

A two-stage hypothesis of fatigue damage initiation and its experimental verification

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THE PAPER deals with the presentation of two-stage model for the fatigue damage development. The fatigue process was divided into two stages: stage I contains changes occurring inside a micro-region, while stage II describes a development of the microcracks until the fatigue fracture arises. A review of certain hypotheses concerning changes occurring in microregions and crack development is given. Basing on these hypotheses an uniform relation between a degree of material damage and number of cycles is presented. For verification of this relation the experimental fatigue tests were performed. As a measure of the material damage the amplitude changes of vibrations of the specimen end loaded by constant amplitude disturbance is assumed. On the basis of the results obtained the coefficients appearing in the equation derived are evaluated.

W artykule przedstawiono dwuetapowy model rozwoju uszkodzenia zmęczeniowego. Proces zmęczeniowy rozdzielono na dwa etapy. Etap pierwszy obejmuje zmiany zachodzące wewnątrz mikroobszaru. Etap drugi to rozwój mikropęknięcia aż do powstania złomu zmęczeniowego. Podano przegląd niektórych hipotez dotyczących zmian zachodzących w mikroobszarach oraz rozwoju pęknięcia. W oparciu o nie przedstawiono jednolitą zależność między stopniem uszkodzenia materiału a ilością cykli. Dla weryfikacji podanej zależności przeprowadzono badania zmęczeniowe. Jako miarę uszkodzenia materiału przyjęto zmianę amplitudy drgań końca próbki poddanej obciążeniu o stałej amplitudzie wymuszenia. Na podstawie otrzymanych wyników wyznaczono współczynniki występujące w równaniu.

В статье представлена двухэтапная модель развития усталостного повреждения. Усталостный процесс разделен на два этапа. Этап I охватывает изменения происходящие внутри микрообласти. Этап II это развитие микротрещины вплоть до возникновения усталостного излома. Дается обзор некоторых гипотез, касающихся изменений происходящих в микрообластях, а также развития трещины. Опираясь на эти гипотезы представлена единая зависимость между степенью повреждения материала и количеством циклов. Для проверки приведенной зависимости проведены усталостные исследования. Как мера повреждения материала принято изменение амплитуды колебаний конца образца подвергнутого нагрузке с постоянной амплитудой вынуждения. На основе полученных результатов определены коэффициенты выступающие в уравнении.

1. Introduction

IN ADDITION to the development of the theory of linear cumulation of fatigue, being quite convenient in practice, the theory of two-stage cumulation damage has made some progress recently. In this theory, the damage cumulation process is assumed to depend on the stage of the fatigue process. The best known and the most typical theory is that of CORTEN-DOLAN [1]. In this theory the function describing the fatigue collapse process is different for each different stage. The first, denoted by G , is treated as a measure of preparation of the material for the occurrence of fatigue damage. It is zero for the initial state and unity when the first (preliminary) stage has been completed. The other function, denoted by D is, similarly to the theory of linear cumulation, a measure of the degree of

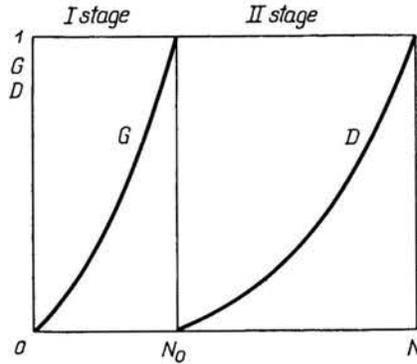


FIG. 1. Damage of the material in function of the number of cycles.

growth of a fatigue crack and is zero for $G < 1$ and unity at the moment of damage (Fig. 1).

Similar consideration dealing with the subdivision of the fatigue process into two stages have been presented by VAGANOV [9], who obtained two characteristic curves describing the growth of the first macrocrack and the collapse process. These curves are represented in Fig. 2.

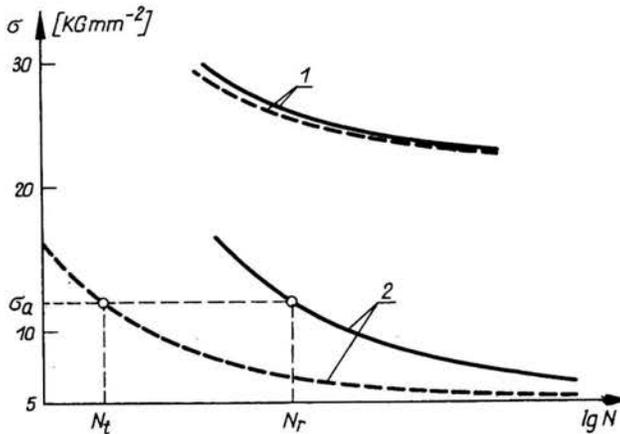


FIG. 2. Influence of stress concentration on the formation of a macrocrack and a fracture:
1 — smooth specimen, 2 — specimen with a groove.

