

The experimental research and hydrodynamical models of a “sultan”

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THE PRESENT paper concerns the experimental and theoretical investigation of two effects observed during an underwater explosion. These effects are a shock wave and gas bubble pulsation with detonation products. The mechanism of “sultan” formation (vertical and lateral fountains on the free surface) is investigated and the role played in this process by gas bubble and shock wave is shown.

AS IS KNOWN, during an underwater explosion two effects are observed — a shock wave and gas bubble pulsation with detonation products. The presence of a free surface near the charge leads to an anomalous growth of the first pulsation amplitude, to the formation of vertical and lateral fountains (“sultans”) on the free surface and of cavitation region as

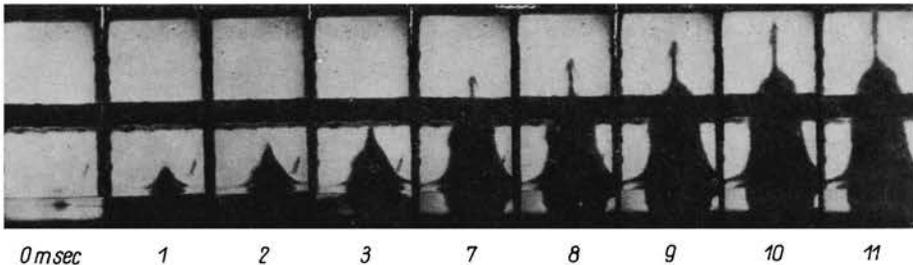


FIG. 1. Explosion of a charge weighing 1.2 g, $H = 5$ cm, near the free surface.

a result of shock wave reflection from this surface. Figure 1 represents different moments of the explosion of the charge weighing 1.2 g at the depth of 5 cm.

The present paper deals mainly with the mechanism of “sultan” formation and the role played in this process by gas bubble and a shock wave. In experimental and mathematical modelling it is expedient to separate these effects, say, to exclude the action of a shock wave. In experiment this is achieved, for example, by modelling an explosion caused by an electrical discharge in liquid, in mathematical models, by assuming the liquid to be ideal and incompressible.

Consider a plane mathematical model of the phenomenon. Let the lower subplane $z \leq 0$ be occupied by an ideal incompressible weightless liquid [region $Q(t)$] in which there is gas bubble $R(t)$ at a distance H from the free surface $\zeta(t)$. The pressure on $\zeta(t)$ is constant and equal to atmospheric p_a , that on $R(t)$ is variable according to the adiabatic law

$$p(t) = p_0 \left[\frac{S(t)}{S_0} \right]^{-\gamma}, \text{ where } \gamma = 3, \text{ the liquid motion being potential.}$$

Let

$$\varphi = \varphi' \sqrt{\frac{p_a}{\rho_0}} a_0, \quad t = t' \sqrt{\frac{\rho_0}{p_a}} a_0, \quad \mathbf{r} = \mathbf{r}' \cdot a_0, \quad p = p' p_a, \quad H = h a_0$$

and $q(t, \mathbf{r}) = 0$ is the boundary equation of $Q(t)$.

The formulation of the problem:

$$(1) \quad Q(t): \quad \varphi \rightarrow 0 \text{ (at } |\mathbf{r}| \rightarrow \infty), \quad \Delta\varphi = 0,$$

$$(2) \quad \zeta(t): \quad p = 1, \quad \varphi_t - \frac{1}{2}(\nabla\varphi)^2 = 0,$$

$$(2') \quad R(t): \quad p(t) = p_0[S(t)/S_0]^{-\gamma}, \quad \varphi_t - \frac{1}{2}(\nabla\varphi)^2 = p(t) - 1$$

at $t = 0$: $\zeta(0)$ is horizontal, $\varphi = 0$ on $\zeta(0)$, $R(0)$ is a circumference, $\varphi = \text{const}$ on $R(0)$,

$$h > 1, \quad p_0 \gg 1, \quad S(0) = S_0 = \pi a_0^2.$$

The combination of the analog method — for equation (1) which is solved by the method of electrohydrodynamic analogy on electroconducting paper — and the difference method — for equation (2) and (2') — reduces the solution of the problem to determining, at the time t_i , the distribution of the velocity potential values φ_{ij} (near the boundaries of $Q(t_i)$) and of their derivatives $\nabla\varphi_{ij}$ on the boundaries $\zeta(t_i)$ and $R(t_i)$. For the next time moment $t_{i+1} = t_i + \Delta t_i$, from (2) and (2') new $\varphi_{i+1,j}$ are obtained and the obtained values $\nabla\varphi_{ij}$ determine

