

The supersonic flow around a sharp-nose elliptic cone at the angle of attack

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THE PAPER deals with the flow around an elliptic cone in a wide range of angles of attack. The conic flow parameters between the body and the shock wave are obtained by means of numerical solution of complete gasdynamic equations. The pressure coefficients obtained coincide with the experimental values with the accuracy 4–6%. In the cross-section plane of elliptic cone there are plotted the lines of constant pressure, Mach number and the constant entropy lines.

Praca omawia opływ stożka eliptycznego w szerokim zakresie kątów natarcia. Parametry przepływu stożkowego pomiędzy ciałem opływającym a falą uderzeniową otrzymano drogą rozwiązania numerycznego pełnego układu równań gazodynamiki. Otrzymane parametry ciśnienia zgadają się z danymi doświadczalnymi z dokładnością do 4–6%. W płaszczyźnie przekroju stożka wykreślono linie stałego ciśnienia, liczby Macha oraz linie stałej entropii.

В работе получена картина обтекания эллиптического конуса в широком диапазоне углов атаки. Параметры конического течения между телом и ударной волной определяются путем численного решения полных уравнений газовой динамики. Полученные коэффициенты давления совпадают с экспериментальными данными с точностью 4–6%. В плоскости поперечного сечения эллиптического конуса построены линии постоянного давления, числа Маха и линии постоянной энтропии.

THE FLOW pattern around elliptic cones at large angles of attack was obtained by A. P. BAZZHIN and co-workers [1] and by A. N. KRYKO, M. Ya. IVANOV by means of numerical solution of complete gasdynamics equations. There are calculation results of P. I. CHUSHKIN [2] for zero and small angles of attack. Experimental data of the supersonic flow past a sharp elliptic cone are given by A. MARTELLUCCI [3] and A. I. SHVETZ [4]. Besides, there are papers in which the flow around an elliptic cone was obtained by approximation methods.

In the present paper, the supersonic flow around an elliptic cone is obtained in a wide range of angles of attack when the stream between the body and the shock wave remains conic. The flow patterns at large and small angles of attack are shown to be considerably different. Calculation results are in good agreement with experimental data.

1. The velocity vector at infinity is in the symmetry plane which passes through the small axis of the ellipse and forms the angle of attack α with the cone axis. The flow pattern is schematically presented in Fig. 1.

Unsteady gasdynamics equations are given by conical variables $t, \eta = Y/X, \xi = Z/X$, where X, Y, Z are Cartesian coordinates. In plane ξ, η coordinates n, β connected with the body surface are introduced (Fig. 2). Here n is the distance from point E to ellipse contour divided by the distance between the body and the shock wave ε ; β is an angle between the normal to the ellipse contour and coordinate η .

The equation system is t -hyperbolic, its solution at zero and small angles of attack is looked for in the region $\Omega(t \geq 0, 0 \leq n \leq 1, 0 \leq \beta \leq \pi)$ limited by the body surface,

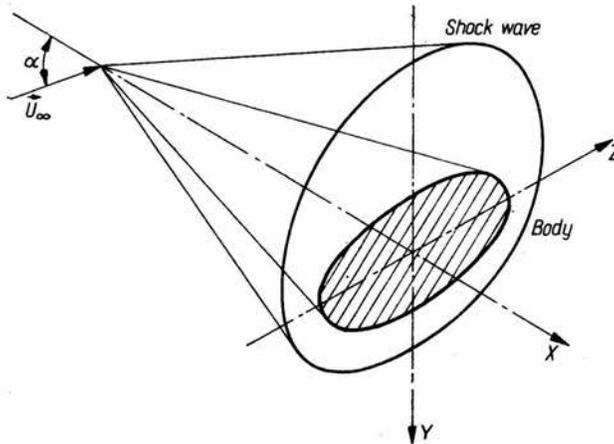


FIG. 1.

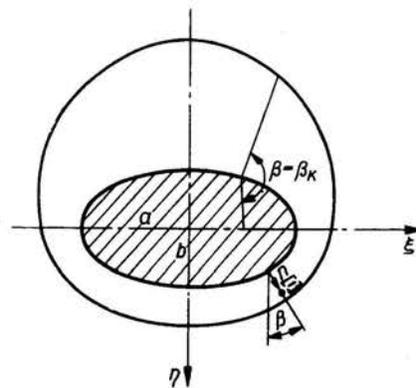


FIG. 2.

the shock wave and the symmetry plane. At $\beta = 0$ and $\beta = \pi$ boundary conditions of symmetry are set; at the body surface ($n = 0$), the boundary condition of the zero normal velocity is set and at the shock wave ($n = 1$), the conditions of conservation of mass, tangential component of velocity, momentum and energy.

Some flow field is to be set as initial condition. The steady problem solution is obtained as the limit of the unsteady problem solution at $t \rightarrow \infty$.

The equation system was solved by a numerical method described by A. N. LUBIMOV and V. V. RUSANOV [5].

