

Hydromagnetic stability of the interface between a gas stream and a liquid

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AN INVESTIGATION is made of the inertial effects of the gas motion upon the stability characteristics of the wave motion at the interface between a gas stream and a conducting liquid in a uniform magnetic field. The analysis considers a body force directed towards the liquid, and the effects of the surface tension of the liquid. First the case with the applied magnetic field parallel to the direction of streaming is considered. For a subsonic gas flow, the inertial effects of the gas motion are found to lead to overstability of the wave motion at the interface, and the magnetic field, if sufficiently strong, will eliminate the instability. For a supersonic gas flow, the inertial effects of the gas motion are found to worsen the instability of the wave motion of the interface, and the magnetic field, however strong, will not eliminate the instability. Next, the case with the applied magnetic field transverse to the direction of streaming is considered, both of the fluids now being taken to be perfectly conducting. For a subsonic gas flow, the magnetic field is found to reduce the instability of the wave motion at the interface, but will not eliminate it. For a supersonic gas flow, the magnetic field is found to worsen the instability of the wave motion at the interface.

Zbadano wpływ efektów wewnętrznych ruchu gazu na cechy stabilności ruchu falowego na powierzchni pomiędzy strumieniem gazu i cieczą przewodzącą w jednorodnym polu magnetycznym. Analiza uwzględnia siły masowe skierowane w stronę cieczy i efekty napięć powierzchniowych cieczy. Rozważono najpierw przypadek z polem magnetycznym przyłożonym równoległe do kierunku przepływu. Dla przepływu poddźwiękowego gazu efekty inercji ruchu gazu wpływają na stabilizację ruchu falowego na powierzchni oddzielającej, a pole magnetyczne, jeżeli jest dostatecznie silne, może wyeliminować niestabilność. Dla przepływu naddźwiękowego gazu efekty inercji pogarszają niestabilność ruchu falowego na powierzchni oddzielającej, a pole magnetyczne, chociaż silne, nie eliminuje niestabilności. Następnie rozważono przypadek z polem magnetycznym poprzecznym do kierunku przepływu, przyjmując obydwie płyny jako idealnie przewodzące. Dla przepływu poddźwiękowego gazu stwierdzono, że pole magnetyczne redukuje niestabilność ruchu falowego na powierzchni oddzielającej ale jej nie eliminuje. Dla przepływu poddźwiękowego pole magnetyczne pogarsza niestabilność ruchu falowego na powierzchni oddzielającej.

Исследовано влияние внутренних эффектов движения газа на свойства устойчивости волнового движения на поверхности между потоком газа и проводящей жидкостью в однородном магнитном поле. Анализ учитывает массовые силы направленные в сторону жидкости и эффекты поверхностных напряжений жидкости. Рассмотрен сначала случай с магнитным полем, приложенным параллельно направлению течения. Для дозвукового течения газа эффекты инерции движения газа влияют на устойчивость волнового движения на разделяющей поверхности, а магнитное поле, если оно достаточно сильное, может исключить неустойчивость. Для сверхзвукового течения газа эффекты инерции ухудшают неустойчивость волнового движения на разделяющей поверхности, а магнитное поле, хотя сильное, не исключает неустойчивости. Затем рассмотрен случай с магнитным полем поперечным направлению течения, принимая обе жидкости как идеально проводящие. Для дозвукового течения газа констатировано, что магнитное поле уменьшает неустойчивость волнового движения на разделяющей поверхности, но не исключает ее. Для дозвукового течения магнитное поле ухудшает неустойчивость волнового движения на разделяющей поверхности.

