

On the quasi-static growth of cracks

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THE GROWTH of a crack modelled by a region of small transversal dimensions is studied on the basis of general thermodynamics of irreversible processes. The non-local model has been selected due to the fact that in applying the traditional model — a cut — some of the essential features of the crack growth process are lost. From the expression of the entropy production density the singular component is isolated which is connected with the crack motion and located at the crack tip. Certain integrals invariant with respect to the choice of the contour of integration and analogous to the well known J -integral can be interpreted as terms describing thermodynamic forces driving the crack. The equations of crack motion are illustrated by an example.

Wzrost szczelin rozważa się na podstawie ogólnych zasad termodynamiki procesów nieodwracalnych, przy czym szczelinę modeluje się za pomocą obszaru o małych wymiarach poprzecznych. Zastosowanie modelu nielokalnego tłumaczy się tym, że przy tradycyjnym modelu szczeliny, mającym postać cięcia, traci się pewne istotne elementy procesu pęknięcia. Z wyrażenia na gęstość produkcji entropii wydzielono składnik osobliwy związany z procesem wzrostu szczeliny i zlokalizowany w jej wierzchołku. Pewne całki niezmiennicze ze względu na wybór konturu całkowania są analogicznie do znanej z teorii pęknięcia całki J i mogą być interpretowane jako wyrażenia opisujące siły termodynamiczne poruszające szczelinę. Równanie wzrostu szczeliny zilustrowano przykładem.

С общих позиций термодинамики необратимых процессов изучается рост трещины, которая моделируется областью с малым поперечным размером. Выбор нелокальной модели обусловлен тем, что в традиционной модели трещины—разрезы—теряются некоторые существенные черты роста трещины. В плотности порождения энтропии выделена сингулярная составляющая, связанная с ростом трещины и локализованная в ее вершине. При этом роль термодинамических сил, движущих трещину, играют инвариантные относительно контура интегрирования интегралы, подобные известному интегралу J . Уравнения роста трещины иллюстрируются примером.

IT IS KNOWN [1, 2] that the crack growth characteristics (trajectory and speed) contain information on the material strength of fracture mechanisms. The process of decoding information is a major problem in the modern theory of strength of materials.

In fracture mechanics the study of crack growth proceeds in two principal directions. The first one consists in determining the stress field components in bodies containing cracks which move according to prescribed laws [3–5]; on the other hand, attempts are made to construct suitable crack growth laws. In the latter case the states of stress and strain are usually assumed to be known (quasi-static approximation), and then the speed of a crack moving along a prescribed trajectory is evaluated [4, 6, 7]. In order to determine the trajectories, certain special principles must usually be applied [5, 8]. Therefore, the relations between the crack growth laws and the material strength characteristics are not considered in those papers.

The aim of this paper is to construct the crack growth equation on the general basis of thermodynamics of irreversible processes, without applying any additional principles,

and to establish the relationship between the crack growth and the material strength characteristics.

The crack is modelled by a region of a small transversal dimension a which is of the order of magnitude of the smallest characteristic dimension of the real material region which still may be considered as statistically homogeneous (i.e. in which the continuous medium model may be applied). The choice of such a model is justified in the light of the fact that in applying the traditional model of a cut some of the essential features of the crack growth process get lost.

The production of entropy in a body with a crack is decomposed into two parts, regular and singular, the latter being localized in the vicinity of the crack tip. The singular component represents a bilinear form in fluxes and forces; the role of forces is played by certain curvilinear integrals (analogous to the well-known J -integral) invariant under the choice of integration contour, and the role of fluxes — by the linear and angular crack tip velocities.

The crack equations are constructed within the framework of linear approximation of the method of approximations, the value of a being assumed as the small parameter. Confining the considerations to the terms linear in a , we make the assumption that the crack tip region does not deform; at the same time the crack is completely characterized by its trajectory and growth rate. It is shown that the trajectory equation may be derived only by replacing the usual crack model, a cut, with another model.

A particular example of motion of a shallow crack in a linear Maxwell medium is considered. It is shown that the non-homogeneity of the material strength properties influences the form of the trajectory.