The curve of fatigue collapse can be represented in the form of a sum of two functions

$$(1.1) \quad N_r = \varphi(\sigma_a) + \psi(Q_a),$$

where Q_a — external load, $\sigma_a = Q_a/F$ — nominal stress of the initial material, $\varphi(\sigma_a) = N_t$ — curve corresponding to the first macrocrack, $\psi(Q_a) = N_r - N_t$ — period from development of the fatigue crack until fracture occurs.

It can be seen from the figure that these periods depend on the stress level and the uniformity of loads acting on each particular microregion of the element. In the case of uniform stress distribution, where all the microregions are subjected to the same load, the

preliminary period is relatively long. As soon as the first macrocrack occurs its development is very rapid because the entire material is uniformly affected. In this case the curves N_r and N_f are near each other.

In elements with notches, in which the stress concentration is high, the most (loaded stressed) microregions undergo decohesion in a very short time and the crack passes through sound regions; this is much slower. The curves N_r and N_f are at a large distance apart.

If the mechanism of formation of fatigue defects is considered in the light of the theory concerning decohesion of a micro-volume subjected to variable loading and the mechanism of propagation of cracks, it seems to be justified to assume a two-stage model of fatigue collapse. The physics of the processes occurring at each particular stage will be the starting point in the attempt to describe the destruction in the first and the second stage of the fatigue process (Sects. 3 and 4).

As a measure of fatigue damage we can assume the changes in the properties of the material which occur in the course of the fatigue process. In the present paper, the parameter determining the degree of damage is the change in the dynamic properties of a specimen. In order to determine experimentally the changes in dynamic properties, specimens of the PA-4 alloy [11] were subjected to the action of a variable bending moment on a NPL-Amsler testing machine. In the course of the action of the variable load the specimen was gradually damaged. Finally the rigidity was varied. A variation in rigidity produced a change in the amplitude of vibration of the specimen, and this was measured, in function of the number of cycles N with the aid of a microscope. The character of the variation of the amplitude in function of the number of cycles is shown in Fig. 3.

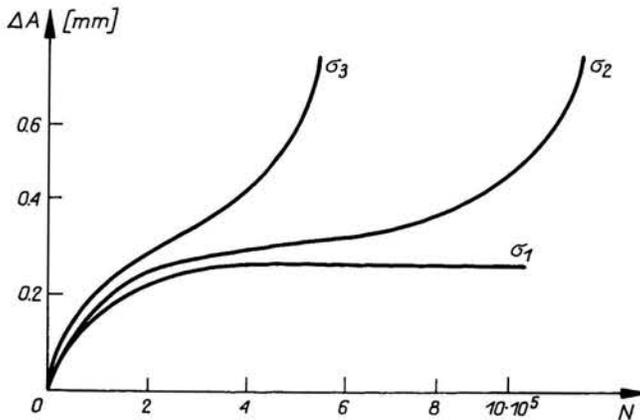


FIG. 3. Amplitude increment in function of the number of cycles ($\sigma_1 < \sigma_2 < \sigma_3$).

It is assumed that in the first stage of fatigue damage there is an increase in the amplitude of vibration which is rapid at the beginning, and then approximately linear. The second stage is characterized by a considerable increase in the amplitude as a result of a considerable degree of damage.

2. A model of the first stage of fatigue damage

The description of the phenomena occurring in metal under the action of variable loads requires the assumption that a stress field σ acts in the region of the body. The symbol σ_p will denote the stress acting on the k -th micro-element. If the micro-strength limit of the k -th element is P_k , the decohesion can be written for that microelement in the form

$$(2.1) \quad \sigma_k \geq P_k.$$

On the basis of the theory of M. ZAKRZEWSKI and L. GOŁASKI [10] on the strength P_k of a micro-volume with dislocations and the theory of D. NEUBERT [7] on the influence of the dislocation density on the value of the local stress, the decohesion condition of a micro-volume subjected to the action of a variable stress can be formulated thus [4]:

$$(2.2) \quad \sigma_{\max} = \frac{GE}{3G-E} \int_T^{T_{\text{top}}} \alpha dT + \left[m - \frac{GE}{3G-E} b^2 \right] \varrho_k,$$

where σ_{\max} — maximum stress in the cycle, G — shear modulus, E — Young's modulus, T_{top} — melting temperature, T — ambient temperature, α — coefficient of thermal dilatation, m — material constant, b — Burger's vector, ϱ_k — dislocation density in the k -th micro-volume.

