additional error of measurement the equations were given, which can be considered as criteria for the range of application of the differential pressure type flowmeters to the measurements of the mean value of a pulsating mass flux. The obtained formulae were also transformed to display the dependence of measuring error on the frequency and amplitude of differential pressure pulsation. Criteria formulated in this way are of great usefulness from the viewpoint of measuring practice. Results of theoretical studies were compared with the available results of measurements. Good conformity was observed.

W pracy sformułowano jedno i dwuwymiarowy model matematyczny pulsującego przepływu lepkiego i nieściśliwego przez rurociąg ze zwężką. Model jednowymiarowy został sformułowany na podstawie całki Cauchy'ego-Lagrange'a, a dwuwymiarowy na podstawie równań Reynoldsa i równań $k-\varepsilon$ modelu turbulencji. Równania Reynoldsa i równania modelu turbulencji rozwiązano numerycznie metodą różnic skończonych. Wyznaczone zmienne w czasie pola prędkości i ciśnienia wykorzystano do analizy przepływu w obrębie zwężki oraz do oceny założeń upraszczających przyjętych przy formułowaniu modelu jednowymiarowego. Stwierdzono ograniczoność opisu przepływu równaniem jednowymiarowym ze względu na zmienność jego współczynników w funkcji czasu. Uzależniono zakres stosowalności tego równania od amplitudy pulsacji strumienia masy i liczby Strouhala. Model jednowymiarowy wykorzystano do oceny własności metrologicznych zwężek pomiarowych przy obecności pulsacji strumienia masy. Uzależniono dodatkowy błąd pomiaru spowodowany pulsacją od jej częstotliwości, amplitudy i kształtu. Przy założonej wartości dodatkowego błędu pomiaru podano równania o charakterze kryterialnym, określające zakres stosowalności zwężek do pomiaru średniej wartości pulsującego strumienia masy. Otrzymane zależności przekształcono również tak, aby uzależnić błąd pomiaru od amplitudy i częstotliwości pulsacji ciśnienia różnicowego. Tak sformułowane kryteria mają dużą przydatność z punktu widzenia praktyki pomiarowej. Rezultaty badań teoretycznych porównano z dostępnymi wyynikami pomiarów, otrzymując dobrą ich zgodność.

Сформулированв одно- и двумерная математическая модель, описывающая неустановившееся течение несжимаемой жидкости через трубопровод с измерительной диафрагмой. Более общая модель применена к анализу явления течения и оценки интервала важности упрощающих предположений, принимаемых при формулировке наиболее часто применяемой одномерной модели. Исследовано влияние амплитуды, частоты и формы пульсаций на ошибки измерения потока массы. Предложены простые, аналитические критерия, позволяющие оценить может ли быть пульсирующее течение трактованным как квазиустановившееся течение.

1. Introduction

FLOWS IN INSTALLATIONS are accompanied by pulsations and transient states concerned with operation of hydraulic systems or with presence of disturbances of random character. Practically it means that if any method of measurement is used, in which an assumption of the steadiness of flow is made, then the additional errors of measurement can occur. In the cases of pulsating flows encountered in practice, values of such errors depend among others on the frequency and amplitude of pulsation. Among the known methods of measurement, discussed in detail in the works by KREMLEVSKI [8] and MILLER [10], the differential pressure type flowmeters method, which is characterized by simplicity and great accuracy, is widely applied. This is reflected in Polish [17] and international [7] normative documents, which describe several selected types of orifices and nozzles recommended for measurements of steady flow. In the literature of the considered subject the term "steady flow" is not defined precisely with respect to the methods of measurement of mass flux. Hence, the range of applicability of the considered method to measurements of flows which are variable in time is not known. Numerous attempts of using orifices and nozzles in the measurements of pulsating mass flows [10, 11, 16, 19] indicated, than in unfavourable conditions, measurement errors could be as large as 50% (e.g. [19]), unless the suitable corrections in calculations or modifications of measuring apparatus had been made. In some papers [3, 4, 13] attempts were made to estimate the range of parameters in which a transient state could be considered as quasisteady and modification of the method of calculation was not required. The majority of investigators assume that the range of application of the theory of quasi-steady flow depends on the amplitude of pulsation of mass flux (or differential pressure) and on the value of criterial Strouhal number, which is a function of pulsation frequency. As for the limit values of these parameters, in the literature there is a great divergence of opinions. In the paper by OPPENHEIM and CHILTON [15] the value $\mathrm{Sh} < 0.002$ is recommendes irrespective of the flowmeter type and pulsation amplitude. On the experimental grounds of the ESTEL [6] stated that at small pulsation amplitudes ($h_{\dot{M}} < 0.2$), error of measurement mean value of mass flow is less than 1%, irrespective of the value of Strouhal number. MOTTRAM and ZAREK [12] related the limit value of the Strouhal number ($\mathrm{Sh} = 0.006 \div 0.1$) to the type of flowmeter and stated that it did not depend on the pulsation amplitude. In the paper by ZAREK [19] a more complex criterial number was proposed, which comprised, beside the Strouhal number, the quotient of the differential pressure pulsation amplitude and its mean value. In the work by SAUER [18] it was stated that for $\mathrm{Sh} < 0.2$ the influence of mass flux pulsation on the measurement of the mean value of flow is negligible. Similar results were presented by PALENCAR and VIEST [16] on the grounds of experiments carried out for Strouhal numbers in the range $0 \div 0.01$. In the paper [3] the dependence of the limit value of Strouhal number on the nozzle type and the amplitude and form of mass flow pulsation was stated. From the above review it follows that in literature there is no agreement neither on the form of criterion nor on the limit values of Strouhal number. Therefore broader analysis of the influence of pulsation on the characteristics of a stream and on the errors of mass flow measurement seems very advisable. It would be profitable to find the theoretically supported criterion for the range of parameters in which a transient flow can be treated as a steady one, the admissible error of measurement being assumed. Considerations will be carried out with the use of one- and two-dimensional mathematical models describing a transient flow throug a pipe orifice. The more general model will be applied in an analysis of admissibility of the simplyfying assumptions generally made in formulation of the one-dimensional model.