1. Energy and entropy balance

In a medium endowed with relaxation properties and lacking chemical reactions and mass transfer, the energy balance equation has the form

$$(1.1) \quad \dot{u} = \partial_i(\sigma_{ij}\varepsilon_{ij}) - \text{div } q.$$

Here u — internal energy density, σ_{ij} , ε_{ij} — stress and strain tensors, q — heat flux vector. u is now written in the usual form as the sum

$$(1.2) \quad u = f + Ts$$

with f denoting the free energy density, T — absolute temperature, s — entropy density. The energy balance in Eq. (1.1) takes the form

$$(1.3) \quad \dot{f} + T\dot{s} + \dot{T}s = \partial_i(\sigma_{ij}\varepsilon_{ij}) - \text{div } q.$$

The time rate of the change of entropy \dot{s} consists of two parts: \dot{s}_i — entropy production due to irreversible processes, and \dot{s}_e — the part due to the internal entropy flux

$$(1.4) \quad \dot{s}_e = -\text{div} \left(\frac{q}{T} \right).$$

Equations (1.3) and (1.4) yield the entropy production

$$(1.5) \quad T\dot{s}_i = T\dot{s} - T\dot{s}_e = \partial_i(\sigma_{ij}\varepsilon_{ij}) - \frac{1}{T}q \operatorname{grad} T - \dot{f} - \dot{T}s.$$

Let us assume that a crack is propagating within the body occupying the region V . The crack is modelled by a region $V_1 \subset V$ characterized by the small transversal dimension a (Fig. 1).

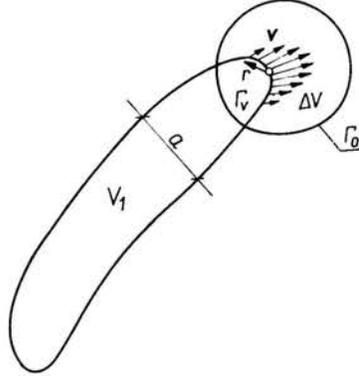


FIG. 1.

Properties of the fractured material in the region V_1 are different from those of the original in $V_2 = V \setminus V_1$. Thus the densities of the free energy, entropy and work inside the regions V_1 and V_2 are also different:

$$(1.6) \quad \begin{aligned} f(x) &= \{1 - \chi(x)\} f_2(x) + \chi(x) f_1(x), \\ s(x) &= \{1 - \chi(x)\} s_2(x) + \chi(x) s_1(x), \\ (\sigma_{ij}\varepsilon_{ij}) &= \{1 - \chi(x)\} (\sigma_{ij}\varepsilon_{ij})_2 + \chi(x) (\sigma_{ij}\varepsilon_{ij})_1, \\ \chi(x) &= \begin{cases} 1, & x \in V_1, \\ 0, & x \in V_2. \end{cases} \end{aligned}$$

Such a choice of the model is connected with the fact that in applying the traditional model of the crack, a cut, the characteristic features of fracture propagation are lost.

Equations (1.5) and (1.6) for a body with a crack V_1 yield in the case of thermal equilibrium, the expression

$$(1.7) \quad \begin{aligned} T\dot{s}_i &= \{1 - \chi(x)\} \{\partial_i(\sigma_{ij}\varepsilon_{ij})_2 - \dot{f}_2 - \dot{T}s_2\} + \chi(x) \{\partial_i(\sigma_{ij}\varepsilon_{ij}) - \dot{f}_1 - \dot{T}s_1\} - \frac{q}{T} \operatorname{grad} T \\ &\quad + \dot{\chi}(x) \{[f_2 + Ts_2 - (\sigma_{ij}\varepsilon_{ij})_2] - [f_1 + Ts_1 - (\sigma_{ij}\varepsilon_{ij})_1]\} \\ &\equiv \{1 - \chi(x)\} T\dot{s}_{i2} + \chi(x) T\dot{s}_{i1} + \dot{\chi}(x) \{h_2 - h_1\}. \end{aligned}$$

