

**AN ELASTIC CUBE SUBJECTED TO ANTI-SYMMETRICAL PRESSURE LOADING.
EXACT 3D ANALYTICAL FORMULAE VERSUS NUMERICAL SOLUTIONS
BASED ON MESHFREE METHOD**

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1. Introduction

Many Finite Element Method (FEM) solvers do not accurately predict stresses, particularly at interfaces of the elements due to the piecewise continuous nature of the displacement field assumed in FEM formulation. Moreover, the well known difficulties in adaptive analysis using FEM (especially in 3D problems) justify attempts to develop alternative numerical methods, e.g. Mesh Free Methods (MFree). This paper refers to the two versions of the MFree algorithms, namely Radial Point Interpolation Method (RPIM) and Moving Least Squares (MLS) method. The hitherto existing versions of 3D MFree suffer from the drawbacks like relatively small number of numerical examples of quantitative analysis (e.g. lack of comparisons of the numerical solutions with the exact 3D solutions) and lack of the fast search procedures in constructing the influence (or support) domains with irregular and non-convex boundaries. The aim of the present paper is to put forward an improved version of the Element Free Galerkin (EFG) formulation for the numerical approximation of the 3D boundary value problems of linear elasticity. The monomial basis functions from the Pascal pyramid used with the radial basis functions in RPIM and with the non-singular weight functions in MLS method are implemented in computing the shape functions and their derivatives. Well known properties, advantages and disadvantages of both the formulations are discussed in many papers and monographs (see e.g. [2], [4]) but most of the work related to the development of EFG has been focused on two-dimensional applications. On the other hand, the numerical results in three-dimensional EFG method (very often coupled with FEM) are rarely presented (see e.g. [1]). In the present paper, for benchmarking purposes, three various numerical solutions for a linear-elastic and isotropic cube subject to an anti-symmetrical pressure loading are shown and compared. First and second numerical result are obtained by RPIM and MLS methods. The Kronecker delta function property in RPIM allows a direct imposition of essential boundary conditions, but the use of non-singular weight functions in MLS approximation does not allow for a direct imposition of essential boundary conditions, hence EFG formulation with Lagrange Multipliers is implemented. Third solution is shown in the analytical form found by G. Jemielita [3].

2. Numerical and analytical solution

Consider a 3D elastic body $\Omega \subset \mathbb{R}^3$. In the meshfree method used, the global interpolation (in RPIM) and the global approximation (in MLS) $\mathbf{u}^h = \mathbf{u}^h(\mathbf{x}) = [u_x^h(\mathbf{x}), u_y^h(\mathbf{x}), u_z^h(\mathbf{x})]^T$ ($\mathbf{x} = [x, y, z]^T \in \Omega$) of the displacement field are calculated from the formula $\mathbf{u}^h(\mathbf{x}) = \sum_I \Phi_I(\mathbf{x}) \mathbf{u}_I$ where $\Phi_I = \Phi_I(\mathbf{x})$ is the diagonal matrix of the shape functions corresponding $N = N(\mathbf{x})$ nodes in the support domain of the point \mathbf{x} and \mathbf{u}_I is the vector of the displacement parameters of the node I . The exact analytical formula for the displacement field $\mathbf{u} = \mathbf{u}(\mathbf{x}) = [u_x(\mathbf{x}), u_y(\mathbf{x}), u_z(\mathbf{x})]^T$ of the elastic isotropic cube $a \times b \times h$ subject to the loading anti-symmetrical with respect to the middle

plane $z = 0$ load $\mathbf{t} = \mathbf{t}(\mathbf{x}) = [t_x(\mathbf{x}), t_y(\mathbf{x}), t_z(\mathbf{x})]^T$ applied on the top and bottom free sides can be written as (see [3])

$$(1) \quad u_x(\mathbf{x}) = \left[\alpha_1 A_1 z \cosh(pz) + B_1^1 \sinh(pz) \right] \sin\left(\alpha_1 x + \frac{\pi}{2}\right) \sin(\alpha_2 y)$$

$$(2) \quad u_y(\mathbf{x}) = \left[\alpha_2 A_1 z \cosh(pz) + B_1^2 \sinh(pz) \right] \sin(\alpha_1 x) \sin\left(\alpha_2 y + \frac{\pi}{2}\right)$$

$$(3) \quad u_z(\mathbf{x}) = \left[p A_1 z \sinh(pz) + B_1^3 \cosh(pz) \right] \sin(\alpha_1 x) \sin(\alpha_2 y)$$

where $t_x = t_y = 0$, $t_z(x, y, h/2) = t_z(x, y, -h/2) = 0.5 q \sin(\pi x/a) \sin(\pi y/b)$. In the above three expressions (1), (2), (3) all coefficients depend (in a rather complicated way) on the known components a , b , h defining the sizes of the cube, material constants E , ν (Young modulus, Poisson ratio) and load parameter q . The cube is simply supported on the remaining (unloaded) four vertical boundary planes along z and x axes or z and y axes. The material parameters used in analysis are: $E = 3.0 \cdot 10^6 [N/m^2]$, $\nu = 0.3$. A uniform nodal distribution with the total number of nodes equal to 1241 and with the $8 \times 8 \times 8 = 512$ background mesh of hexahedron cells for integration is employed. The basis functions of quadratic order from the Pascal pyramid are used. The original search procedure guarantees that exactly 15 from among at least 20 nodes are visible from each integration point. Dimensionless lengths of the sides of the support domain in all x , y and z directions are set as equal to 3 and the 6-point Gauss integration scheme is adopted. The value of the load parameter $q = 1.0 \cdot 10^6 [N/m^2]$. The length a , width b and height h of the cube are equal to 0.9 [m].

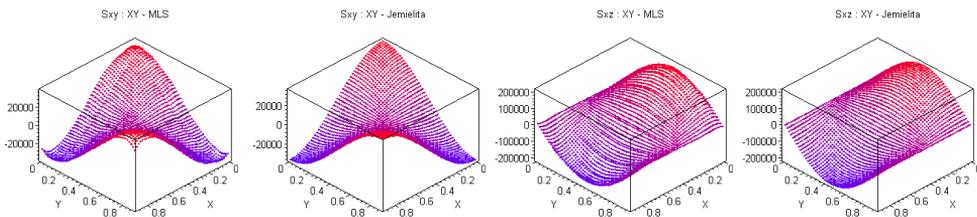


Fig. 1. Shear stress distributions σ_{xy} , σ_{xz} through a cross section $z = 0.225$ [m] of the square block – MLS (first and third figs) and exact analytical solution (second and fourth figs).

The proposed version of the EFG formulation clearly demonstrates robustness of the algorithm and its ability to produce accurate and numerically reliable results.

Acknowledgement. The paper was prepared within the Research Grant no T07A 038 30.

3. References

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