Using the results of experimental works by A. V. GUREV and G. STOLJAROV [3] and S. KOCAŃDA [6] the following relation was found between the dislocation during ϱ_k , the value of maximum and minimum load (σ_{\max} and σ_{\min}), the tensile strength R_m , the number of load cycles N and the initial values of the dislocation density ϱ_0 [5]:

$$(2.3) \quad \varrho_k = \varrho_{0N} \left[\left(\exp \frac{\sigma_{\max}}{R_m} - \exp \frac{\sigma_{\min}}{R_m} \right) \ln + 1 \right].$$

If we substitute the relation (2.2) and (2.3) into (2.1), we can establish the decohesion condition of a micro-volume in the form

$$(2.4) \quad \sigma_{\max} = A + B\varrho_k,$$

where

$$A = \frac{GE}{3G-E} \int_T^{T_{\text{top}}} \alpha dT, \quad B = m - \frac{GE}{3G-E} b^2.$$

The Eq. (2.4) is a mathematical model of the first stage of the fatigue process.

As can be seen from the relation (2.4) there exists for each value of σ_{\max} a critical value of the dislocation density, for which there occurs decohesion of the micro-volume considered. Thus, as a measure of damage in the first stage of the fatigue process we can assume a quantity U_1 proportional to the dislocation density (2.3), that is

$$(2.5) \quad U_1 = k_1 \ln N + k_2,$$

where

$$k_1 = a\varrho_0 \left(\exp \frac{\sigma_{\max}}{R_m} - \exp \frac{\sigma_{\min}}{R_m} \right), \quad k_2 = a\varrho_0.$$

The variation of damage U_1 in the course of the first stage of the fatigue process is represented, in function of the number of load cycles, in Fig. 4.

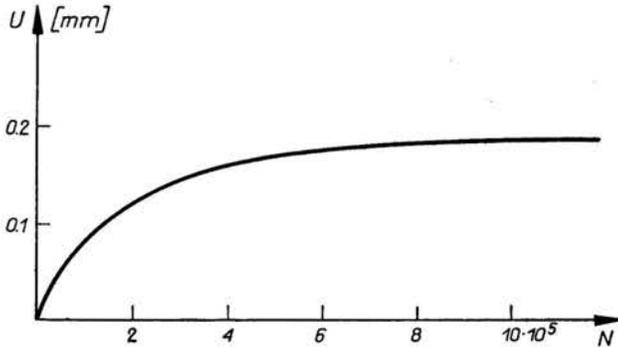


FIG. 4. Damage of the material in the first stage of the fatigue process.

3. A model of the second stage of the fatigue damage

The second stage begins with the occurrence of a macro-crack and ends with a fatigue rupture. The development of the crack is determined by the degree of destruction of the crystal lattice and the state of load. In this period the influence of the loading conditions (constant load or constant strain) is essential. The material subjected to a fatigue load shows varying parameters such as the modulus of elasticity $E(n)$, the width of the hysteresis loop $\delta(n)$, the property of damping of ultrasonic waves and other properties [9]. The influence of the loading conditions is clearly seen in Fig. 5 [8].

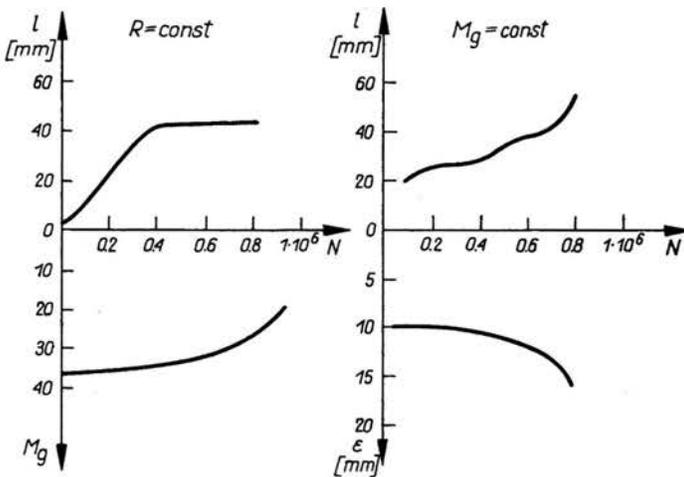


FIG. 5. Variation of the crack length in the neck of a connecting rod [8].