2. One-dimensional mathematical model of flow

Reduction of the flow in a nozzle to a one-dimensional motion is an approach which is the most frequently found in the theoretical papers concerning the discussed problem [1, 12 and 14]. Considerations are usually limited to the main stream, and corrections accounting for two-dimensionality of motion (and other factors) are made later.

A transient flow of an incompressible fluid through a pipeline with a orifice is considered (Fig. 1). By writing the Cauchy–Lagrange integral for Sects. 1 and 2 of the main

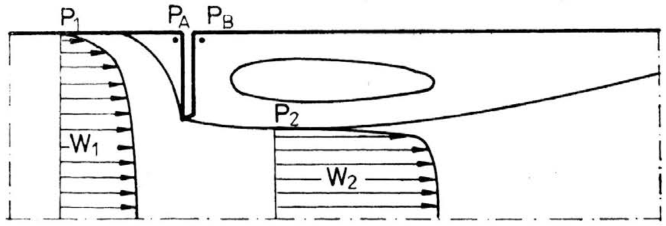


FIG. 1. Flow system with a pipe orifice.

stream, with energy losses taken into account, the relation between the difference of pressures in the Sects. 1 and 2 and the mass flux can be obtained:

$$p_1 - p_2 = \frac{\dot{M}^2}{2\varrho\alpha^2 A_0^2} + \varrho \int\limits_{z_1}^{z_2} \frac{\partial w}{\partial t} dz, \tag{2.1}$$

where $\dot{M}$ is the mass flux, w — average fluid velocity in the section, ϱ — density, A_0 — cross-sectional area of the orifice and α is the discharge coefficient. If we introduce a factor $\psi = \dfrac{p_1 - p_2}{p_A - p_B}$ accounting for the fact that the points A and B in which pressure is measured do not have to coincide with the Sects. 1 and 2, and if we use the relation

$$w = \frac{\dot{M}}{\varrho A},$$

Eq. (2.1) will take the form

$$p_A - p_B = \frac{\dot{M}^2}{2\varrho\alpha^2 A_0^2} + \frac{1}{\psi} \int\limits_{z_1}^{z_2} \frac{dz}{A(z, t)} \frac{d\dot{M}}{dt} - \frac{\dot{M}}{\psi} \int\limits_{z_1}^{z_2} \frac{1}{A^2(z, t)} \frac{\partial A(z, t)}{\partial t} dz, \tag{2.2}$$

in which the discharge coefficient is described by the formula

$$\alpha = \frac{\varkappa\sqrt{\psi}}{\sqrt{C_2+\xi-C_1 m^2\varkappa^2}}. \tag{2.3}$$

In this formula $\varkappa$ is the contraction coefficient, m — area ratio, C_1 and C_2 — Coriolis coefficients, ξ — coefficient of energy losses. Under the assumption that changes in time of the cross-sectional area of the main stream are small, the last term in (2.2) can be neglected. Formula $\frac{1}{\psi}\int\limits_{z_1}^{z_2}\frac{dz}{A(z,\,t)}$ is replaced with the quotient l_e/A_0 and $p_A-p_B = \Delta p$. Then, Eq. (2.2) is as follows:

$$\Delta p = \frac{\dot{M}^2}{2\varrho\alpha^2A_0^2}+\frac{l_e}{A_0}\frac{d\dot{M}}{dt}. \tag{2.4}$$

Quantity l_e is called the effective length of the orifice. On the grounds of acoustic investigations, MOTTRAM and ZAREK [12] presented relations between l_e and the geometry of a confuser for orifices and nozzles and for Venturi tubes. Equation (2.4) expresses the dependence of the current value of mass flux $\dot{M}(t)$ on the differential pressure $\Delta p(t)$ measured in the orifice. The mean value of mass flux in the pulsation period T can be found from

$$\overline{\dot{M}} = \frac{1}{T}\int\limits_0^T \dot{M}(t)\,dt, \tag{2.5}$$

where the function $\dot{M}(t)$ is the solution of Eq. (2.4) for the given (e.g. measured) function $\Delta p(t)$. Examples of calculations were presented in the paper [2].

From the point of view of the aim of this paper, however, it is necessary to carry out a broader analysis and estimation of the simplifications assumed in deriving the formula (2.4). This concerns particularly the estimation of the range of changes of the flow number α and of the cross-section of stream behind the nozzle as a function of amplitude and frequency of pulsation. As these quantities are closely related to multidimensionality of the phenomenon of flow, the more general mathematical model is required.

3. Two-dimensional mathematical model

3.1. Equations of motion and boundary conditions

A two-dimensional, time-dependent, turbulent flow of an incompressible viscous fluid through a segment of the pipeline with an orifice is considered. The scheme of the hydraulic system is shown in the Fig. 1. Axial symmetry of the stream is assumed. Average parameters of the stream can be described by the Reynolds equation, complemented by the semi-empirical model of turbulence [20, 21]. In the cylindrical coordinate set, the general form of equations of transport is as follows:

$$\frac{\partial\phi}{\partial t}+\frac{\partial}{\partial z}(U\phi)+\frac{1}{r}\frac{\partial}{\partial r}(rV\phi) = \frac{\partial}{\partial z}\left(\Gamma_\phi\frac{\partial\phi}{\partial z}\right)+\frac{1}{r}\frac{\partial}{\partial r}\left(r\Gamma_\phi\frac{\partial\phi}{\partial r}\right)+S_\phi, \tag{3.1}$$

where by the symbol ϕ the following quantities are denoted in turn: 1 (equation of continuity), axial U and radial V components of the velocity vector, kinetic energy of turbulence k, and rate of dissipation of kinetic energy of turbulence ε. Coefficients of the set of Eqs. (3.1) are collected in the Table 1. In the above approach, the influence of turbulent fluctuations of velocity on the average parameters of stream is modelled by the coefficient of effective viscosity

Table 1. Coefficients in Eq. (3.1).