Here \dot{s}_{i1} and \dot{s}_{i2} denote the entropy productions due to irreversible processes occurring within the regions V_1 and V_2 , and h_1, h_2 denote the enthalpies in the respective regions V_1, V_2 . The term $\dot{\chi}\{h_2 - h_1\}$ corresponds to the production of entropy due to the change of the region V_1 (crack growth), and $\dot{\chi}(x)$

$$(1.8) \quad \dot{\chi}(x) = \frac{\partial \chi(x)}{\partial x_k} \frac{dx_k}{dt} = \delta_k(\partial V_1) v_k.$$

Here ∂V_1 is the boundary of V_1 , v_k — components of the velocity vector of the boundary points, ∂V_1 and $\delta_j(\partial V_1)$ is calculated in the following manner [9]: for any bounded function $\psi(x)$

$$(1.9) \quad \int \psi(x) \delta_j(\partial V_1) dV = \oint_{\partial V_1} \psi(x) n_j d\Omega,$$

n_j are components of the external unit normal to ∂V_1 .

Let us confine our considerations to the plane problem in which the boundary of a region V_1 is represented by a contour Γ . It is assumed that the velocity v_j differs from zero only in the small neighbourhood Γ_v of the crack tip and it may be represented by a power in the small parameter a (Fig. 1).

$$(1.10) \quad \mathbf{v} = \mathbf{v}_0 + \mathbf{w} \times \mathbf{r} + \mathbf{v}_1.$$

The first two terms in Eq. (1.10) correspond to the rigid motion of the crack tip, and \mathbf{v}_1 is the velocity connected with the crack tip deformation.

The production of entropy in the region V due to the growth of the region V_1 may be written, in view of Eqs. (1.8) to (1.10), in the form

$$(1.11) \quad \int_V \dot{\chi}(x) \{h_2 - h_1\} dV = -v_0(\mu_0 - I) - w(\mu_1 - D) + O(a^2),$$

$$\mu_0 = e_j \int_{\Gamma_v} h_1 n_j d\Gamma, \quad \mu_1 = \int_{\Gamma_v} (x_1 n_2 - x_2 n_1) h_1 d\Gamma,$$

$$I = e_j \int_{\Gamma_v} h_2 n_j d\Gamma, \quad D = \int_{\Gamma_v} (x_1 n_2 - x_2 n_1) h_2 d\Gamma, \quad e_j = \frac{v_{0j}}{v_0}.$$

Here Γ_v — the moving portion of the contour Γ (Fig. 1); μ_0 — surface density of enthalpy for a rectilinear crack; μ_1 — the complementary surface density of enthalpy connected with the crack curvature, v_0 — rectilinear velocity of the tip; w — angular crack tip velocity; I — enthalpy variation of the region V_2 due to a unit increment of the crack length; D — the complementary variation of enthalpy in the region V_2 due to the curvature of the crack increment.

Parameters μ_0 and μ_1 depend solely on the state of the fractured material and are determined according to the model of fracture of the body.

Parameters I and D depend on the state of the medium surrounding the fractured region V_1 and are functionals of the fracture process.

2. Invariant integrals μ_0 , μ_1 and I , D

In a homogeneous isotropic medium μ_0 , μ_1 , I , D are expressed by curvilinear integrals independent of the contour of integration. In order to establish the invariance properties in the case of homogeneity and isotropy, the equations of state, equilibrium and temperature field homogeneity must be used.

In view of homogeneity of the medium, enthalpy depends on the coordinates indirectly through the state parameters which may be assumed as the stresses σ_{ij} and entropy s . Then the conditions of invariance of enthalpy under arbitrary translations are reduced to the equations

$$(2.1) \quad \tilde{\partial}_k h = \partial_k h - \frac{\partial h}{\partial \sigma_{ij}} \partial_k \sigma_{ij} - \frac{\partial h}{\partial s} \partial_k s = 0.$$

Here $\tilde{\partial}_k$ is the operator of differentiation of the explicit functions of x_k , and ∂_k — the total derivative operator.

