Arch. Mech., 38, 1–2, pp. 185–190, 1986

The effect of crack front irregularity on the fracture toughness of brittle materials

E. SMITH (MANCHESTER)

IF THE STRUCTURAL elements of a brittle material have a wide distribution of failure stresses, a macroscopic crack will have an irregular front. This irregularity produces an increase in the fracture toughness of the material, and the paper models this irregularity in a very simple manner. The associated theoretical analysis shows that the fracture toughness can be raised appreciably as a result of the irregularity.

Jeśli elementy konstrukcyjne wykonane z kruchego materiału mają znaczny rozrzut naprężeń niszczących, to makroskopowa szczelina będzie miała nieregularne czoło. Ta nieregularność powoduje wzrost odporności materiału na pękanie. W pracy w prosty sposób zmodelowano taką nieregularność. Związana z tym analiza teoretyczna wykazuje, że odporność na pękanie może znacznie wzrosnąć przy nieregularności czoła szczeliny.

Если конструкционные элементы изготовленные из хрупкого материала имеют значительный разброс разрушающих напряжений, то макроскопическая щель будет иметь нерегулярный фронт. Эта нерегулярность вызывает увеличение устойчивости материала к растрескиванию. В работе простым способом моделируется такая регулярность. Связанный с этим способом теоретичнский анализ показывает, что устойчивость к разтрескиванию можем значительно возрастать при нерегулярности фронта щели.

1. Introduction

IF A BRITTLE material, such as a rock-like material, contains a macroscopic crack, crack extension is associated with the formation of microcracks in a zone ahead of the crack tip [1, 2]. The microcrack size is typically that of the structural element, e.g. the crystal size, and the microcracks form because the structural elements have a distribution of failure stresses. It has been surmised [1] that the microcracks around a primary crack are contained within two zones: (a) an inner zone very close to the crack tip where the microcracks interact or link with the primary crack, and (b) an outer zone in front of the crack tip where the microcracks change the effective elastic modulus within that zone.

EVANS, HEUER and PORTER [1] argue that this modulus change within the outer microcrack zone what is a source of toughness enhancement, i.e. it is responsible for an increase in J_{IC}. To support their case, they rely on the theoretical results [3, 4] for a crack penetrating, and included within an elastic inclusion of lower moduls. These results indicate that, for a given applied stress and crack length the crack opening decreases as the inclusion modulus decreases. Thus Evans, Heuer and Porter presume that the crack opening decreases as the microcrack density in the outer zone and the size of this zone increases, thereby causing a corresponding increase in J_{IC}. However, when considering the effects of this outer zone, the present author believes that it is inappropriate to use the results for a model in which a crack penetrates and is included within an elastic inclusion of lower modulus. It is more appropriate to use the results from a model in which an elastic inclusion of lower modulus lies ahead of the crack tip, i.e., the crack tip does not penetrate the inclusion. In this context TIROSH and TETELMAN [5] have analysed the Mode I plane strain model of a solid containing a circular cylindrical hole ahead of a crack tip, with the centre of the hole lying on the crack plane. Their numerical results clearly show that the hole's presence leads to an increase in the crack tip stress intensity, the magnitude of this increase being greater the greater the hole radius is and the nearer the hole is to the crack tip; similar results are obtained with the corresponding Mode III model for which an analytical solution is possible [6]. On this reckoning, the author [7] takes the alternative view to that of Evans, Heuer and Porter, and believes that the presence of the outer microcrack zone is, in a direct sense, a source of weakness rather than toughness enhancement, as regards its effect on J_{IC}. However, as indicated earlier, the formation of microcracks ahead of a macroscopic crack is indicative of a distribution of failure stresses for the material elements. The front of a macroscopic crack will therefore have an irregular character and crack extension should be more diffcult, i.e. J_{IC} should increase as a result of this irregularity. The degree of irregularity will increase with the width of the distribution of failure stresses, and this should lead to an increase in J_{IC}. It is because of this irregularity effect, rather than a reduced modulus within the outer microcrack zone, that the author believes that microcracking is responsible for J_{IC} values that are larger than one would expect for a brittle material.