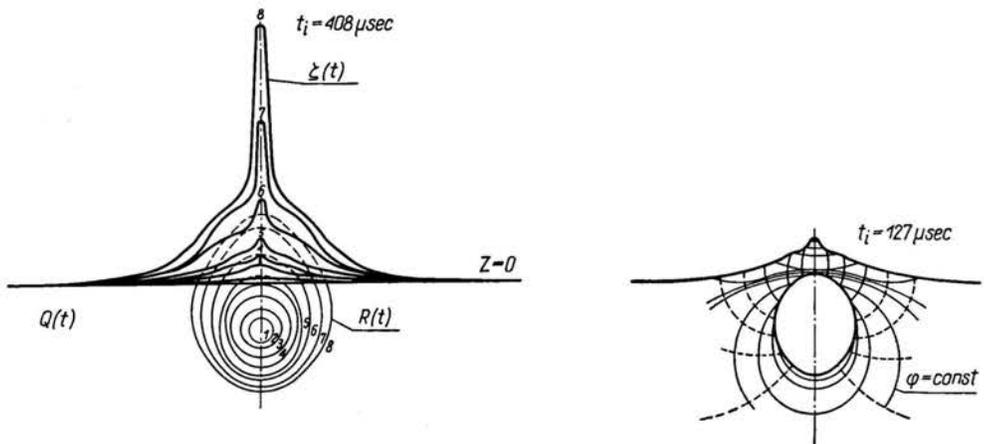


FIG. 2. Calculation of a plane "sultan" for $h = 4$, $p_0 = 4 \cdot 10^4$.

the new location of $\zeta(t_{i+1})$ and $R(t_{i+1})$. Figure 2 depicts the result of calculation for $h = 4$ and $p_0 = 4 \cdot 10^4$. The analysis of the calculation results has shown that part of the liquid above the gas bubble at the initial moments of time obtains an upward vertical impulse with a maximum velocity along the axis, at the subsequent moments moves inertially and is elongated into a jet (sultan). This result can be modelled mathematically and experimentally as follows.

Let at $t = 0$ the solid cylinder with the radius a_0 be placed in the liquid at a depth $H > a_0$ under the horizontal free surface and obtain instantly a velocity V_0 along the normal. Consider, in a similar formulation, an unsteady inertial motion of the liquid after the cylinder stops ($t = t_0$). As a result, the formation of a vertical jet is observed. This is confirmed by experiments (axi-symmetrical and plane) in which a solid semi-sphere (semi-cylinder) placed close to the free surface of the liquid is pushed by an electrical discharge under it — Fig. 3. The above results make it possible to conclude that the mechanisms of sultan formation in both cases are identical.

The role of a shock wave in the process under consideration is illustrated by Fig. 4 with sufficient completeness. A charge with a radius of 2mm is placed between two flat solid walls (plane formulation) at 7.5cm from the free surface. The part of the wall which is above the free surface is made transparent. In the figure, the breaking-off dome with

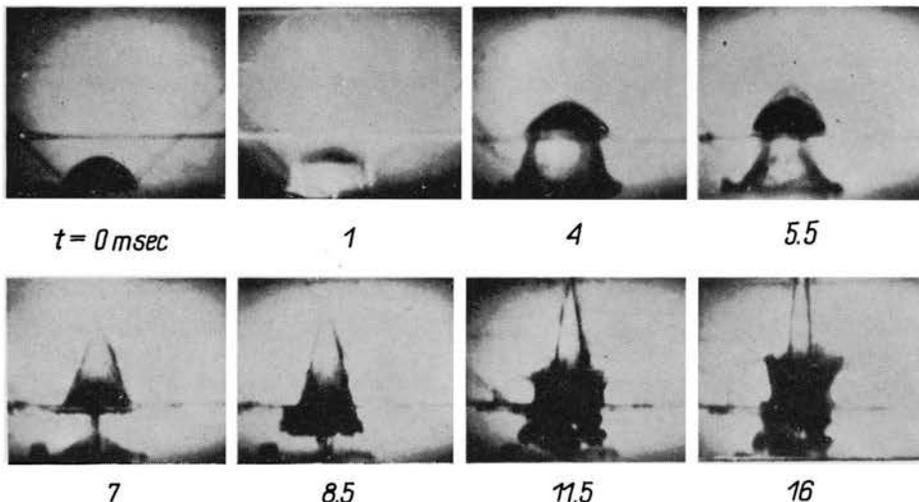


FIG. 3. Experimental model of a "sultan".

the cavitation zone in which the bubble pulsation causes the dome to disperse is distinctly separated from the sultan which is formed under the dome and then pierces it. At least part of the sultan is made up of homogeneous liquid. A considerable thickening of the dome at the axis suggests that a cumulative cavity can be formed on the liquid surface in the breaking-off process which will lead to the intensification of the effect considered in the first model.