In view of the temperature field homogeneity $\partial_k T = 0$ and $\partial_k h$ are reduced to the divergence form

$$(2.2) \quad \tilde{\partial}_k h = \partial_j \{ \delta_{jk} (h - Ts + \sigma_{il} \varepsilon_{il}) - \sigma_{ij} u_{i,k} \},$$

where u_i is the displacement field.

From Eqs. (2.2) and (2.1) it follows that in a homogeneous medium and for any $\Delta V \subset V$ bounded by Γ_0 (Fig. 1) we have

$$(2.3) \quad \int_{\Delta V} \tilde{\partial}_k h dV = \oint_{\Gamma_0} n_j \{ \delta_{jk} f - \sigma_{ij} u_{i,k} \} d\Gamma = 0.$$

For a non-homogeneous medium (containing a crack) the enthalpy density may be represented in a form analogous to Eq. (1.6),

$$(2.4) \quad \begin{aligned} h(x) &= h_2(x) \{ 1 - \chi(x) \} + h_1(x) \chi(x), \\ \tilde{\partial}_k h &= \delta_k(\Gamma) (h_2 - h_1). \end{aligned}$$

From Eqs. (2.2) and (2.4) we now obtain, instead of Eq. (2.3), the relation

$$(2.5) \quad \begin{aligned} \int_{\Delta V} \tilde{\partial}_k h dV &= \int_{\Gamma_0} (h_2 - h_1) n_k d\Gamma = \oint_{\Gamma_0} n_j \{ \delta_{jk} f - \sigma_{ij} u_{i,k} \} d\Gamma \\ &= \int_{\Gamma_2} n_j \{ \delta_{jk} f_2 - (\sigma_{ij} u_{i,k})_2 \} d\Gamma - \int_{\Gamma_1} n_j \{ \delta_{jk} f_1 - (\sigma_{ij} u_{i,k})_1 \}, \\ \Gamma_0 &\subset \Gamma \cap \Delta V, \quad \Gamma_1 = \Gamma_0 \cap V_1, \quad \Gamma_2 = \Gamma_0 \cap V_2. \end{aligned}$$

Similarly, in isotropic media the enthalpy should be invariant under the group of rotations, and hence in a polar coordinate system it should not explicitly depend upon the angle φ ,

$$(2.6) \quad \tilde{\partial}_\varphi h = \partial_\varphi h - \frac{\partial h}{\partial \sigma_{ij}} \partial_\varphi \sigma_{ij} - \frac{\partial h}{\partial s} \partial_\varphi s = 0,$$

$\partial_\varphi h$ may be reduced to a divergence form similar to Eq. (2.2); this yields the invariance property of the integrals μ_1 and D ,

$$\begin{aligned} \int_{\Delta V} \tilde{\partial}_\varphi h dV &= \int_{\Gamma_0} (h_2 - h_1)(x_1 n_2 - x_2 n_1) d\Gamma = D - \mu_1 \\ &= \oint_{\Gamma_0} \{ (x_1 n_2 - x_2 n_1) f - \sigma_{ij} n_j (x_1 \partial_2 - x_2 \partial_1) u_i - (\sigma_{i1} u_2 - \sigma_{i2} u_1) n_i \} d\Gamma, \\ (2.7) \quad D &= \int_{\Gamma_2} \{ (x_1 n_2 - x_2 n_1) f_2 - [\sigma_{ij} n_j (x_1 \partial_2 - x_2 \partial_1) u_i]_2 - [(\sigma_{i1} u_2 - \sigma_{i2} u_1) n_i]_2 \} d\Gamma \\ &\quad - \int_{\Gamma_1} \{ (x_1 n_2 - x_2 n_1) f_1 - [\sigma_{ij} n_j (x_1 \partial_2 - x_2 \partial_1) u_i]_1 - [(\sigma_{i1} u_2 - \sigma_{i2} u_1) n_i]_1 \} d\Gamma. \end{aligned}$$

Thus it follows from Eqs. (2.3), (2.5), (2.6) and (2.7) that the integrals I , D , μ_0 , μ_1 are invariant with respect to the choice of contours Γ_1 , Γ_2 . In view of that invariance the region of integration ΔV may be contracted and reduced to the size of an immediate neighbourhood of the crack tip, its diameter becoming of the order of magnitude of a .