The effect of crack tip irregularity on macroscopic crack extension can be appreciated by appealing to the results [8] of a Mode III model (Fig. 1) in which a long crack has two

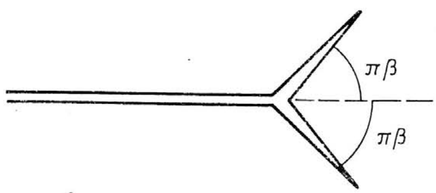

FIG. 1. The model [8] of a bifurcated crack, with two very short and identical segments making angles $\pi\beta$ with the major crack.

very short segments of equal length at its tip, each segment making an angle $\pi\beta$ with the major crack. The crack tip stress intensity K_B at the tips of the segments is related to the overall crack tip stress intensity K_A by the expression [8]

$$K_B = \frac{K_A}{\sqrt{2}} \left[\frac{1-\beta}{\beta} \right]^{\beta/2} \tag{1}$$

showing that crack extension is more difficult (i.e. $K_B < K_A$) when a crack front is irregular. This particular model is somewhat special in that there is no prolongation of the major crack, but instead two small segments form symmetrically at the end of the major crack. The present paper presents a more general model to describe the irregularity at a crack tip, and the results of a theoretical analysis complement those from the analysis of the bifurcation model in Fig. 1.

2. Theoretical analysis

The model is illustrated in Fig. 2. A semi-infinite crack exists within an infinite solid that deforms according to Mode III loading conditions, the overall stress intensity due to the applied loadings being K_A. This crack has two identical segments of length a, and these are perpendicular to the main crack, being situated at a distance l behind the main crack tip. With a Mode III problem, the displacement w, which is parallel to the figure normal at all points of the solid, satisfies Laplace's equation

$$\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}=0 \tag{2}$$

the appropriate stress components being

$$p_{xz}=\mu\frac{\partial w}{\partial x} \quad \text{and} \quad p_{yz}=\mu\frac{\partial w}{\partial y}, \tag{3}$$

where μ is the shear modulus of the material. Accordingly, there exists some complex function $F(z)$ where $z = x+iy$, such that the displacement is given by the relation

$$\mu w = \mathrm{Re}[F(z)] \tag{4}$$

and the stresses by the expression

$$\frac{dF}{dz}=p_{xz}-ip_{yz}=\mu\frac{\partial w}{\partial x}-i\mu\frac{\partial w}{\partial y}; \tag{5}$$

$F(z)$ must satisfy the appropriate boundary conditions, and in order to determine $F(z)$, the z plane is mapped into the $t = \varepsilon+i\eta$ plane, where a solution can be readily obtained using the relation

$$\frac{dF}{dt}=\frac{dF}{dz}\cdot\frac{dz}{dt}=\frac{\partial w}{\partial \varepsilon}-i\mu\frac{\partial w}{\partial \eta}. \tag{6}$$

The conformal transformation that maps the region in the upper half of the z plane outside the crack (Fig. 2) into the upper half of the t plane (Fig. 3) with corresponding points transforming as shown in Figs. 2 and 3 is

$$z+l=a\left[\frac{(t+L+s)^2}{s^2}-1\right]^{\frac{1}{2}}. \tag{7}$$

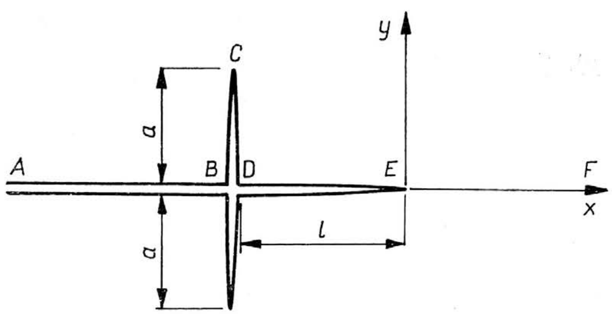

FIG. 2. The $z = (x+iy)$ plane containing a semi-infinite crack with identical segments near the main crack tip. The overall stress intensity due to the applied loadings is K_A.

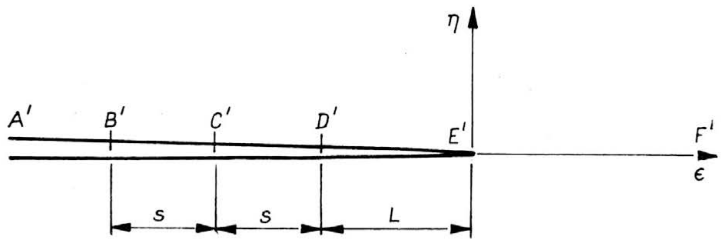

FIG. 3. The $t = \varepsilon + i\eta$ plane obtained from the z plane by the conformal transformation (7); corresponding points are as indicated (see also Fig. 2).