ϕ	Γ_ϕ	S_ϕ
U	ν_{ef}	$\frac{\partial}{\partial z}\left(\nu_{ef}\frac{\partial U}{\partial z}\right)+\frac{1}{r}\frac{\partial}{\partial r}\left(r\nu_{ef}\frac{\partial V}{\partial z}\right)-\frac{1}{\rho}\frac{\partial p}{\partial z}$
V	ν_{ef}	$\frac{\partial}{\partial z}\left(\nu_{ef}\frac{\partial U}{\partial r}\right)+\frac{1}{r}\frac{\partial}{\partial r}\left(r\nu_{ef}\frac{\partial V}{\partial r}\right)-\frac{2\nu_{ef}V}{r^2}-\frac{1}{\rho}\frac{\partial p}{\partial r}$
k	$\frac{\nu_{ef}}{\sigma_k}$	$G-\varepsilon$
ε	$\frac{\nu_{ef}}{\sigma_k}$	$\frac{\varepsilon}{k}(c_{\xi 1}G-c_{\xi 2}\varepsilon)$
1	0	0

$$G=\nu_{ef}\left\{2\left[\left(\frac{\partial U}{\partial z}\right)^2+\left(\frac{\partial V}{\partial r}\right)^2+\left(\frac{V}{r}\right)^2\right]+\left(\frac{\partial U}{\partial r}\quad\frac{\partial V}{\partial z}\right)^2\right\}.$$

$$\nu_{ef} = \nu_t+\nu,$$

where ν_t is a turbulent viscosity, which can be found on the basis of the Laundner and Spalding $k-\varepsilon$ model of turbulence,

$$\nu_t = C_\mu \frac{k^2}{\varepsilon}.$$

The following values of empirical constants were taken:

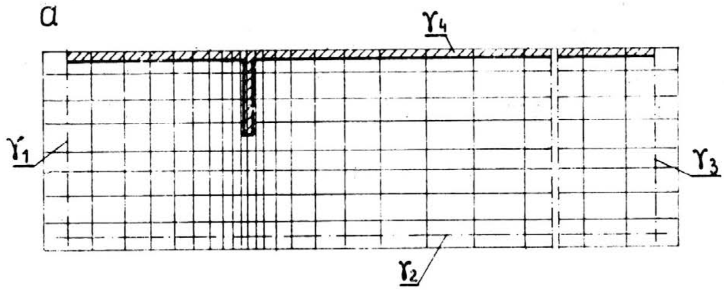
$$C_\mu = 0.09, \quad C_{\xi 1} = 1.43, \quad C_{\xi 2} = 1.92, \quad \sigma_k = 1.0, \quad \sigma_\varepsilon = 1.3.$$

They correspond with the recommendations from the paper [9]. As it was demonstrated in numerous applications [21], the set (3.1) properly describes the average parameters of a broad class of turbulent flows.

Because of the character of equations of the mathematical model and the aim of calculations, it is necessary to formulate the boundary conditions for all the dependent variables of the set (3.1).

In the inlet section γ_1 (Fig. 2a) the fully developed pulsating flow is assumed. Then, $V = 0$, $\partial\phi/\partial z$ and the set (3.1) is reduced to

$$\frac{\partial \phi}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\Gamma_\phi \frac{\partial \phi}{\partial r}\right)+S_\phi, \tag{3.2}$$



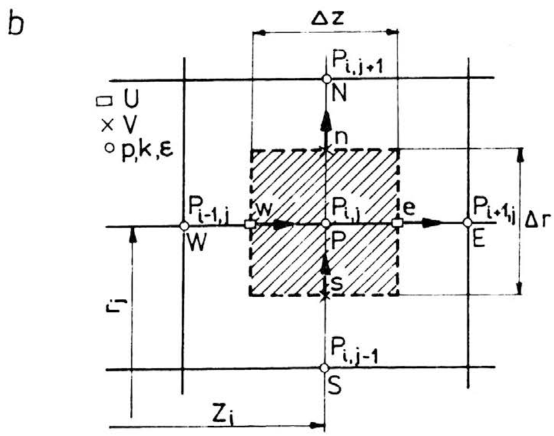


FIG. 2a. Difference mesh with marked boundaries of the region of calculations. b. Fragment of the difference mesh with denotations.

Table 2. Coefficients in Eq. (3.2).

ϕ	Γ_ϕ	s_ϕ
U	ν_{ef}	$-\frac{1}{\rho}\frac{\partial p}{\partial z}$
k	$\frac{\nu_{ef}}{\sigma_k}$	$\nu_{ef}\left(\frac{\partial U}{\partial r}\right)^2 - \varepsilon$
ε	$\frac{\nu_{ef}}{\sigma_e}$	$\frac{\varepsilon}{k}\left(c_{\xi 1}\nu_{ef}\left(\frac{\partial U}{\partial r}\right)^2 - c_{\xi 2}\varepsilon\right)$

where ϕ denotes U, k, ε, in turn. Coefficients of the set (3.2) are collected in the Table 2. The set (3.2) is solved under the following assumption on the form of pressure gradient:

$$\frac{\partial p}{\partial z} = A + B\sin\omega t,$$

where A and B are constants. As a result, the required distributions of all the variables in the section γ_1 are obtained, for every value of time.

On the axis of symmetry of the tube γ_2 the conditions $V = 0$ and $\partial\phi/\partial r = 0$ are assumed to account for axial symmetry of the stream. In the section γ_3 the boundary conditions are assumed arbitrarily

$$V = \frac{\partial\phi}{\partial z} = 0, \qquad \dot{M}\Big|_{\gamma_1} = \dot{M}\Big|_{\gamma_3},$$

where $\dot{M}$ denotes the mass flux of the fluid, as the information about distribution of variables in this section is missing. The influence of the boundary conditions in the section γ_3 on the fields of pressure and velocity in the region of the orifice is small provided the section is far enough from the nozzle.

In the nodes of the difference mesh which are adjacent to the walls of the orifice and the pipe (γ_4) boundary conditions are formulated according to the model $k-\varepsilon$ for large Reynolds numbers [9]. If it is assumed that the point P is in the region of developed turbulence, then the component U_P of the velocity vector, parallel to the wall, is described by the logarithmic formula

$$\frac{U_P}{(\tau|\varrho)_w^{1/2}} = \frac{1}{\varkappa}\ln\left(\frac{Ey_P(\tau_w\varrho)_w^{1/2}}{\mu}\right),$$

where y_P is the distance of point P from the wall, $\varkappa = 0.42$ and $E = 9.7$ are the empirical constants. Assumption that the shear stress τ are constant along the segment connecting point w on the wall and point P in the boundary layer, implies

$$\tau_p = \tau_w = \varrho C_\mu^{1/2} k_p.$$

Compiling the above formulae we obtain the identity

$$\tau_p = \frac{\varrho\varkappa C_\mu^{1/4} k_p^{1/2} U_P}{\ln\{Ey_P C_\mu^{1/4}\varrho k^{1/2}/\mu\}}$$

which relates the shear stress on the wall to the kinetic energy of turbulence and the component of velocity vector parallel to the wall.