Density of the entropy production in a body with a crack V_1 is equal to

$$(2.8) \quad T \dot{s}_i = T \dot{s}_{i0} - \delta_a(x-M) \{ (\mu_0 - I) v_0 + (\mu_1 - D) w + O(a^2) \}.$$

Here \dot{s}_{i0} — entropy production in a body with a stationary crack; $\delta_a(x-M)$ — δ -function concentrated in the neighbourhood of the crack tip M , its diameter being of the order of a [10].

Equation (2.8) shows that the entropy production in a body containing a crack consists of two components, the regular and singular ones, the latter being concentrated in the small neighbourhood of the crack tip. Both the singular and regular components allow for representations by bilinear forms in fluxes and forces; v_0 and w play the parts of fluxes in the singular production process, while the invariant integrals $(\mu_0 - I)$ and $(\mu_1 - D)$ represent the forces.

It should be noted that the fluxes and forces written explicitly in Eq. (2.8) correspond to the linear part of expansion of the singular production into a power series of the small characteristic parameter a . Taking into account the remaining terms of the expansion we can determine the additional production connected with the deformation of the crack tip in the process of its motion.

3. Equations of quasi-static growth of cracks

Pursuant to the second law of thermodynamics

$$(3.1) \quad \dot{s}_i \geq 0.$$

The equality sign holds true only in reversible processes.

Let us consider the ideal process of crack growth in which no irreversible deformations occur; this means that by reversing the sign of loads the crack is sealed, the system and the thermostat returning to their initial states. Such an ideal growth may be termed per-

fectly brittle. Under non-zero fluxes ($v_0 \neq 0, w \neq 0$) the entropy production $\dot{s}_i = 0$ in such processes, and due to the independence of v_0 and w , it follows that

$$(3.2) \quad \begin{aligned} I - \mu_0 &= 0, \\ D - \mu_1 &= 0. \end{aligned}$$

In the case of linear approximation in a , the crack is characterized by two parameters: the length and curvature of its middle line. Taking into account that I and D are functionals of the process, it is concluded that Eqs. (3.2) completely determine the brittle crack growth.

However, in most of the real structural materials the crack growth is accompanied by irreversible deformations and is not a reversible process. In such cases the simplest equations governing the quasi-static growth of quasi-brittle cracks will be the linear phenomenological relations between the fluxes and forces, similar to the traditional relations of linear thermodynamics of irreversible processes.

$$(3.3) \quad \begin{aligned} v_0 &= L_{11}(\mu_0 - I) + L_{12}(\mu_1 - D), \\ w &= L_{21}(\mu_0 - I) + L_{22}(\mu_1 - D). \end{aligned}$$

Here L_{ij} denote certain phenomenological coefficients.

4. Shallow crack in a Maxwell medium

Consider a crack extending in an infinite medium subject to tension p applied at infinity. The medium is assumed to obey the linear Maxwell model rules

$$(4.1) \quad \dot{\epsilon}_{ij} = \dot{\epsilon}_{ij}^e + \frac{1}{\tau} \text{dev } \epsilon_{ij}^e.$$

Here ϵ_{ij}^e — elastic part of ϵ_{ij} , τ — relaxation time. It is moreover assumed that the crack is shallow and the stress intensity factors may be determined according to Eq. (A.4). The

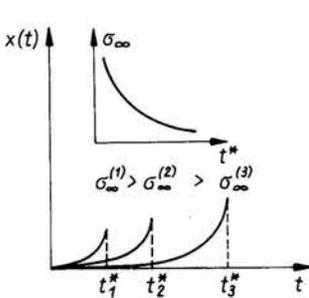


FIG. 2.