The complex function that satisfies the boundary conditions in Fig. 2 has the form

$$F = iAt^{\frac{1}{2}}, \tag{8}$$

where A is a real constant. For large t, z, the relation (7) gives $z = at/s$, and if K_A is the overall stress intensity due to the applied loadings, it follows from the relations (5) and (8) that

$$A = -K_A\sqrt{\frac{2a}{\pi s}}. \tag{9}$$

For small t, z, relation (7) gives

$$\frac{l}{a} = \left[\frac{(L+s)^2}{s^2} - 1\right]^{\frac{1}{2}} \tag{10}$$

and

$$z = \frac{a(L+s)t}{s^2\left[\frac{(L+s)^2}{s^2} - 1\right]^{\frac{1}{2}}}, \tag{11}$$

whereupon it follows from the relations (5), (8) and (11) that the stress intensity K_L at the tip E in the original model is

$$K_L = -A\sqrt{\frac{\pi}{2}}\left[\frac{(L+s)^2}{s^2} - 1\right]^{\frac{1}{4}} \frac{s}{a^{\frac{1}{2}}(L+s)^{\frac{1}{2}}} \tag{12}$$

and the relations (9), (10) and (12) give

$$\frac{K_L}{K_A} = \frac{\left(\frac{l}{a}\right)^{\frac{1}{2}}}{\left[1+\left(\frac{l}{a}\right)^2\right]^{\frac{1}{4}}}. \tag{13}$$

To obtain the stress intensity K_B at the tip C in the original model, consider the situation in the immediate vicinity of C, i.e. $z = -l + ai + z_0$, where z_0 is small. The relations (5), (7) and (8) then show that

$$K_B = -A\sqrt{\frac{\pi}{2}}\frac{s}{(L+s)^{\frac{1}{2}}(2a)^{\frac{1}{2}}}, \tag{14}$$

whereupon the relations (9), (10) and (14) give

$$\frac{K_B}{K_A} = \frac{1}{\sqrt{2}\left[1+\left(\frac{l}{a}\right)^2\right]^{\frac{1}{4}}}. \tag{15}$$

For the special case where $l = 0$, $K_B = K_A/\sqrt{2}$ a result that checks, for $\beta = \frac{1}{2}$, with the bifurcation result (1) in the Introduction.

3. Discussion

The preceding section has analysed a very simple model that simulates the irregularity at the tip of a macroscopic crack in a brittle material. The irregularity is due to the material elements having a distribution of failure stresses, which allows for the formation of microcracks in the vicinity of the macroscopic crack tip. The relation (13) shows that the irregularity is responsible for an increase in the macroscopic fracture toughness of the material since the stress intensity K_L at the tip E is decreased as a consequence of the irregularity, i.e. the presence of the microcrack segments. For example with $l/a = \frac{1}{2}$, the ratio K_L/K_A is equal to 0.67, and becomes even smaller as l/a decreases. These results therefore complement the bifurcation model results described in the Introduction, and support the view that the macroscopic fracture toughness can be raised appreciably as a result of the crack tip irregularity. Of course, this increase has to be balanced against the decrease due to the microcracks which form ahead of the primary crack, and which are not accounted for in the present paper's model. The overall fracture toughness is therefore a composite stemming from two effects that act in opposing directions: (a) an effect due to the microcracks ahead of the primary crack and (b) an effect due to the crack irregularity. Both effects may be incorporated within a very simple descriptive model by assuming that there is a Dugdale–Bilby–Cottrell–Swinden DBCS zone [9, 10] of nonlinear material ahead of a macroscopic crack tip. The average stress σ_{AV} within the zone is governed by the density of microcracks, while the failure displacement u_F at the tip is governed by the crack tip irregularity effect; the overall resistance to macroscopic crack extension is $J_{IC} \equiv \sigma_{AV}\, u_F$.

Finally it is worth mentioning that, although this paper has concentrated on the effect of crack front irregularity on the onset of crack extension, irregularity also influences the subsequent growth of a macroscopic crack. Thus as indicated by WNUK and MURA [2], with some brittle materials such as for example Westerly granite, the onset of macroscopic crack extension occurs at a critical value J_{IC}, and further extension requires that J be increased. Crack front irregularity will obviously increase the material's resistance to continued crack extension, i.e. the slope of the material's J-crack growth resistance curve.

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DEPARTMENT OF METALLURGY AND MATERIALS SCIENCES
JOINT MANCHESTER UNIVERSITY/UMIST MANCHESTER, UNITED KINGDOM.

Received April 29, 1984.