Value of energy dissipation ε_p in the boundary point can be found from the relation

$$\varepsilon_p = C^{3/4} k_p^{3/2}/\varkappa y_P.$$

Value of kinetic energy of turbulence k_p in the point adjacent to the wall is found from the general equation of balance with diffusion neglected. The component representing dissipation $\varrho\varepsilon$ in the equation of transport is estimated as follows:

$$\varrho\varepsilon = C_\mu^{3/4}\frac{kp^{3/2}}{\varkappa}\ln\left\{\frac{Ey_P C_\mu^{1/4} kp^{1/2}}{\nu}\right\}.$$

The obtained boundary value problem for the set of five nonlinear differential equations was solved by means of the finite difference method. As the initial conditions, the distributions of dependent variables corresponding to the steady flow were used.

3.2. Finite-difference equations and computational algorithm

Discretization of the differential equations of transport was based upon integrating the components of these equations on the difference mesh (Fig. 2a), the fragment of which is shown in Fig. 2b. As a result of integration, the following finite-difference equation was obtained:

$$\frac{r_p}{\Delta t}A_p^\phi(\phi_p^{n+1}-\phi_p^n)+\left\{rU\phi-r\Gamma_\phi\frac{\partial\phi}{\partial z}\right\}_e^{n+1}A_e^\phi$$
$$-\left\{rU\phi-r\Gamma_\phi\frac{\partial\phi}{\partial z}\right\}_w^{n+1}A_w^\phi+\left\{rV\phi-r\Gamma_\phi\frac{\partial\phi}{\partial r}\right\}_n^{n+1}A_n^\phi$$
$$-\left\{rV\phi-r\Gamma_\phi\frac{\partial\phi}{\partial r}\right\}_s^{n+1}A_s^\phi=(S_p^\phi\phi_p+S_U^\phi)_p^{n+1}r_pA_p^\phi,$$

where the indices e, w, n and s denotes the nodes lying half-way from the central point P to one of the adjacent main nodes E, W, N and S. Quantities A^ϕ are related to the areas of surfaces formed by edges of a control surface, when it is rotated by a unit angle, and $A_p^\phi=\Delta z\cdot\Delta r$. Components S_p^ϕ and S_U^ϕ are concerned with the linearization of source terms of the set (3.1).

Since in the general difference equation the unknowns in the main nodes of difference mesh must appear, the estimation of shares of the convection and diffusion fluxes through the control surfaces is required. In the paper [20] it was demonstrated that the stability of algorithm is ensured by a hybrid scheme, which is a combination of a central scheme and a „stream oriented" scheme. Considering the surface A_w^ϕ we obtain

$$\left(U\phi-\Gamma_\phi\frac{\partial\phi}{\partial z}\right)_w r_wA_w^\phi=\begin{cases}U_wr_wA_w^\phi(\phi_w+\phi_p)\\ \qquad -\Gamma_\phi r_wA_w^\phi(\phi_p-\phi_w)\delta_z & \text{for}\quad |\mathrm{Re}^\delta|<2,\\ U_wr_wA_w^\phi\phi_w\quad\text{for}\quad|\mathrm{Re}^\delta|\geqslant 2,\\ U_wr_wA_w^\phi\phi_p\quad\text{for}\quad|\mathrm{Re}^\delta|\leqslant -2,\end{cases}\tag{3.3}$$

where $\mathrm{Re}^\delta=\varrho U_w\Delta z/(\Gamma_\phi)_w$ is a mesh Reynolds number. After applying the hybrid scheme to the remaining elementary surfaces we obtain the difference equation in a general form:

$$a_p^\phi\phi_p^{n+1}=a_e^\phi\phi_E^{n+1}+a_w^\phi\phi_W^{n+1}+a_N^\phi\phi_N^{n+1}+a_s^\phi\phi_s^{n+1}+\Gamma_pA_p^\phi S_U^\phi+M_p\phi_p^n,\tag{3.4}$$

where

$$a_p^\phi=a_E^\phi+a_w^\phi+a_N^\phi+a_s^\phi-r_pA_p^\phi S_p^\phi+M_p,$$
$$M_p=\frac{r_p}{\Delta t}A_p^\phi$$

and the upper index ϕ is related to the variable ϕ. The difference equation (3.4) is valid in every interior point of a difference mesh. It relates the value of variable ϕ in the central point P of the mesh to the corresponding values in four neighbouring main points. Coefficients a^ϕ refer to the joint share of convection and diffusion estimated according to the scheme (3.3). As the difference scheme for variable t is implicit, the sets of difference equations of type (3.4) are solved iteratively, for every time step $n+1$. Solution of the set of difference equations is based on the method SIMPLE [20]. Algorithms are the extension of methods applied in paper [1] to the problem of transient flow. A single interior iteration cycle consists of the solution of the set of equations of motion

$$a_p^VV_p^*=\sum_j a_j^VV_j^*+S_U^V+A_s^V(p_s-p_p),$$
$$a_p^UU_p^*=\sum a_j^UU_j^*+S_U^U+A_w^U(p_w-p_p),$$

where quantities denoted ( )* are the approximate values and solution of a Poisson equation for the pressure correction p'

$$a_P^p p_P' = \sum_j a_j^p p_j' + S_U^p$$

for satisty continuity equation the correction for velocity field

$$U_p = U_p^* + DU(p_w - p_p'),$$
$$V_p = V_p^* + DV(p_{s'} - p_{p'})$$

is carried out for the current values of pressure correction p'. Component S_U^p depends sources of mass resulting from the fact that the approximate velocity field does not satisfy on the the equation of continuity. Equations of the turbulence model are solved simultaneously with the equations of motion.