2. Steady equations with conical variables ξ, η are considered below. The type of equation is determined by Mach number M_s , calculated by the velocity vector component, orthogonal to the position vector plotted from the cone nose and the local sound velocity. If the cone ellipticity is not large, the whole flow is conically subsonic ($M_s < 1$) at zero and small angles of attack and the equation system is of an elliptic type throughout the region. In this case it is necessary to consider the whole region between the body and the shock wave. With increasing incidence, the conically supersonic zones ($M_s > 1$) appear in the stream, where the equation system is of the hyperbolic type. At large

angles of attack, these zones extend up to body surface and the flow "choking" takes place when disturbances from the leeward side of the cone do not extend to the windward one.

At flow choking, the calculation region Ω is restricted along coordinate β by some value $\beta_k < \pi$, ray β_k being in the hyperbolic region and corresponding plane $\beta(X, Y, Z) = \beta_k$ being the space-like surface in the X, Y, Z -space. At the ray $\beta = \beta_k$ no boundary conditions are necessary, beside this the formulation of the problem is the same as mentioned above.

3. Using the results of A. I. GOLUBINSKII [6], analytic investigation of the flow near the spreading line on the elliptic cone surface was carried out in the analysis. The main result of this investigation is: if the minimum of the velocity component along the ruling is achieved at some ruling of the elliptic cone, then the maximum of pressure and density occurs on this ruling and it is the spreading line.

The flow in the plane of conic variables ξ, η is considered. At the flow around the elliptic cone at zero angle of attack, two spreading points are located on its surface in the symmetry plane containing the major axis of the ellipse and in another symmetry plane there are two singular points of FERRI (branch points), where all the streamlines converge. At small angles of attack, both spreading points are displaced to the windward side of the cone and the branch points remain in the symmetry plane. As the angle of attack increases, the spreading points approach each other and at some angle of attack they merge with the branch point. Thus at large angles of attack as well as at the flowing of a circular cone, there is only one spreading point on the windward side.

Irregularity of the flow in branch points was taken into consideration as by K. I. BABENKO and co-workers [7]. The entropy in the branch point when approaching it along the body surface was assumed to be equal to the entropy in the spreading point which was located by the minimum of the velocity component along the ruling (maximum of pressure). At large angles of attack, the entropy on the cone surface coincided with that in the symmetry plane on the windward side and the branch point on the windward side disappeared automatically. The algorithm suggested made it possible to obtain the solution without knowing beforehand which of the flow regimes would realize.

For more exact calculation of the vertical layer, coordinate n was stretched near the cone surface.

4. In Figs. 3-5, calculation results of elliptic cone flow by perfect gas are presented in plane ξ, η . This cone has a semi-apex angle of the cone in the plane of a major axis $\vartheta = 21.97^\circ$ and the ratio of semidiameters of ellipse $a/b = 1.788$. Mach number at infinity M is equal to 6.0. The trace of the velocity vector at infinity is marked by a square on the diagrams.

Lines $M_s = \text{const}$ are shown in Fig. 3, the values M_s on the neighbouring curves vary by 0.2. At $\alpha < 8^\circ$ the whole flow behind the shock wave is conically subsonic and at $\alpha = 8^\circ$ the conically supersonic flow zones arise near the shock wave. As the angle of attack increases, these zones approach to the body surface and are closed to the body at $\alpha = 10^\circ$. At $\alpha > 10^\circ$ the calculation region was limited by angle $\beta_k < \pi$. Further increase of the angle of attack caused the displacement of the line of parabolicity ($M_s = 1$) to the windward side.