1. Introduction

CHANG and RUSSEL [3] made a study of the stability characteristics of the wave motion at the interface between a liquid layer and a gas stream adjacent to it and found that the nature of the waves generated at the interface depends markedly on the state of flow of the gas. For supersonic gas flow, the gas pressure at the interface is out of phase with the surface tension so that a purely oscillatory constant-amplitude motion of the interface is not possible. For a subsonic gas flow, however, the stabilising effect of the surface tension gives rise to cut-off frequencies. BUTI *et al.* [1] extended this analysis to the case wherein the two fluids are electrically conducting and are subject to a uniform magnetic field. For the case with the applied magnetic parallel to the direction of streaming, BUTI *et al.* [1] found that even the nature of the effect of the magnetic field on the stability of the wave motion at the interface depends markedly on whether the gas flow is subsonic and supersonic. In the present paper the latter result is shown to be true also for the case with the applied magnetic field transverse to the direction of streaming. Both CHANG and RUSSELL [3], and BUTI *et al.* [1] ignored the inertial effects of the gas motion. Since the latter become important for waves with speeds of propagation comparable with the gas speed, the present paper incorporates them and extends the previous calculations.

2. The case with a uniform magnetic field parallel to the direction of streaming

Consider the wave motion at the interface between a nonconducting gas stream and a perfectly conducting liquid in a uniform magnetic field H' parallel to the interface (see Fig. 1). The following analysis considers a body force g' directed towards the liquid,

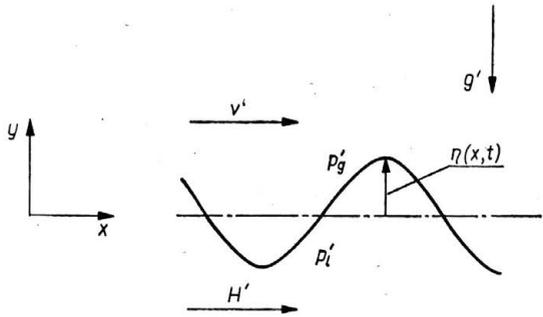


FIG. 1.

and the effects of the surface tension T' of the liquid. The liquid is assumed to be initially quiescent and of infinite depth whose mean level of contact with gas flowing past it is the plane $y = 0$, (see Fig. 1). Both the liquid and the gas are assumed to be inviscid and the effects of the viscous boundary layer at the interface are ignored. If the motion of the whole system is supposed to start from rest, it may be assumed to be irrotational. If a typical interfacial disturbance is characterised by a sinusoidal travelling wave with an amplitude a' and wavelength λ' , (the primes denote dimensional quantities), then all the quantities

in the following are nondimensionalised with respect to a reference length $\lambda'/2\pi$, a time $(\lambda'/2\pi g)^{1/2}$, and the inertial effects of the gas motion are characterised by the ratio of the wave speed to the gas speed. The gas density ρ'_g is small so that the corresponding body force is negligible. The potential function of the motions of the liquid and the gas are taken to be, respectively,

$$(2.1) \quad \text{and} \quad \begin{aligned} & (g')^{1/2} \left(\frac{\lambda'}{2\pi} \right)^{3/2} \varphi(x, y, t) \\ & \left(\frac{\lambda'}{2\pi} \right) V' [x + \phi(x, y, t)]. \end{aligned}$$

The latter are governed by the following linearised equations:

$$(2.2) \quad y = \eta: \varphi_{xx} + \varphi_{yy} = 0,$$

$$(2.3) \quad y > \eta: \phi_{yy} - (M^2 - 1)\phi_{xx} - M^2(2\delta\phi_{xt} + \delta^2\phi_{tt}) = 0,$$

where

$$\begin{aligned} M^2 & \equiv \frac{U'^2}{c'^2} \\ \delta & \equiv \left(\frac{\lambda' g'}{2\pi} \right)^{1/2} \frac{1}{U'} \sim \frac{\text{wave speed}}{\text{gas speed}} \ll 1, \end{aligned}$$

c' denotes the speed of sound in the gas, V' the ambient gas velocity, and $y = \eta(x, t)$ the disturbed shape of the interface.

If \mathbf{h}' is the perturbation in the magnetic field, one has

$$(2.4) \quad y < \eta: \mathbf{h}'_t = H' \varphi_{xx}.$$

One has the following linearised boundary conditions at the interface:

(i) kinematic condition:

$$(2.5) \quad y = 0: \quad \varphi_y = \eta_t,$$

$$(2.6) \quad \phi_y = \delta\eta_t + \eta_x;$$

(ii) dynamic condition:

$$(2.7) \quad y = 0: \quad \varphi_t + \eta + \frac{H'h'_x}{\rho'_l U'^2} = k^2 \eta_{xx} - \frac{k\sigma}{2} C_p,$$

where

$$\begin{aligned} k^2 & \equiv \left(\frac{2\pi}{\lambda'} \right)^2 \frac{T'}{\rho'_l q'}, \quad \sigma \equiv \frac{\rho'_g U'^2}{\sqrt{\rho'_l g' T'}}, \\ C'_p & \equiv -2\phi_x - 2\delta\phi_t, \end{aligned}$$

ρ' being the mass density, and the subscripts l and g refer to the liquid and the gas, respectively.