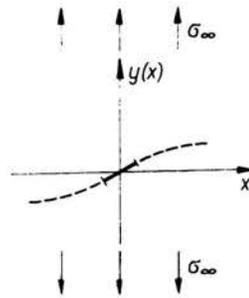


FIG. 3.

abscissa of the crack vertex is selected as the natural parameter. The stress field is obtained by superposition of uniform and asymptotic fields. Thermal effects are disregarded.

The first Eq. (2.2) takes the form

$$(4.2) \quad \left\{ 1 + \frac{c}{\dot{x}(t)} \right\} J_0(t) + R(t) = \mu_0,$$

$$(4.2) \quad \begin{aligned} t \geq t_1 \geq t_0, \quad x(t_1) = a, \\ \text{[cont.]} \quad J_0(t) = \frac{1}{E} \{K_I^2(t) + K_{II}^2(t)\} = \frac{\pi p^2}{2E} x(t), \end{aligned}$$

$$R(t) \simeq \frac{1+\nu}{E} p^2 c(t-t_0), \quad c = \frac{a}{2\tau}, \quad \mu_0 = h_1 a.$$

Here $K_I(t)$, $K_{II}(t)$ — stress intensity factors, E — Young's modulus, ν — the Poisson ratio, t_0 — the time of application of loads, t_1 — time of creation of the macrocrack.

Equations (4.2) yield the crack growth velocity $\dot{x}(t)$,

$$(4.3) \quad \begin{aligned} \dot{x}(t) &= \frac{cx(t)}{l^* - l(t) - x(t)}, \\ l^* &= \frac{2E\mu_0}{\pi p^2}, \quad l(t) = \frac{2(1-\nu)}{\pi} c(t+t_0). \end{aligned}$$

The solution in Eq. (4.3) is shown in Fig. 3. The necessary condition of crack stability with respect to the external load $\partial x(t)/\partial p = (\infty)$ yields the critical length $x^* = x(t^*)$,

$$(4.4) \quad x^* = l^* - l(t^*).$$

The above expression shows that in a viscoelastic medium the critical length x^* differs from the Griffith critical value by the second term $l(t^*)$ which is connected with the energy dissipation.

The equations similar to Eq. (4.3) have been derived in [6].

In order to satisfy the second Eq. (3.2), it is necessary to specify the form of the crack tip. For the sake of simplicity let us assume it to have a wedge form. We obtain

$$(4.5) \quad \begin{aligned} \frac{2r_0}{E} \left\{ 1 + \frac{c}{\dot{x}(t)} \right\} K_I(t) K_{II}(t) + \frac{r_0^2}{3} \partial_y R(t) + \mu_1 = 0, \\ r_0 \sim a. \end{aligned}$$

The following equation (cf. also Eq. (A.4)) for the crack ordinates is now obtained:

$$(4.6) \quad y(x) + \frac{2}{\pi} \int_0^x \sqrt{\frac{S}{x-S}} \left\{ \frac{y(x)-y(S)}{x-S} - \frac{y(S)}{S} \right\} dS = \frac{r_0}{3} x \frac{\partial_y(\mu_0 - R)}{\mu_0 - R}.$$

The right-hand side of Eq. (4.6) is a function of the non-homogeneity characteristics of the strength parameters, in accordance with the assumed model of material fracture.

Non-homogeneity of the strength properties produces the undulation of the crack trajectory (fracture surface roughness). In real materials the strength properties are usually described by random fields and Eq. (4.6) is transformed into a stochastic equation of the fracture surface (integrations and differentiations should be considered in the stochastic sense).

It should be pointed out that by comparing the solutions of such stochastic equations with real fracture surfaces obtained experimentally, we may be able to determine the micro-nonhomogeneity of material properties of the body.

Appendix

The analytical determination of intensity factors for curvilinear cracks was discussed in [5, 11, 12]. In this paper we shall use the method presented in [12].