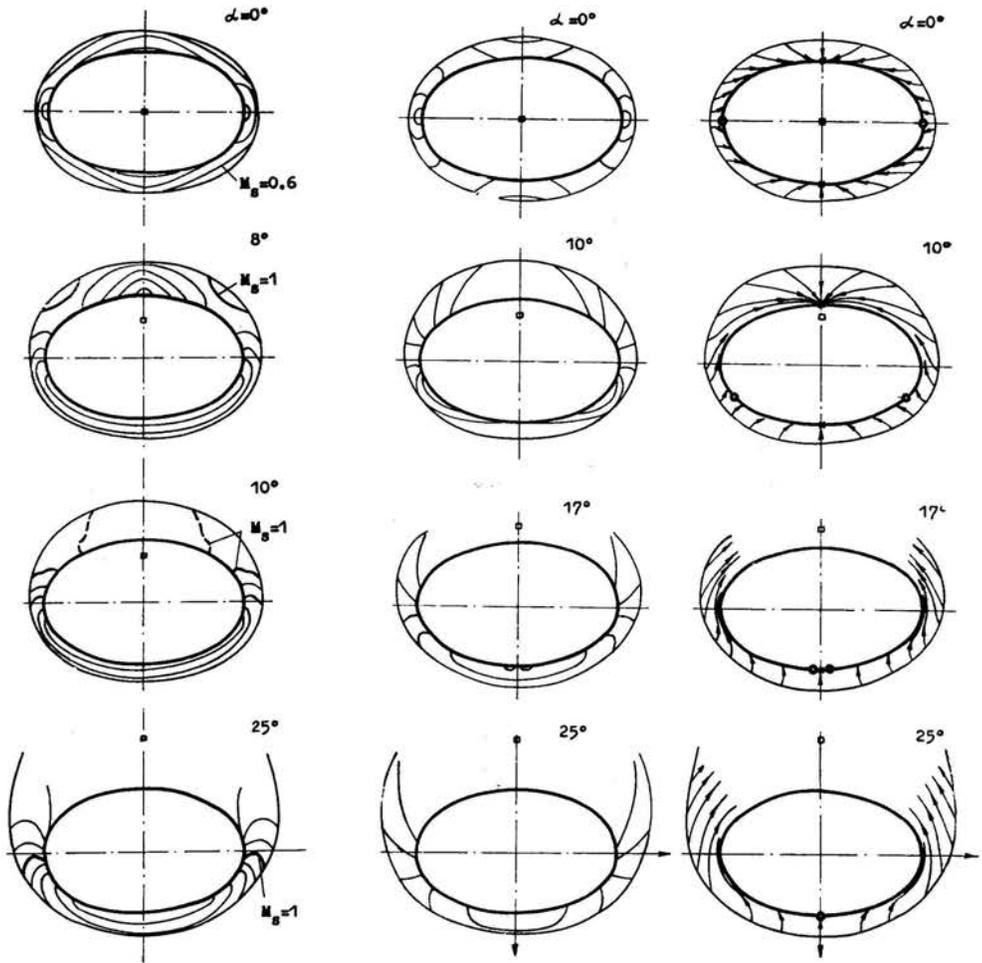


FIG. 3.

FIG. 4.

FIG. 5.

Isobares and intersections of stream surfaces with the plane $X = \text{const}$ which in this paper are referred to as "streamlines" on plane ξ, η are shown in Figs. 4 and 5. Here branch points are marked by crosses and the spreading points by circles. As it is seen from the diagrams, the pressure maximum on the body surface corresponds to the spreading point which is displaced to the windward side as the angle of attack increases. At $\alpha = 18^\circ$ both spreading points merge with the branch point and at $\alpha \geq 18^\circ$ the flow regime with one spreading point is realized.

Pressure coefficient C_p distribution on the elliptic cone surface is plotted against the central angle ω in Figs. 6-8. Experimental points taken from the papers of A. MARTELLUCCI [3] and A. I. SHVETZ [4] are given for comparison in the same figures. As can be seen from the plots, calculation results agree with experimental data quite satisfactorily. The flow fields obtained in the paper coincide well with the calculations of A. P. BAZZHIN [1] and P. I. CHUSHKIN [2].

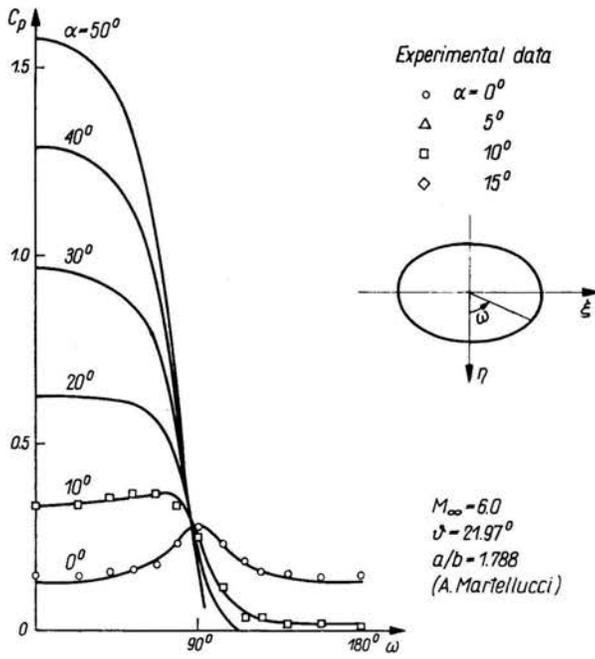


FIG. 6.

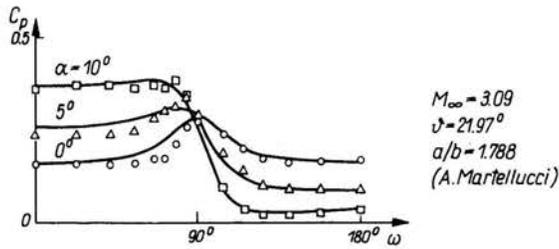


FIG. 7.

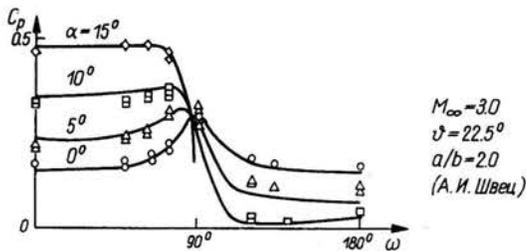


FIG. 8.

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