The internal iteration process is continued until the convergence criterion is fulfilled

$$\max\{\mathrm{Res}(\phi)\} \leqslant \lambda, \tag{3.5}$$

where

$$\mathrm{Res}(\phi) = \max_{i,j}\Big\{a_P^\phi \phi_P - \sum_{N,s,E,w} a_j^\phi \phi_j - S_U^\phi\Big\}.$$

When condition (3.5) is satisfied, the calculations concerning the next time step Δt are undertaken. Value of Δt was chosen experimentally. No limitation on the value of Δt concerned with the stability of algorithm was encountered.

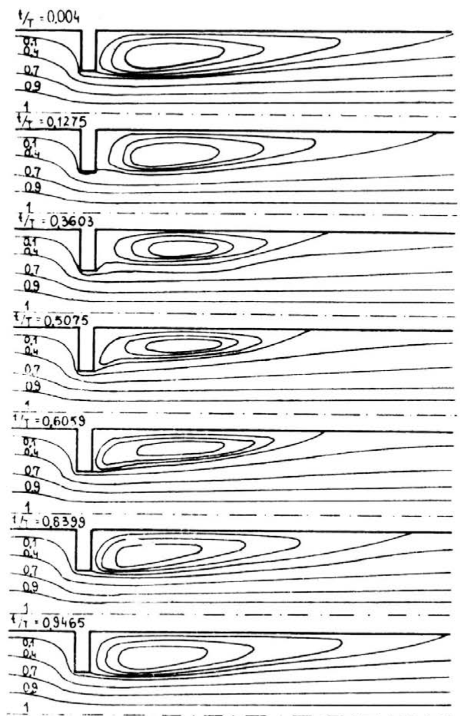


FIG. 3. Isolines of stream-function for various times in one pulsation period.

Periodic solutions were obtained by integration of the set of Eqs. (3.1) in the time interval which was much larger than the one period of pressure pulsation T. Full oscillation of dependent variables was obtained after 2 cycles.

4. Discussion of simplyfying assumptions in the one-dimensional model

On the grounds of the mathematical model presented in Sect. 3 the calculations for turbulent pulsating flows were carried out. Solution of the set (3.1) determines the velocity and pressure fields in the orifice area. Knowing these fields one can examine the function $\Delta p = f(\dot{M})$ as well as analyze the simplyfying assumptions made in the one--dimensional model.

Results of the calculations for a flow sinusoidally pulsating with a frequency equal to 210 rad/s and amplitude $h_{\dot{M}}$ equal to 0.175 is displayed in the form of isolines of the stream function for several time points in one pulsation cycle. Calculations were made for the fluid with density $\varrho = 10^3$ kg/m^3 and viscosity $\nu = 10^{-6}$ m^2/s flowing through a nozzle with modulus $m = 0.25$. Influence of the flux pulsation on the of fluid motion in the region before the orifice is comparatively small, and it is significant only for large mass flux amplitude $h_{\dot{M}}$. Behind the orifice a periodical growth and decrease of the length of recirculation zone can be noticed. Numerical calculations demonstrated that for large

Table 3. Comparison of the values of the terms of Eq. (2.2) for various amplitudes and frequencies of pulsation.

(I) $\left(\frac{\dot{M}^2}{2\varrho\alpha_s^2 A_0^2}\right)/\Delta p$ Re $\cong 2.5\cdot 10^5$, m = 0.4

(II) $\frac{1}{\Psi}\left(\frac{d\dot{M}}{dt}\int_{z_1}^{z_2}\frac{dz}{A(z,t)}\right)/\Delta p$

(III) $\frac{1}{\Psi}\left(\dot{M}\int_{z_1}^{z_2}\frac{1}{A^2(z,t)}\frac{\partial A(z,t)}{\partial t}dz\right)/\Delta p$

$$(\bar{x}) = \frac{1}{T}\int_0^T |x|\,dt$$

		$h_{\dot{M}} = 0.1$			$h_{\dot{M}} = 0.3$		
ω rd/s	Sh	$(\bar{\mathrm{I}})$	$(\bar{\mathrm{II}})$	$(\bar{\mathrm{III}})$	$(\bar{\mathrm{I}})$	$(\bar{\mathrm{II}})$	$(\bar{\mathrm{III}})$
9	0.024	1.007	0.008	$9\cdot10^{-6}$	1.014	0.032	$2.94\cdot10^{-5}$
20	0.053	1.016	0.015	$2.9\cdot10^{-5}$	1.020	0.060	$9.6\cdot10^{-5}$
150	0.396	1.013	0.11	$2\cdot10^{-3}$	1.025	0.63	$1.86\cdot10^{-3}$

values of ω and $h_{\dot{M}}$ a „separation" from the orifice and segmentation of the recirculation zone can take place. This testifies to the strong dependence of the processes taking place behind the orifice on the pulsation character and, consequently, to the limited range of applicability of Eq. (2.4). Therefore, it is an important problem to estimate the changes of coefficients in Eq. (2.4) concerned with the processes taking place in the orifice.

In the Table 3 the comparison is presented of the terms of Eq. (2.2) (with coefficient ψ taken into account) for two different mass flux pulsation amplitudes and pulsation frequencies equal 9, 20 and 150 rad/s, respectively. The average Reynolds number for the tested flows was equal 2.5×10^5. Values of the terms have been related to the instantaneous value of differential pressure, and then absolute values of the obtained quotients have been averaged in time. The results indicate that the value of the inertia term II increases as ω and $h_{\dot{M}}$ increases and for $\omega = 150$ rad/s it is comparable with the term $M^2/2\varrho\alpha^2 A_0^2$. Value of the third term in the considered range of changes of ω and $h_{\dot{M}}$ is smaller by several orders of magnitude than the other terms and the assumption consisting in neglecting it is correct.

