

PROBABILISTIC RESPONSE OF NONLINEAR SYSTEMS VIA PI: NORMAL, POISSONIAN AND COMBINED WHITE NOISES

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1. Introduction

In Engineering field several actions are random in nature and then estimation of the response, of linear or nonlinear systems under these agencies, has to be developed through statistics. As well known random excitations may be simulated by two kinds of noises: normal and non-normal white noises. For systems under normal or non-normal white noise, the response statistics may be obtained by solving the Fokker-Plank Kolmogorov (FPK) equation or the kolmogorov-Feller equation, respectively. However, exact solutions of the partial differential equations governing the evolution of the response probability density function (PDF) are known only for very few cases. Alternatively, several approximate solutions techniques have been developed including variational methods, finite element method, and the path integration approach. The path integral solution (PIS) is an effective tool for evaluating the response in terms of PDF at each time instant [1-4].

2. General Features on Path Integration method

For summarizing the general features of the (PIS) method, it is better resorting to the case of a half oscillator driven by a white noise, whose equation of motion is given in the form:

$$(1) \quad \begin{cases} \dot{X}(t) = -\alpha X(t) + f(X,t) + W(t) \\ X(0) = X_0 \end{cases}$$

where $f(X,t)$ is a deterministic nonlinear function of $X(t)$ and t, α is a parameter that must be positive and $W(t)$ is a white noise (zero-th order memory Markov process) and X_0 is the attendant initial condition (deterministic or random variable). The PIS allows us to capture the entire evolution of the response process in terms of PDF, having an assigned initial condition.

The starting point is the Chapman-Kolmogorov equation, that holds true, because of the Markovianity of the input and of the response:

$$(2) \quad p_X(x, t + \tau) = \int_D p_X(x, t + \tau | \bar{x}, t) p_X(\bar{x}, t) d\bar{x}$$

The latter, for τ small may be interpreted as a step-by-step procedure, that means, if we suppose that the PDF of the response at the generic time instant (t) is already known, we may evaluate the PDF of the response at the close time instant ($t + \tau$). Regarding the numerical implementation of the PIS method a computational domain D has to be selected. It is convenient to select a symmetrical computational domain with a maximum size equal to x_1 , i.e. $-x_1 \leq x \leq x_1$. The size of the domain is identified by, first, running a Mont Carlo Simulation (MCS) with a low number of samples. Then, dividing the domain in a number n_x of intervals, for each grid point, the path integral from Eq. (2)

can be evaluated. By looking at this equation (2) it is apparent that the crucial point is to evaluate the kernel, where a conditional joint PDF is present. Considering the physical significance of this conditional joint PDF that is: from the whole trajectories of the response process $X(t)$, we take those assuming in t the deterministic value \bar{x} , hereafter labelled $\bar{X}(\rho)$ solution of the following differential equation:

$$(3) \quad \begin{cases} \dot{\bar{X}}(\rho) = -\alpha \bar{X}(\rho) + f(\bar{X}, \rho) + W(t + \rho) \\ \bar{X}(0) = \bar{x} \end{cases}$$

being \bar{x} a deterministic initial condition and $0 \leq \rho \leq \tau$. It is worth stressing that: the CPDF of Eq.(1) coincides with the unconditional PDF of Eq.(3a) evaluated in τ , that is

$$(4) \quad p_X(x, t + \tau | \bar{x}, t) = p_{\bar{X}}(x, \tau)$$

These are the general features of PIS, now the problem is to particularize the kernel and this will be dependent on the system and on the type of white noise.

It will be introduced an excursus to asses how versatile is PIS for evaluating probabilistic response of linear and nonlinear systems. Firstly the case of systems under normal white noise will be examined, secondly the case of systems under Poissonian white noise and lastly the case under combined noises that is normal and non-normal white noises acting simultaneously. The accuracy of the method is assessed using Monte Carlo simulation and the exact solution when the latter is available.

3. References

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