The infinity conditions are

$$(2.8) \quad y \Rightarrow -\infty: \quad \varphi_y \Rightarrow 0$$

and if the gas flow is subsonic,

$$(2.9) \quad y \Rightarrow \infty: \quad \phi_y \Rightarrow 0.$$

2.1. Subsonic gas flow

Since we are looking for travelling waves, introduce

$$(2.10) \quad \xi = x - ct$$

so that Eqs. (2.2)–(2.9) become

$$(2.11) \quad y < 0: \quad \varphi_{\xi\xi} + \varphi_{yy} = 0,$$

$$(2.12) \quad y > 0: \quad \phi_{yy} + \gamma_1^2 \phi_{\xi\xi} = 0,$$

$$(2.13) \quad y = 0: \quad \varphi_y = -c\eta_\xi,$$

$$(2.14) \quad \phi_y = (1 - \delta c)\eta_\xi,$$

$$(2.15) \quad \eta = c\varphi_\xi + k\sigma(1 - \delta c)\phi_\xi - \frac{N^2}{c}\varphi_\xi + k^2\eta_{\xi\xi},$$

$$(2.16) \quad y \Rightarrow -\infty: \quad \varphi_y \Rightarrow 0,$$

$$(2.17) \quad y \Rightarrow +\infty: \quad \phi_y \Rightarrow 0,$$

where

$$\gamma_1^2 \equiv m_1^2 + 2\delta cM^2,$$

$$m_1^2 \equiv 1 - M^2,$$

$$N^2 \equiv \frac{H'^2}{4\pi\rho_1' U'^2}.$$

Let

$$(2.18) \quad \eta = A \cos \xi,$$

A being an arbitrary constant; then from Eqs. (2.11)–(2.14), (2.16) and (2.17), one obtains

$$(2.19) \quad \varphi = A c e^y \sin \xi.$$

$$(2.20) \quad \phi = \frac{A}{\gamma_1} (1 - \delta c) e^{-\gamma_1 y} \sin \xi.$$

Using Eqs. (2.18)–(2.20) in Eq. (2.15), one obtains the dispersion relation

$$(2.21) \quad c^2 + \frac{k\sigma(1 - \delta c)^2}{\gamma_1} - (k^2 + 1 + N^2) = 0.$$

For $\delta \Rightarrow 0$, this reduces to what was reduced by BUTI *et al.* [1] in the proper limit.

Recalling that $\delta \ll 1$, Eq. (2.21) gives

$$(2.22) \quad c = k\sigma\delta \left(\frac{1}{m_1} + \frac{M^2}{2m_1^3} \right) \pm \sqrt{k^2 + 1 + N^2 - \frac{k\sigma}{m_1}}.$$

It is obvious that the inertial effects of the gas motion ($\delta \neq 0$) lead to overstability (CHANDRASEKHAR [2], Chapter I). Also, note that if the applied magnetic field is sufficiently strong, it will eliminate the instability.

The cut-off wavenumbers correspond to

$$(2.23) \quad k_{c_{1,2}} = \frac{\sigma}{2m_1} \mp \sqrt{\frac{\sigma^2}{4m_1^2} - (1 + N^2)}.$$

Thus there are two cut-off wavenumbers, and all disturbances with wavenumbers above or below these value propagate without growth or decay. Note that the effect of the applied magnetic field is to reduce k_{c_2} and increase k_{c_1} , i.e. to narrow the unstable window — further indication of the stabilising nature of the applied magnetic field.