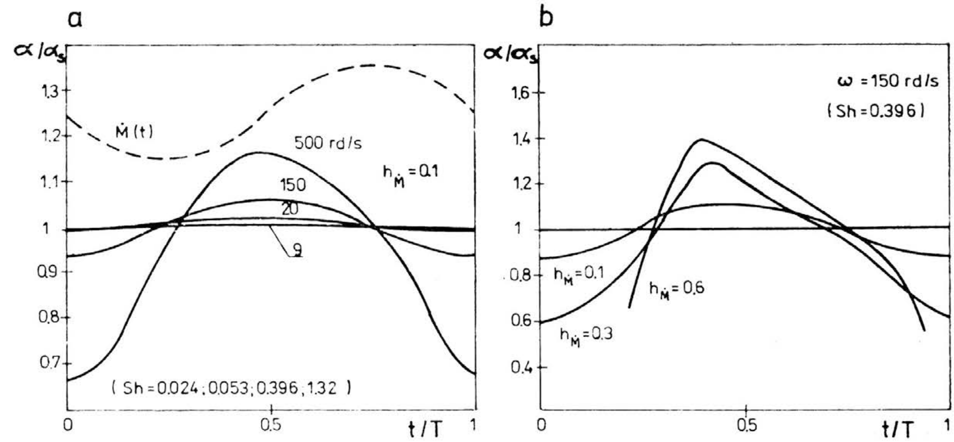


FIG. 4a. Influence of pulsation frequency on the range of changes of discharge coefficient. b. Influence of pulsation amplitude on the range of changes of discharge coefficient number as a function of time.

In Figs. 4a and 4b the influence of amplitude and frequency of flux pulsation on the range of changes of the discharge coefficient was plotted as a function of time. The value of α was found from Eq. (2.2), in which the required quantities were calculated on the basis of known fields of velocity and pressure. The discharge coefficient has been related to the value of α_s was found for a steady flow, the parameters of which correspond to the average values for the pulsating flow. Results of the calculations indicate that together with the increase of pulsation amplitude and frequency (Strouhal number), the range of changes of the discharge coefficient as a function of time increases. Broader analysis demonstrated that, among the quantities determining the internal structure of the discharge coefficiente (formula (2.3)), changes in time of the coefficient ψ play the greatest role. For large values of ω, pressure gradient in the axial direction periodically assumes negative values. In the nearest neighbourhood of the orifice a considerable radial pressure gradient also appears. For these reasons, for large amplitudes and frequencies of flux pulsations, the value of ψ can periodically assume negative values. In this case the one-dimensional model (2.4) may not be applied to the description of the considered flow. For instance, for frequency $\omega = 150$ rad/s and amplitude $h_{\dot{M}} = 0.6$, in a certain range of pulsation cycle it is impossible to find the discharge coefficient from Eqs. (2.2) and (2.3). On the other hand, the results indicate that there exists a range of pulsation amplitudes and frequencies, in which the changes of α are negligible.

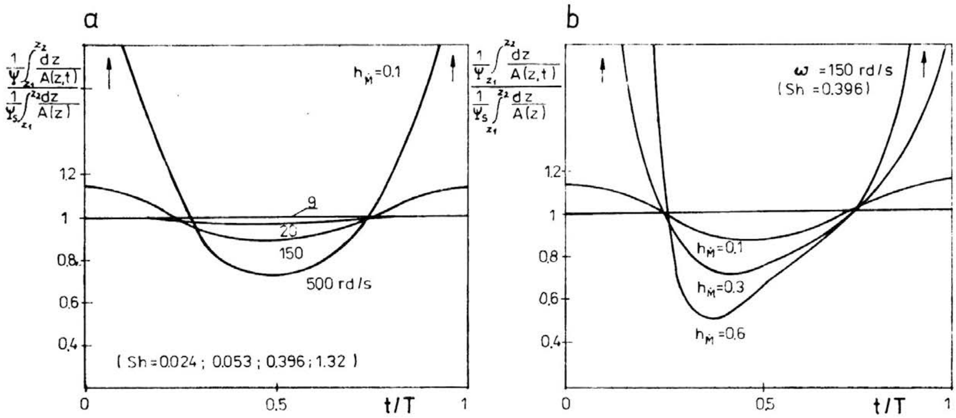


FIG. 5a. Influence of pulsation frequency on the range of changes of the integral as a function of time. b. Influence of pulsation amplitude on the range of changes of the integral as a function of time.

Figures 5a and 5b display the influence of amplitude and frequency of pulsation on the range of changes of the expression $\frac{1}{\psi}\int\limits_{z_1}^{z_2}\frac{dz}{A(z,t)}$ as a function of time. The integral in the above expression was calculated numerically, and the cross-sectional area of the stream $A(z, t)$ was found from the known stream function. The range of changes in time of the considered expression grows together with the amplitude and frequency of pulsations of flux. Changes of the coefficient ψ as a function of $h_{\dot{M}}$ and ω are essential here,

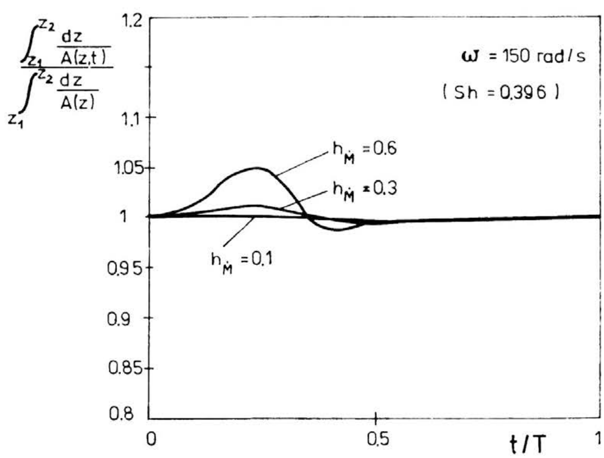


FIG. 6. Influence of pulsation on the range of changes of the integral as a function of time.

whereas the influence of these parameters on the value of the integral $\int\limits_{z_1}^{z_2} \frac{dz}{A(z,t)}$ as a function of time is comparatively small. Results shown in the Fig. 6 confirm this statement. Calculations demonstrated that for $h_{\dot{M}} \leqslant 0.1$ and $\omega \leqslant 20$ rad/s the influence of $h_{\dot{M}}$ and ω on the value of the expression under consideration is negligibly small. This corresponds to the range of parameters, in which also the flow number is practically constant. This means that the Eq. (2.4) can be used to calculate the instantaneous values of mass flux, on the grounds of measurement of the function $\Delta p\ (t)$. On the other hand, for larger values of ω and $h_{\dot{M}}$, Eq. (2.4) does not describe correctly the pulsating flow through the orifice meter, because of the strong dependence of coefficients on time and character of pulsations.

