

## FE-SIMULATIONS OF SIZE EFFECTS IN GRANULAR AND QUASI-BRITTLE MATERIALS

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### 1. Introduction

One of the most important properties of the behaviour of many engineering materials is a size effect phenomenon, i.e. experimental results vary with the size of the specimen [1]. Thus, the results from laboratory tests which are scaled versions of the actual structures cannot be directly transferred to them. Two main size effects can be defined: deterministic (energetic) and statistical. The first one is caused by strain localization which cannot be appropriately scaled during laboratory tests. Thus, the specimen strength increases with increasing ratio  $l_0/L$  ( $l_0$  – characteristic length of microstructure influencing both the thickness and spacing of strain localization,  $L$  – specimen size). This feature is strongly influenced by the pressure level in granular bodies; i.e. shear resistance and dilatancy decrease with increasing pressure. A statistical effect (called also a stochastic effect) is caused by the spatial variability/randomness of local material strength. According to the Weibull's theory (Weibull 1951), this effect is caused by weak spots whose amount usually grows with increasing specimen size. Thus, the specimen strength diminishes with increasing specimen size. Up to now, the size effects are still not taken into account in the specifications of most of design codes for engineering structures. The understanding of the physical mechanism of a size effects is of a major importance for civil engineers who are forced to extrapolate experimental outcomes at the laboratory scale to results which can be used in real situations. Since large specimens or structures are far beyond the range of testing in laboratories, their design has to rely on a realistic extrapolation of testing results with small specimens or structures.

### 2. Size effects in granular bodies

The size effects in granular bodies were investigated with plane strain footings on sand. To describe a mechanical behaviour of a cohesionless granular material during a monotonous deformation path, a micro-polar hypoplastic constitutive model was used. Non-polar hypoplastic constitutive models formulated at the Karlsruhe University describe the evolution of the effective stress tensor depending on the current void ratio, stress state and rate of deformation by isotropic non-linear tensorial functions obtained according to the representation theorem. The constitutive models were formulated by a heuristic process considering the essential mechanical properties of granular materials undergoing homogeneous deformation. A striking feature of hypoplasticity is that the constitutive equation is incrementally linear in deformation rate. The hypoplastic models are capable of describing a number of significant properties of granular materials: non-linear stress-strain relationship, dilatant and contractant volumetric change, stress level dependence, density dependence and material softening. A further feature of hypoplastic models is the inclusion of critical states, i.e. states in which a grain aggregate can deform continuously be deformed at constant stress and a constant volume. In contrast to elasto-plastic models, a decomposition of deformation components into elastic and plastic parts, the formulation of a yield surface, plastic potential, flow rule and hardening rule are not needed. The hallmark of these models are their simple formulation and procedure for determining material parameters with standard laboratory experiments. A further advantage lies in the fact that one single set of material parameters is valid for a wide range of pressures and densities. Hypoplastic constitutive models without a characteristic length can describe only realistically the onset of shear localization, but not its formation. A characteristic length can be introduced into hypoplasticity by

means of a micro-polar, non-local and second-gradient theory. In this paper, a micro-polar theory was adopted [2]. A micro-polar model makes use of rotations and couple stresses which have clear physical meaning for granular materials. The rotations can be observed during shearing and but remain negligible during homogeneous deformation. The presence of the couple stresses gives rise to a non-symmetry of the stress tensor and a presence of a characteristic length.

In the paper, a deterministic (energetic) and statistical size effect were carefully analysed [2], [3]. The deterministic calculations were carried out with an uniform distribution of the initial void ratio for 3 different footing's widths. The numerical results with respect to the load-displacement curve and strain localization were compared with corresponding laboratory tests at Tokyo University [5]. Various properties of granular bodies may be considered as random. In the present work, only the initial void ratio was of primary interest. To investigate a statistical size effect, the distribution of the initial void ratio was assumed to be spatially correlated. In order to reduce the number of realizations without losing the accuracy of the calculations, a Latin hypercube method was applied. Initially, truncated Gaussian random fields were generated in a granular specimen using a conditional rejection method [3] for a weakly and strongly correlated random fields and a large and low standard deviation.

### 3. Size effects in brittle materials

The size effects were investigated in concrete elements subject to uniaxial tension or bending. The analysis was carried out with a finite element method based on an elasto-plastic crack model with non-local softening [6]. A linear Drucker-Prager criterion with an isotropic hardening and softening and a non-associated flow rule was defined in a compressive regime, and a Rankine criterion with an isotropic softening and an associated flow rule was adopted in a tensile regime. To ensure the mesh-independence, to capture properly localized zones and to investigate a deterministic size effect, both criteria were enhanced in a softening regime by a characteristic length of micro-structure with the aid of a non-local theory [6]. The deterministic calculations were carried out with different specimen sizes. They were confronted with corresponding experimental results (e.g. [7]). In the statistical calculations, the tensile strength of concrete was assumed to be random (spatially correlated). In order to reduce the number of realizations without losing the accuracy of the calculations, a Latin hypercube method was again applied.

### 4. References

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