According to [12], in an isotropic body containing smooth curvilinear cuts L_j ($j = 1, 2, \dots, K$) the evaluation of intensity factors may be reduced to determining the functions $p(t)$ at L_j satisfying the equation

$$(A.1) \quad \frac{1}{\pi_i} \int_L \left(\frac{e^{i\psi_0}}{t-t_0} - \frac{e^{-i\psi_0}}{t-t_0} \right) p(t) dt - \frac{1}{\pi_i} \int_L \left(\frac{e^{i\psi_0}}{t-t_0} + \frac{t-t_0}{(t-t_0)^2} e^{-i\psi_0} \right) \overline{p(t)} \overline{dt} = \phi(t_0),$$

$$\int_{L_j} p(t) dt = 0, \quad j = 1, 2, 3, \dots, k,$$

$$\phi(t) = -\{X_n(t) + iY_n(t)\},$$

$$t, t_0 \in L, \quad L = \bigcup_{j=0}^K L_j, \quad i = \sqrt{-1}.$$

Here $\psi_0 = \psi(t_0)$ — the angle between the external normal to the left-hand edge of the cut and the Ox -direction, $X_n(t)$, $Y_n(t)$ — the given functions defined by the loads applied to the crack edges and at infinity.

The intensity factors are then calculated from the equations

$$(A.2) \quad K_I(b_j) - iK_{II}(b_j) = -\sqrt{\frac{8\pi}{b_j - a_j}} \lim_{t \rightarrow b_j} \{p(t) \sqrt{(t-a_j)(t-b_j)}\}.$$

Here a_j and b_j — origins and ends of the arcs L_j .

Under the assumption of $K = 1$, the contour L is analytical and shallow

$$\sup_{t, t_0 \in L} |\kappa(t)(t-t_0)| \ll 1$$

with $\kappa(t)$ denoting the curvature of L at point t , and Eqs. (A.1) and (A.2) yield the expressions for the intensity factors of the cracks shown in Fig. 3

$$(A.3) \quad K_I - iK_{II} = \frac{2\sigma_\infty}{\sqrt{2\pi(b-a)}} \int_L \sqrt{\frac{t-a}{b-t}} dL.$$

From Eq. (A.3) we obtain K_I and K_{II}

$$(A.4) \quad K_I \simeq \frac{\sigma_\infty}{\sqrt{2}} \sqrt{\pi x},$$

$$K_{II} \simeq \frac{K_I}{\pi x} \int_0^x \sqrt{\frac{S}{x-S}} \left(\frac{y(x)-y(S)}{x-S} - \frac{y(S)}{S} \right) dS + K_I \frac{y(x)}{x}.$$

Discussion

1. The choice of the non-local crack model follows from the fact that in the limiting case of $a \rightarrow 0$ the integrals D and μ_1 (representing the thermodynamic forces connected with the angular velocity ω) tend to zero; consequently, in the case of a cut-type model of a crack, the trajectory equation becomes degenerate. In the author's opinion it is necessary

to introduce an additional length parameter so as to give a complete description of the crack growth process.

2. As it has been mentioned in Sect. 1 quadratic terms in the expansion of the entropy production are responsible for the deformation of the crack tip. This factor may be of major importance in the analysis of brittle-viscous transitions in crack propagation processes or for fatigue crack analysis, and thus it would be most useful to construct the relations connecting the crack tip velocity and the corresponding thermodynamical forces. Similar attempts made in the theories of plasticity and creep indicate that serious difficulties may be encountered in solving that problem.

3. It is seen from Eq. (1.7) that the generalized thermodynamic flux χ corresponds to a thermodynamic force ($h_2 - h_1$) — the jump of enthalpy (chemical potentials) of the fractured and unfractured phases. It is known that the phase equilibrium (stationary crack) corresponds to the equality of chemical potentials. By means of the thermodynamical analysis the process of fracture is reduced in a natural way to a problem of phase transition.

4. The possibility of evaluating the strength properties (which cannot be determined directly) by investigating the statistical properties of the fracture surface seems very promising. It can be seen from the solution of the stochastic equation of crack surface that its properties depend not only on the non-homogeneous behavior of strength characteristics but also on the crack length and velocity. Reconstruction of the strength properties will then be possible only under the condition of most accurate experimental investigation of growing cracks.

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