5. Estimation of the range of application of nozzles to transient flows

Analysis carried out in Sect. 4 demonstrated that the simplyfying assumptions, commonly made when deriving the equation (2.4), can lead to significant errors for pulsations with large amplitude and frequency. This results from the fact that the coefficients α and l_e depend on time and on the form of pulsation. In consequence, the orifice method of measurement of mass flux can not be applied for transient flows if the value of Sh is large, even when calculations are carried out on the basis of Eqs. (2.4) and (2.5).

When the Eq. (2.4) correctly describes the transient flow through a flowmeter, the mean value of mass flux can be found from Eqs. (2.4) and (2.5) for measured values of the function $\Delta p(t)$. However, this involves significant complication of the methodology of calculations. From the practical point of view, it is advisable to find the range of application of the formula recommended for calculating the steady flows

$$\overline{\dot{M}} = \alpha A_0 \sqrt{\overline{2\varrho \Delta p}}, \tag{5.1}$$

which results from Eq. (2.4) by setting $d\dot{M}/dt = 0$. Two cases should be distinguished here. In the first case, the mean value of a pulsating mass flux is found from Eq. (5.1) on the basis of the mean value of differential pressure, i.e.

$$\overline{\dot{M}^*} = \alpha A_0 \sqrt{2\varrho \overline{\Delta p}}, \tag{5.2}$$

where

$$\overline{\Delta p} = \frac{1}{T} \int\limits_0^T \Delta p(t)\, dt.$$

In this case the error of measurement σ is defined as

$$\sigma = \frac{\overline{\dot{M}^*} - \overline{\dot{M}}}{\overline{\dot{M}}}. \tag{5.3}$$

Value of σ can be estimated if the character of pulsations of mass flux is known. Assuming that the function $\dot{M}(t)$ can be expressed as

$$\dot{M}(t) = \overline{\dot{M}}(1+h_{\dot{M}}f(t)) = \overline{\dot{M}}\left(1+h_{\dot{M}}\sum_{i=1}^{n} A\sin\omega it\right),$$

where $f(t)$ denotes an arbitrary periodic function, normalized in an interval $\langle -1, 1\rangle$, we obtain that σ is expressed by the formula

$$\sigma = \sqrt{1+\frac{h_{\dot{M}}^2}{2}\sum A_i^2}-1. \tag{5.4}$$

Expanding the radicand into a series, we obtain an approximate formula for the error.

$$\sigma = \frac{h_{\dot{M}}^2}{4}\sum A_i^2. \tag{5.5}$$

Coefficients A_i depend on the shape of mass flux pulsation, expressed as a sine series. For constant discharge coefficient, the value of σ is independent of frequency of mass flux pulsation.

In the second case, the mean value of mass flux is found on the basis of the mean value of the root of differential pressure, i.e.

$$\overline{\dot{M}} = \alpha A_0\frac{1}{T_0}\int^{T}\sqrt{2\varrho\Delta p(t)}\,dt. \tag{5.6}$$

Then, for the assumed form of pulsation of flux, using Eqs. (2.4), (5.3) and (5.6), the error σ can be expressed by the formula

$$\sigma = \frac{1}{T}\int_0^T\sqrt{(1+h_{\dot{M}}f(t))^2+2h_{\dot{M}}\alpha^2\frac{l_e}{w_0}\frac{df(t)}{dt}}\,dt-1. \tag{5.7}$$

Examples of calculations for various amplitudes, frequencies and forms of pulsation of the mass flux were presented in the paper [3]. Under the assumption that the following relation is satisfied

$$\left|2h_{\dot{M}}f(t)+h_{\dot{M}}^2f^2(t)+2h_{\dot{M}}\alpha^2\frac{l_e}{w_0}\frac{df(t)}{dt}\right| < 1 \tag{5.8}$$

the integrand can be expanded into a series, what yields

$$\sigma = -\frac{h_{\dot{M}}^2}{4}\alpha^4\mathrm{Sh}^2\left(\sum_{i=1}^{\infty}A_i^2i^2\right). \tag{5.9}$$

Relations (5.5) and (5.9) are the approximate formulae which allow for estimating analytically the error of measurement, when the form, amplitude and frequency of pulsation of the mass flux is known. Differences in structures of these formulae result from the difference in defining the mean value of mass flux in the period of pulsation.

From the point of view of measuring practice, information on the flow is obtained from the measured values of Δp as a function of time. It is therefore useful to relate the value of σ to the characteristic parameters of differential pressure. This can be obtained by transforming the relations (5.4), (5.5) and (5.9) and replacing the mass flux pulsation amplitude with the quantities characteristic for the pulsation $\Delta p(t)$. By linearizing the second term of Eq. (2.4), for every harmonic a relation between the differential pressure pulsation amplitude and mass flux pulsation amplitude can be obtained,

$$h_{\Delta p_i} = 2h_{\dot{M}} A_i \sqrt{1+\alpha^4 \mathrm{Sh}^2 i^2}, \tag{5.10}$$

where the circular frequency in the Strouhal number is the fundamental frequency. Substituting (5.10) into (5.5) we obtain

$$\sigma = \frac{1}{16} \sum_{i=1}^{\infty} \frac{h^2_{\Delta p_i}}{1+\alpha^4 \mathrm{Sh}^2 i^2} \tag{5.11}$$

and in the second case, for which Eq. (5.9) has been derived, σ is expressed by

$$\sigma = -\frac{\alpha^4 \mathrm{Sh}^2}{16} \sum_{i=1}^{\infty} \frac{h_{\Delta p_i} i^2}{1+\alpha^4 \mathrm{Sh}^2 i^2}. \tag{5.12}$$

In the case of pulsation of sinusoidal or nearly sinusoidal shape, assuming that the additional error of measurement should be smaller than a specified number (say 0.1), from (5.11) and (5.12) the simple criterial relations can be derived

$$h_{\Delta p} < 0.4 \tag{5.13}$$

when $\dot{M}$ is calculated from Eq. (5.2) and

$$h_{\Delta p} \alpha^2 \mathrm{Sh} < 0.4 \tag{5.14}$$

when $\dot{M}$ is found from Eq. (5.6).

