

PROPAGATION OF ULTRASONIC WAVES IN INHOMOGENEOUS MATERIALS

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1. Introduction

The problem of ultrasonic wave interaction with continuous inhomogeneity of material is of great importance for theory and applications. Such materials are commonly present in living systems, nature, building engineering and industry. They are often strongly micro inhomogeneous, form multicomponent systems and processes of their growth, production or destruction often lead to their macro inhomogeneity. The techniques of experimental investigation of such materials, widely developed in recent years, are ultrasonic methods. They allow non-destructive determination of materials coefficients and parameters, evaluation of their state and proceeding processes. However, there is a lack of papers devoted to description and analysis of interaction of ultrasonic waves with continuous inhomogeneity of the material, and existing monographs (e.g. [1], [2]) consider this problem in simple acoustical systems characterized only by the wave number.

The aim of this paper is to present the new method of description and analysis of one dimensional problems of ultrasonic wave interaction with continuous inhomogeneity of materials characterized by dependence of the wave number and the impedance of a medium on the spatial coordinate.

2. Formulation of the problem

We consider one dimensional problem of plane harmonic wave propagation of frequency ω in an arbitrary material with inhomogeneity of acoustical properties that are locally characterized by the impedance z and the wave number k . These parameters, in general, are complex functions of spatial coordinate x and wave frequency ω (Fig. 1).

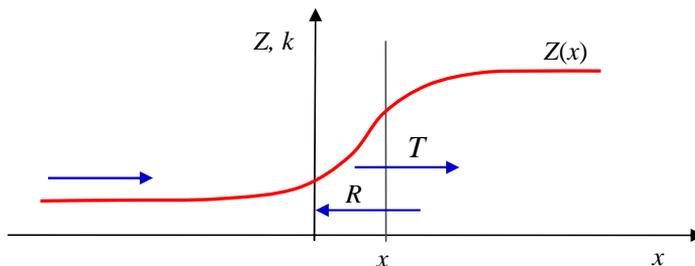


Fig.1. Exemplary distribution of local impedance in material with continuous inhomogeneity of acoustical properties.

Due to interaction with material inhomogeneity each wave propagating in such material generates the coupled backward wave. Therefore, the acoustical field in inhomogeneous material is defined by amplitudes T and R of the forward and backward waves, respectively.

3. Solution of the problem

Equations describing the acoustical field in inhomogeneous material have been obtained considering such medium as a system of homogeneous infinitesimal layers with stepwise changeable acoustical properties approximating functions $k(z)$ and $Z(x)$. In this case wave interaction with continuous inhomogeneity can be considered as multiple reflections and transitions of wave through the boundaries of infinitesimal layers and the acoustical field in the material, characterized by amplitudes T and R , as the superposition of waves propagating in each direction. The obtained equations have the form

$$\frac{dR}{dx} + ikR = \frac{dI}{dx}(2T + R) \quad , \quad \frac{dT}{dx} - ikT = \frac{dI}{dx}(2R + T)$$

where $I = \ln(Z_0/Z)/2$ and Z_0 is constant. From these equations results that only inhomogeneity of medium impedance induce the backward wave.

3. Analysis of exemplary problem

To illustrate the wave interaction with continuous inhomogeneity of the material the special case of inhomogeneous material has been considered. It is composed of two homogeneous halfspaces separated by a layer of inhomogeneous material of thickness L (Fig.3).

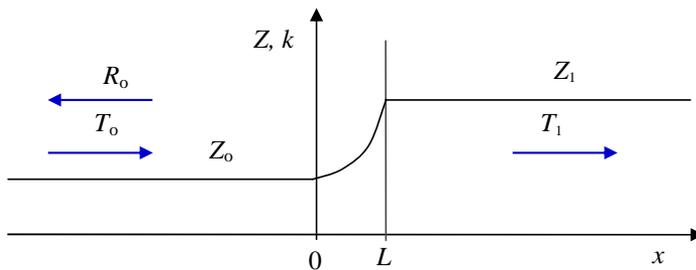


Fig.2. The exemplary medium with continuous inhomogeneity of impedance.

We assume that wave number k is constant in the whole system and impedance of the medium is continuous and changes only within the layer according to the expression

$$Z = Z_0 \exp(\alpha x) = Z_0 (Z_1 / Z_0)^{x/L} \quad , \quad \alpha = \ln(Z_1 / Z_0) / L \quad .$$

In this case the system of equations becomes linear, and due to continuity of impedance, a wave propagating in such medium is reflected only by impedance inhomogeneity in the layer. It enables the analysis of influence of this inhomogeneity on acoustical characteristics of reflected and transmitted waves.

4. References

- [1] L. M. Brekhovskikh (1980). *Waves in Layered Media*. Academic Press, New York.
- [2] P. Filippi, D. Habault, J.P. Lefebvre, A. Bergassoli (1999). *Acoustics. Basic Physics, Theory and Methods*, Academic Press, San Diego.