It represents the development of a crack in the neck of a connecting rod. The load, by a constant moment M , produces a rapid increase in strain ε and the fatigue resistance is low. In tests with constant strain, in connection with the drop in rigidity, the loads drop also. If the load drop is too rapid in function of n , transient stress may fall below the critical stress and cleavage fracture will not occur.

The development of a fatigue crack in function of the number of load cycles has been considered by many authors. Their starting points was the phenomenon of cracking of solids and the dislocation vacancy theory.

The scientists whose considerations are based on the mechanics of cracking of a solid (FROST and DUDGALE [2], T. YOKOBORI [12] and others) consider the relation between the growth of the crack per one cycle and the length of the crack in the form of the equation

$$(3.1) \quad \frac{dl}{dn} = kl,$$

where n — number of cycles, l — length of crack, k — coefficient depending on the stress and the state of the material.

The relation [7] is valid for a purely elastic fracture and also if the existence of a plastic material about the gap is considered. The coefficient k is variable.

Other investigations were concerned with the relation between the degree of development of the crack per one cycle with the coefficient of intensity of stress ΔK (corresponding to the range of the stress applied) or K_{\max} (corresponding to the maximum stress). We obtain a relation similar to (3.1)

$$(3.2) \quad \frac{dl}{dn} = C(\Delta K)^m \quad \text{or} \quad \frac{dl}{dn} = C_0 K_{\max}^{m_1},$$

or

where m, m_1 — power exponent depending on the kind of material.

The recent tests by YOKOBORI [12] to [15] were intended to find the relationship between the coefficients of Eq. (3.2) and the material parameters determining the state of the material: the density of dislocations and vacancy, the strain-hardening coefficient and others. In accordance with his kinetic theory of development of a crack we can write:

$$(3.3) \quad \frac{dl}{dn} = B \left(\frac{\Delta K}{\sqrt{2\delta\varepsilon_1} \sigma_{cy}} \right)^\delta,$$

where ε_1 — region of increased dislocation density, σ_{cy} — initial yield limit,

$$\delta = \frac{2\beta}{1+\beta}(m+1) + \frac{1}{1+\beta}.$$

On integrating the relations describing the increase in length of the crack [Eqs. (3.1)–(3.3)] we obtain an exponential relationship between the length of the crack and the number of cycles. Henceforth, we shall use in the present paper an exponential relation of the type:

$$(3.4) \quad U_2 = k_3 n^{k_4},$$

where U_2 — the defect occurring in the second stage, k_3 , k_4 — parameters constituting, in general, functions of state of the material and the conditions of tests. In particular, the exponent k_4 depends considerably on whether the tests are performed under constant load or constant strain.

Assuming that the total damage of the material is the sum of the damages which occurred in the first and the second stage, that is, taking into consideration the relations (2.5) and (3.4), we obtain:

$$(3.5) \quad u = u_1 + u_2 = k_1 \ln n + k_2 + k_3 n^{k_4}.$$

This relation is represented in the graph of Fig. 6.

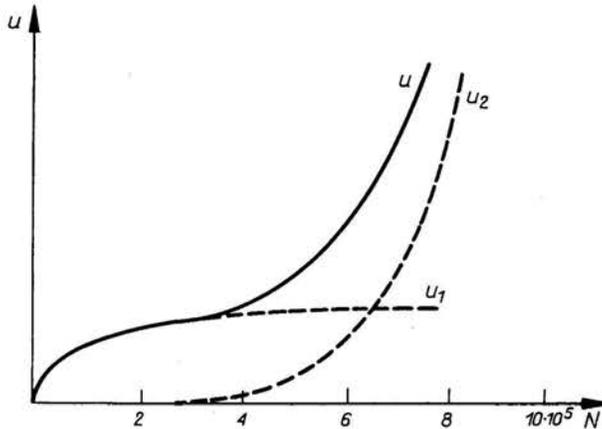


FIG. 6. Variation of damage in the first and second stage.

4. Experimental verification of the theory

In order to verify the agreement between the relation (3.5) and the experimental results, some tests were carried out on the NPL-Amsler testing machine which produces plane bending under conditions of dynamic excitation. The specimen is loaded by a constant force evoked by the unbalance of a rotating disc attached to the end of the specimen and the spring. A diagram of how the machine operates is shown in Fig. 7. The load changes by varying the unbalance m of the disc. The deformation of the specimen depends on the value of the unbalance and the rigidity of the specimen. The tests were carried out under conditions of constant speed equal to the resonance speed of the system with a zero specimen (with zero rigidity). This speed is lower than the resonance speed for the system with a real test-piece. In the course of the test, a variation in the rigidity of the specimen made the resonance curve move in the direction of rotation. This produces an additional increase in amplitude of deformation of the specimen.