A number of authors [1] have noted that the maximum amplitude of the first pulsation pressure is increased anomalously at an underwater explosion near the free surface. The experimental study of this effect has shown that the variation in the maximum pressure of the first pulsation is determined by the variation in the shape of the gas bubble collapsing near the free surface (Fig. 5). As the initial position of the charge approaches the free surface, the maximum amplitude of the first pulsation first decreases (the collapsing gas bubble at these depths flattens taking the shape of an ellipsis) and then sharply increases up to a value exceeding the maximum amplitude for infinite liquid (at these depths gas bubble

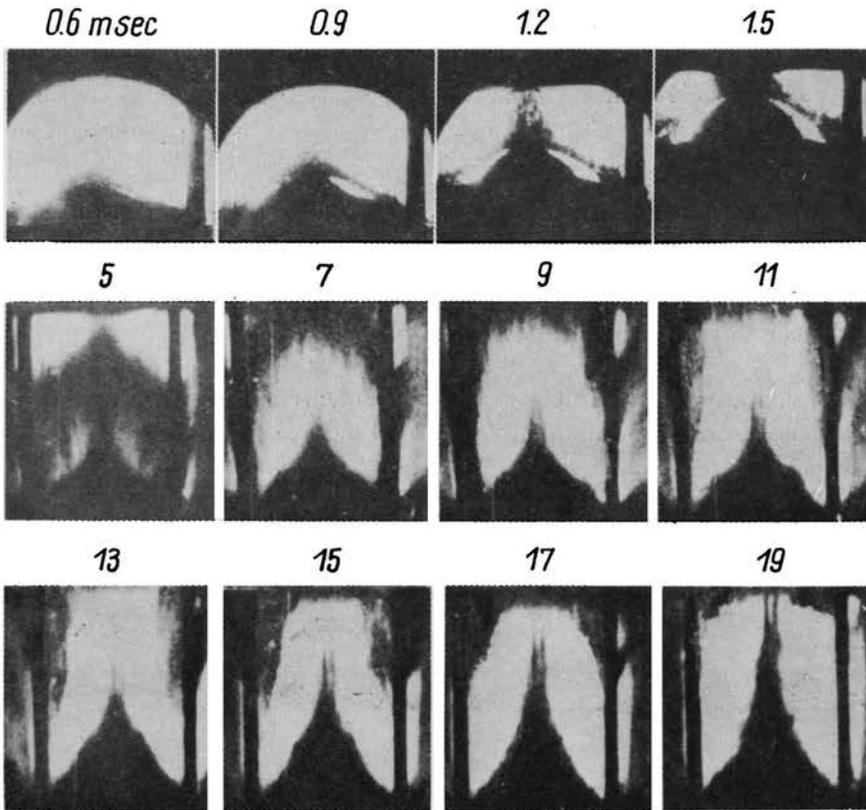


FIG. 4. Development of a breaking-off dome and of a sultan (plane case).

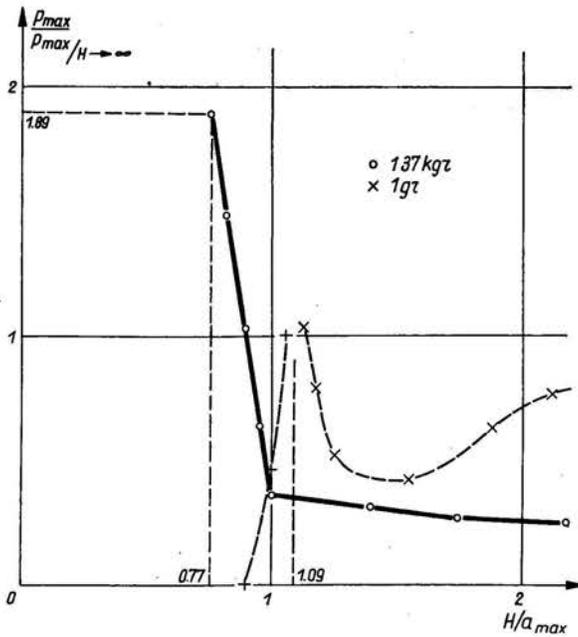


FIG. 5. Variation in the maximum amplitude of the first pulsation with the depth of the charge submersion.

collapses are accompanied by the formation of a cumulative jet directed from the free surface downwards). Figure 6 shows the result of calculating (by above method) the collapse of an empty bubble near the free surface. The initial $\zeta(0)$ and $R(0)$ are shown in the figure, on $\zeta(t) p = p_a = \text{const}$.

At still lesser submersion of the charge, the detonation products force their way up to the atmosphere — the gas bubble turns out to be connected with the free surface of the tubular region — and the first pulsation completely disappears.