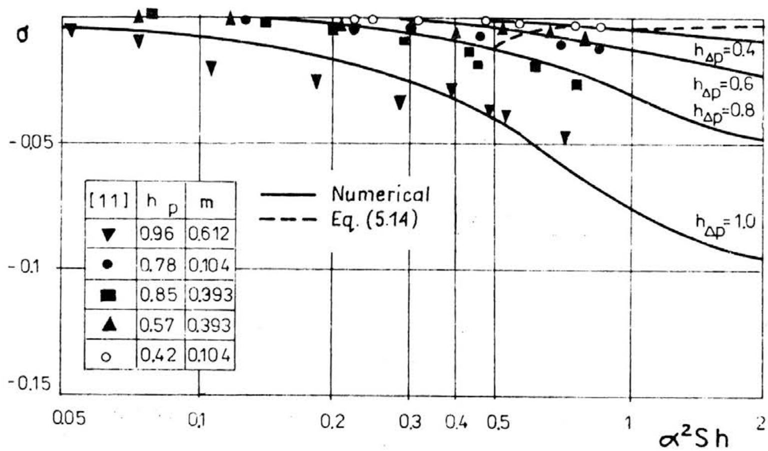


FIG. 7. Measurement error resulting from the assumption of quasi-steadiness of flow (——— numerical calculations, – – – – – Eq. (5.14)).

In Fig. 7 the results of numerical calculations are presented, in which the values of the error σ, defined by Eq. (5.3), were found by solving Eq. (2.8) for sinusoidal pulsations of differential pressure

$$\Delta p(t) = \Delta \bar{p}(1+h_{\Delta p} \sin \omega t).$$

Results of calculations have been related to the experimental data obtained by MOHAMAD and MOTTRAM [11] for the similar, but not strictly sinusoidal, form of pulsation. The satisfactory quantitative and qualitative conformity of calculations with experiments can be noticed. The range of parameters for which $|\sigma| \leqslant 0.01$, with the condition (5.8) taken into consideration, has been marked by the dotted line found from Eq. (5.14). In this range it is assumed that the pulsating flow can be treated as a quasi-steady one. Figure 8

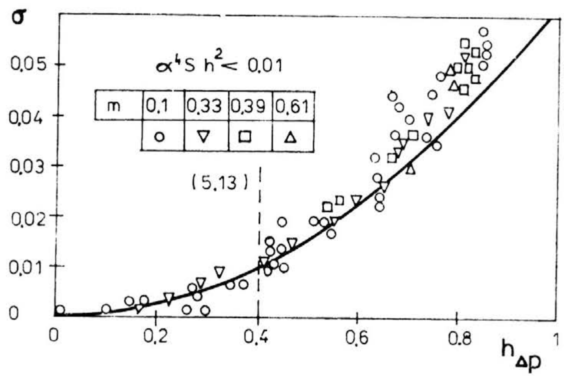


FIG. 8. Measurement error vs differential pressure pulsation amplitude for orifices of different area ratios.

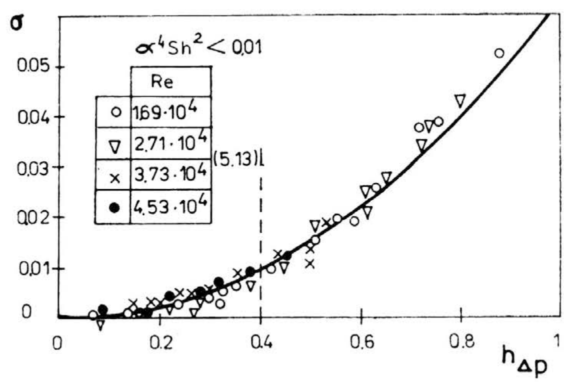


FIG. 9. Measurement error vs differential pressure pulsation amplitude for various Reynolds numbers.

shows the measurement error σ found experimentally by MOHAMMAD and MOTTRAM [11] for orifices with various area ratios, and Fig. 9 displays the results of investigations by MOTTRAM and ZAREK [12] for an orifice with area ratios equal to 0.1, and for flows with various Reynolds numbers. The solid line displays the relation (5.11) in which the expression $\alpha^4 \mathrm{Sh}^2 i^2$ in denominator has been neglected, accounting for its small value. Very good quantitative conformity of the results of experiment with Eq. (5.11) can be noticed. The vertical line illustrates the condition (5.13). It can be stated that for $h_{\Delta p} < 0.4$, the value of error σ does not exdceed 0.01. For the assumed value of σ this condition can also be treated as the criterion of separation between the quasi-steady and pulsating flow.

6. Conclusions

On the grounds of the one- and two-dimensional mathematical models, the influence of the pulsation of stream on the errors of measurement of the mass flux was analysed. The two-dimensional model was applied in the analysis of the flow phenomenon. It was also used to estimate the range of changes of the coefficients in a one-dimensional model. It was stated that the generally used one-dimensional model is limited and can be applied to the description of flows for large frequencies and amplitudes of pulsation of the mass flux. This results from the dependence of discharge coefficient α and effective length l_d on time. As it was demonstrated by numerical investigations, when the amplitude of pulsation of the mass flux $h_{\dot{M}}$ exceeds 0.1, then $h_{\Delta p} > 0.2$, and when the value of Strouhal number Sh is greater than 0.4, the coefficients in the one-dimensional model can not be considered constant. In consequence, this model only approximately describes the phenomenon of flow.

From the one-dimensional model the analytical relations were derived which express the error of measurement of the mean value of mass flux as a function of amplitude, frequency and shape of pulsation. The critical conditions which allow for estimation of the range of application of the differential pıessure type flowmeters to measurements of pulsating flows were formulated. It was stated in the case of pulsations of sinusoidal shape with pulsation amplitude $h_{\dot{M}} < 0.1$ ($h_{\Delta p} < 0.2$) and Strouhal number $\mathrm{Sh} < 0.1$, the coefficients in the model (2.8) are practically constant and the error of measurement of the mean value of mass flux σ is less then 1%. Results of the calculations were compared with the available experimental data and satisfactory conformity was oberved.

It seems advisable to extend the scope of investigations by considering the system: confuser-differential manometer or confuser-differential pressure converter to account for the fact that dynamical properties of an instrument used to measure the pressure difference can have significant influence on the results of measurement.

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