The variation in the rigidity of the test-piece was assumed to be a measure of damage. The variation in the rigidity of the specimen produced a variation in the amplitude of the support of the specimen with the rotating disc. The measurement of the amplitude of vibration was carried out by means of a microscope with an accuracy of approximately

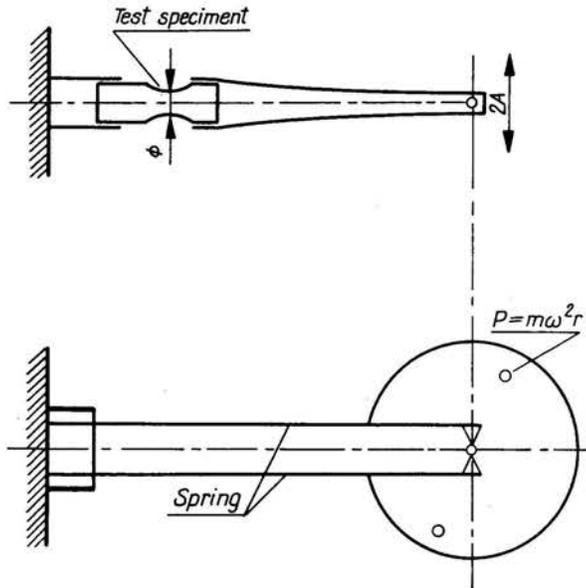


FIG. 7. Diagram of the test stand.

0.01 mm. The increase in amplitude after a definite number of cycles is given in Table 1 and in the diagram of Fig. 8. Round specimen of aluminium alloy has been used. The chemical composition and the mechanical properties are given in Tables 2 and 3.

Table 1

No.	n	ΔI_r	ΔI_c
1	40 000	0.07	0.098
2	120 000	0.15	0.119
3	410 000	0.17	0.143
4	700 000	0.17	0.157
5	1 500 000	0.17	0.179
6	1 960 000	0.24	0.263
7	2 100 000	0.30	0.323
8	2 150 000	0.35	0.352
9	2 210 000	0.39	0.393
10	2 310 000	0.65	0.482

Table 2

Cu	Mg	Mn	Si	Fe	Zn	Al
0.07	0.97	0.94	0.98	0.23	0.06	rest

Table 3

	kGmm ⁻²
$R_{0.02}$	19.3
$R_{0.2}$	21.0
R	30.9
E	7 250

The coefficients k_1 to k_4 in the Eq. (3.5) have been determined on the basis of the results obtained for the amplitude variation $\Delta(2A)$. The familiar method of least squares was used for the computing these coefficients.

As a result of the computation performed, the following equation was obtained:

$$(4.1) \quad 2A = 0.0195 \ln N - 0.1090 + 1.337N^{7.6}$$

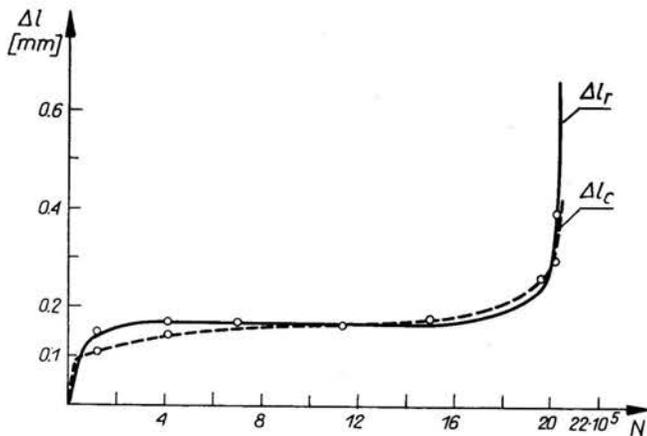


FIG. 8. Variation of the vibration amplitude at the end of the specimen.

and superposed over the experimental diagram Fig. 8. As can be seen from the graph, the relation (3.5) describes in a satisfactory manner the character of the changes in a micro-region and the development of microcracks. This relation also enables us to give a simple relation describing the general fatigue process. An important problem now is to determine the relation between the coefficients k_1 to k_4 and the parameters determining the state of the material and the stress. Once these relationships are known, it will be possible to appraise the state of fatigue damage of a structural element by performing a controlling test under resonance conditions.

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