

FREE VIBRATIONS OF SMOOTH CYLINDRICAL SHELLS

G.D. Gavrylenko and V.I. Matsner

S.P. Timoshenko Institute of Mechanics, Ukrainian Academy of Sciences, Kiev, Ukraine

1. Theoretical results and comparison with experimental data.

The method of calculation of the problem and full notation are presented in [1]. The equations of motion are derived using the principle of stationary action.

Using the standard procedure of the energy method we obtained the analytical solution for finding the vibration frequencies of the cylindrical shell. They can be represented in the following form

$$\omega_{ij}^2 = \frac{K}{r^2 h \rho_0} \frac{A_{33}(A_{11}A_{22} - A_{12}^2) + 2A_{12}A_{13}A_{23} - A_{11}A_{23}^2 - A_{22}A_{13}^2}{(A_{11}A_{33} - A_{13}^2) + (A_{22}A_{33} - A_{23}^2) + (A_{11}A_{22} - A_{12}^2)},$$

where ω_j – frequency, $K=Eh/(1-\mu^2)$, h, r – thickness and radius of the shell, E – modulus of elasticity, μ – Poisson's ratio, ρ_0 – the mass density of the material.

In the general case from three different values of ω^2 one can determine ω_{min} , where ω_{ij} – parameters of frequency. Shells with the following data were calculated and tested [2]: length – 450mm, diameter – 400mm, thickness – 0,5mm. The shells had boundary conditions corresponding to simple supported edges.

In Figure 1 the dependence $f_{ij}=\omega_j/2\pi$ on i is presented, where i – the number of waves in the circumferential direction, j – the number of half-waves in the longitudinal direction.

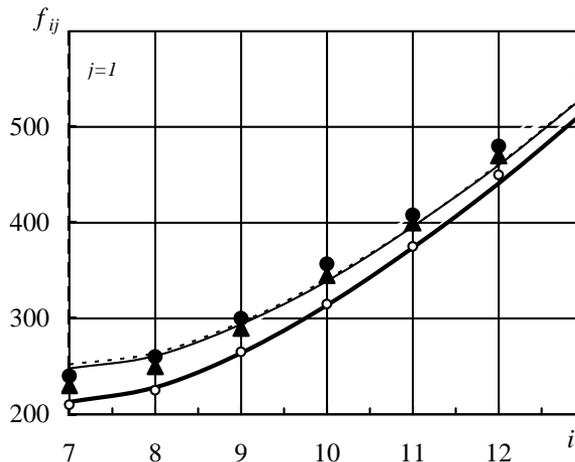


Fig. 1.

In Fig.1 the theoretical curve for an ideal shell is shown, calculated by the above method (solid dark line). The theoretical results [2] are represented by (o) while the experimental data [2] are shown by badges (●, ▲) and the theoretical results for shells having initial deflections are given by the dotted line and thin solid line. The last two shells had deflections with 3 half-waves with amplitude $w_0/h=0,5$ and the zone their placing along the length of the shell is $d/\ell=\ell_0/\ell=0,25$ (in the middle part). The first shell had two dents and one bulge, the second one had two bulges and one dent. The form of deflections was close to $w=w_0 \sin(j\pi x/\ell)$ and j equals 3. The initial imperfections increase by 15% the value of the minimal eigen frequency.

As can be seen from Figure 1 the test data (●, ▲) are close to the theoretical values (black curve). If we take in account axisymmetrical imperfections of the form, the corresponding results (dotted and thin lines) almost coincide with the test data.

From Fig. 1 is clear that regular deflections of such type increase the eigen frequencies of vibration and bring them closer to experimental ones of the defected [2].

2. Conclusions.

A new analytical approach for the analysis frequencies of vibration of the smooth cylindrical shells having initial deflections of the form is used. The analytical solution and the results of calculations are presented. At present there exists a number of methods to calculate vibrations of frequency of ideal shells. But real shells usually have nonideal forms.

The suggested method allows one to define more precisely the eigen frequencies of vibration of incomplete shells. This approach will be used to analyse ribbed cylindrical shells having axisymmetric imperfections under the compressive axial force.

The new numerical approach has been developed at last time for ribbed shells of rotation [3]. The method presented here allows to define more quickly the eigen frequencies than the numerical method [3].

3. References.

- [1] G.D. Gavrylenko & V.I. Matsner (2007). Free vibration of smooth cylindrical shells with local axisymmetrical deflections, *Reports of NAS of Ukraine*, No 12, 54 – 60.
- [2] A.I. Kukarina, V.I. Matsner and E.F. Sivak (1982). On the effect initial deflections on eigen vibrations of ribbed cylindrical shells, *Int. Appl. Mech.*, **18**, No 4, 58 – 63. (*In Russian*).
- [3] G.D. Gavrylenko & O.A. Trubitsina (2008). *Vibrations and Stability of Rotation Ribbed Shells (In Russian)*, Barviks, Dnepropetrovsk.