The character of the variation field of the first pulsation pressure can be estimated simply within the range of ideal incompressible liquid. For an infinite liquid (the plane variant)

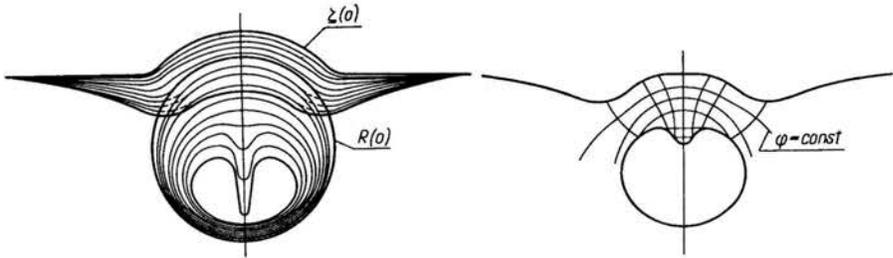


FIG. 6. Calculation of empty bubble collapse near the free surface at $H = 12$ cm, $a_0 = 16$ cm

the maximum gas pressure in the collapsed cylinder (with zero initial velocity) is determined by the expression

$$(3) \quad p_* = p_0 [(\gamma - 1) p_a / p_0]^{\frac{\gamma}{\gamma - 1}},$$

where p_0 is initial gas pressure in the bubble, γ — adiabatic indicator. The collapse of the gas bubble near the free surface can be modelled, taking into account its ellipsoidal shape (in the plane variant), by collapsing, in an infinite liquid, an ellipsis with the poles $+\alpha$, $-\alpha$ at the initial values of the semi-axis $a_0 \approx b_0 \gg \alpha$. This implies that the initially almost cylindrical bubble loses its one-dimensional form during the collapse and its boundary assumes successively the shapes of cofocal ellipses. The maximum gas pressure in the collapsed ellipse (with zero initial velocity) and its semi-axis (a_{\min} and b_{\min}) are determined by (3). The parameter α is determined from some additional considerations or from experiment.

The pressure field formed in the liquid when the cumulative jet penetrates into it can be estimated on the basis of the solution known for a plane stationary problem of two colliding jets one of which has an infinite width. In this case the problem of penetration proves to be identical with that of streamlining a solid whose shape is single-valued and whose size is defined through by the width of the penetrating jet.

In this case the distribution of liquid particles velocities along the symmetry axis is determined by the expression

$$(4) \quad \frac{x}{a} = \frac{1}{\pi} \left[\ln \frac{\beta}{2 - \beta} - \frac{2}{\beta} - \frac{2}{\beta^2} + 4 \right],$$

where a is the semi-width of the jet, β — the ratio of liquid particle velocity to the half of the penetrating jet velocity. The pressure coefficient at sufficiently great distances from the

jet is defined as $\bar{p} \approx 2\beta$. Solving the plane unsteady problem of an empty bubble collapse near the free surface one can estimate the velocity and the geometrical dimensions of the jet. The velocity of the front part of the jet in the position fixed in Fig. 6 is about 300 m/sec at an average bubble wall velocity of 20 m/sec. The further collapsing of the bubble will lead to a sharp increase in the velocity of its boundary and, hence, of the jet.

As a result, the bubble turns out to be divided into two bubbles with the pressure $p_0 \gg p_a$. Consider the problem of the subsequent motion of the liquid in a formulation similar

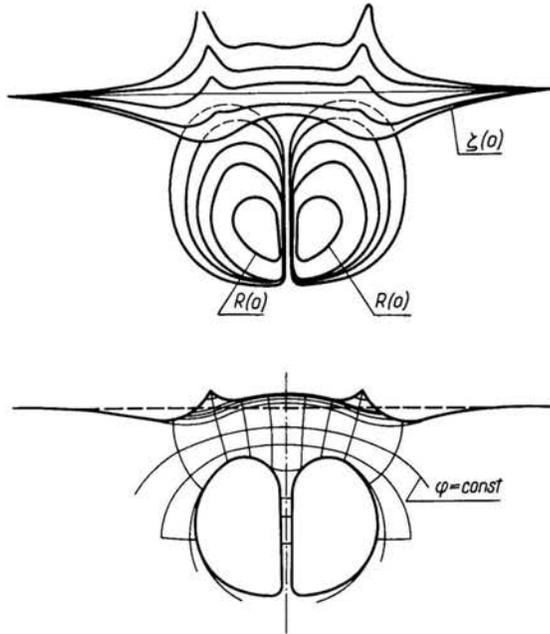


FIG. 7. Calculation of a plane lateral "sultan" at $p_0 = 10^4$.

to the first one with $\zeta(0)$ and $R(0)$ represented in Fig. 7. The figure shows lateral sultans to be developing at the stage of the second expansion of the gas bubble.

Reference

1. ROBERT H. COLE, *Underwater explosion*, Princeton, New Jersey 1948.

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