2.2. Supersonic gas flow

Introducing

$$(2.10) \quad \xi = x - ct,$$

again, the system (2.2)–(2.8) becomes

$$(2.24) \quad y < 0: \quad \varphi_{\xi\xi} + \varphi_{yy} = 0,$$

$$(2.25) \quad y > 0: \quad \phi_{yy} - \gamma_2^2 \phi_{\xi\xi} = 0,$$

$$(2.26) \quad y = 0: \quad \varphi_y = -c\eta_\xi,$$

$$(2.27) \quad \phi_y = (1 - \delta c)\eta_\xi,$$

$$(2.28) \quad \eta = c\varphi_\xi + k\sigma(1 - \delta c)\phi_\xi - \frac{N^2}{c} \varphi_\xi + k^2\eta_{\xi\xi},$$

$$(2.29) \quad y \Rightarrow -\infty: \quad \varphi_y \Rightarrow 0,$$

where

$$\gamma_2^2 \equiv m_2^2 - 2\delta cM^2, \quad m_2^2 \equiv M^2 - 1.$$

Let,

$$(2.30) \quad \eta = Ae^{i\xi}$$

then from Eqs. (2.24)–(2.27), (2.29), one obtains

$$(2.31) \quad \varphi = -iAe^{y+i\xi},$$

$$(2.32) \quad \phi = -\frac{iA}{\gamma_2} (1 - \delta c)e^{i(\xi - \gamma_2 y)}.$$

Using Eqs. (2.30)–(2.32) in Eq. (2.28), one obtains the dispersion relation

$$(2.33) \quad c^2 + \frac{ik\sigma(1 - \delta c)^2}{\gamma_2} - (k^2 + 1 + N^2) = 0.$$

For $\delta \Rightarrow 0$, this reduces to that deduced by BUTI *et al.* [1], in the proper limit.

Recalling that $\delta \ll 1$, Eq. (2.33) gives

$$(2.34) \quad c = \frac{ik\sigma\delta}{2m_2^3} (M^2 - 2) \pm \sqrt{k^2 + 1 + N^2 - \frac{ik\sigma}{m_2}}.$$

It is seen that the physical condition is always one of instability, regardless of the presence of surface tension and the applied magnetic field. The inertial effects of the gas motion worsen this instability.

3. The case with a uniform magnetic field transverse to the direction of streaming

Consider the previous model with both of the fluids perfectly conducting this time, and an applied magnetic field along the z -direction. FEJER [4] found from consideration of the Kelvin-Helmholtz instability of a compressible plasma in a transverse magnetic field that the effect of the latter is to increase the sound speed from c^2 to $(c^2 + N^2)$. Thus, for a subsonic gas-flow the dispersion relation, from Eq. (2.22), gives

$$(3.1) \quad c = k\sigma\delta \left(\frac{1}{m_1^*} + \frac{M^{*2}}{2m_1^{*3}} \right) \pm \sqrt{k^2 + 1 - \frac{k\sigma}{m_1^*}},$$

where

$$M^{*2} \equiv \frac{U^2}{c^2 + N^2}, \quad n_1^{*2} \equiv 1 - M^{*2}.$$

Clearly the transverse magnetic field reduces the instability of the wave motion at the interface, but will not eliminate it, for a subsonic gas-flow.

For a supersonic gas-flow the dispersion relation, from Eq. (2.34), gives

$$(3.2) \quad c = \frac{ik\sigma\delta}{2m_2^{*3}} (M^{*2} - 2) \pm \sqrt{k^2 + 1 - \frac{ik\sigma}{m_2^*}},$$

where

$$m_2^{*2} \equiv M^{*2} - 1.$$

Clearly the transverse magnetic field worsens the instability of the wave motion at the interface for a supersonic gas-flow.

References

1. B. BUTI, G. L. KALRA and S. N. KATHURIA, *Phys. Fluids*, **13**, 364, 1970.
2. S. CHANDRASEKHAR, *Hydrodynamic and hydromagnetic instability*, Clarendon Press, Oxford 1961.
3. I-DEE CHANG, P. E. RUSSELL, *Phys. Fluids*, **8**, 1018, 1965 (Erratum in **10**, 2218, 1967).
4. J. A. FEJER, *Phys. Fluids*, **7**, 1293, 1964.

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