

Topology of surface shear-stress lines

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THE TOPOGRAPHY of surface shear-stress lines on a three-dimensional body topologically equivalent to a sphere has been studied on the basis of simple topological rules. The main points discussed and the concepts introduced are the topological similarity among different patterns and their standard representations on a plane, the structural stability as a discriminating principle of selection among different patterns and the gradual merging principle as an intuitive way of constructing sequences of patterns connected to the variation of external conditions. A brief discussion of some theoretical and experimental patterns deduced from the literature has been done in order to illustrate these points.

Na podstawie prostych zasad topologicznych przeanalizowano topografię linii powierzchniowych naprężeń ścinających dla ciała trójwymiarowego topologicznie równoważnego sferze. Podstawowe problemy tu omówione i wprowadzone koncepcje to podobieństwo topologiczne różnych struktur i ich standardowych reprezentacji na płaszczyźnie, stateczność strukturalna jako kryterium wyboru spośród różnych struktur i zasada stopniowej fuzji jako intuicyjny sposób konstruowania ciągów struktur związanych ze zmianami warunków zewnętrznych. Po omówieniu tych zagadnień przedstawiono krótką dyskusję teoretycznych i obliczeniowych aspektów problemu wynikających z literatury.

На основе простых топологических принципов проанализирована топография поверхностных линий напряжений сдвига для трехмерного тела топологически эквивалентного сфере. Основные здесь обсужденные проблемы и введенные концепции это топологическое подобие разных структур и их стандартных представлений на плоскости, структурная устойчивость как критерий выбора среди разных структур и принцип постепенного синтеза как интуитивный способ построения последовательностей структур, связанных с изменениями внешних условий. После обсуждения этих вопросов представлена короткая дискуссия теоретических и расчетных аспектов проблемы, вытекающих из литературы.

1. Introduction

DURING the last years interest for the structure of complex flows has increased considerably. This is due not only to the large interest that phenomena like coherent structures, separation, vortex breakdown have practically, but also to a new way of thinking about flows. The urgency for quantitative analyses has very often been the reason why qualitative inspections have been neglected: it is more and more evident that in many cases and particularly when the flow is very complex, the first objective is to sketch in outline the pattern of the motion.

Qualitative analysis means essentially topology on the theoretical side and flow visualization on the experimental side, and the interest for these old and newly revisited tools for the physical inspection of complex flows is testified by the growing number of articles dedicated both to topology applied to fluid motions and to new methods of visualization. The analysis of surface shear-stress lines on a surface invested by an external

flow is a powerful tool for studying the separation. The structure of these patterns is determined by the nature and the location of the critical points, usually saddles and nodes, and their number is subjected to rules related to the topological kind, the genus, of the surface.

Apart from the pioneering works of Lighthill [1] and Legendre [2], we notice at present the need for a systematic development of the subject and fundamental in this respect is the recent work of Peake and Tobak [3] where the task of providing a flow grammar based on simple singular points by which to create sequences of plausible flow structures has been tackled. In our work we have in view some main objects. First of all we stress the importance of a clear description of the patterns, preserving their main topological properties, and we propose a *standard representation* based on simple symbols for the singular points and on projections on the plane. In many cases the representation of patterns are obscure and conceal symmetries or similarities with other patterns.

Second we discuss the static topological properties of the patterns, based on the index theorem and on the criteria of *structural stability*, and we discuss the concept of *topological similarities* between patterns. The third point is relative to the topological dynamics of patterns, and we consider the bifurcations from one structure to another with reference to the variation of some parameter like the incidence or the Reynolds number of the external flow. We introduce the concept of *local gradual merging* of new singular points in a pattern and finally we discuss some examples in order to illustrate all these points.

2. General remarks. Standard representation

Let us consider a surface in three-dimensional space, and let us introduce on it a system of curvilinear coordinates α, β . If we indicate with τ_α and τ_β the shear stress components parallel to the coordinate lines, the pattern of the surface shear stress lines is given at a fixed instant t by the system

$$(2.1) \quad \frac{d\alpha}{d\lambda} = \tau_\alpha(\alpha, \beta, t), \quad \frac{d\beta}{d\lambda} = \tau_\beta(\alpha, \beta, t),$$

where λ is a parameter, so that the solutions are curves on the given surface. When at least one of the two components of the shear stress is different from zero, we say that the point is regular; on the contrary, we are in presence of a critical, or singular, point. The nature of a critical point is given by its topological index which is defined in the following way. Let P be a singular point (Fig. 1) and \mathcal{C} a circle around it such that no

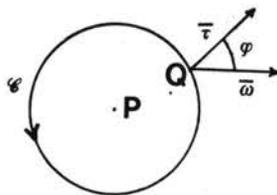


FIG. 1.

other singular point lies within \mathcal{C} or on the boundary. At each point Q of \mathcal{C} we consider the angle ϕ formed by the local vector $\bar{\tau} \equiv (\tau_\alpha, \tau_\beta)$ and a fixed direction of reference $\bar{\omega}$. If we move around \mathcal{C} , ϕ will vary and when we return to the starting point its initial value will be increased or decreased by a multiple of 2π , say $2k\pi$, where k , the index, is an integer, positive or negative.

There are global index theorems relative to the sum of indices of singular points [4, 5] and depending on the topological genus of the surface [6, 7]. In particular, if g is the genus of the surface, (its degree of connection), the sum s of the indexes of the singular point is given by the expression

$$(2.2) \quad s = 2 - 2g,$$

where $g = 0$ for a surface topologically equivalent to a sphere, $g = 1$ for a surface topologically equivalent to a torus, and so on. With regard to the nature of the singular points on a surface we can classify them starting from the simplest ones as saddles (Fig. 2), index -1 , and focuses, nodes and centers (Fig. 3) index 1.

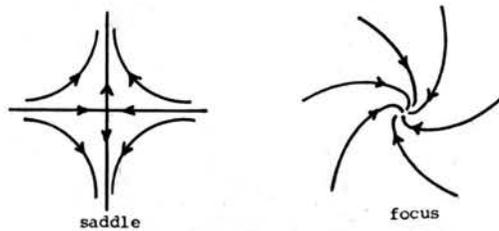


FIG. 2.

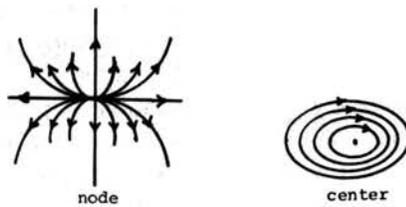


FIG. 3.

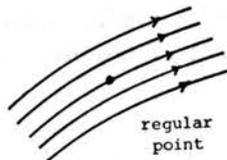


FIG. 4.

The index of a regular point is zero (Fig. 4) and we stress the fact that the index is always an integer characterizing from a topological point of view more and more complex, (degenerated), singularities.

Let us now consider a three-dimensional body topologically equivalent to a sphere invested by an oncoming uniform external flow. In this case the genus g of the surface

is zero, and from the relation (2.2) we have $s = 2$. At a very low angle of attack and Reynolds number the pattern of the skin friction lines is the simplest one, characterized by only two singular points with index 1. According to the definition stated in [3], we call this flow an attached flow, which generally is characterized by two nodal points, an attachment one and a separation one (Fig. 5), or two focuses or centers (Fig. 6).

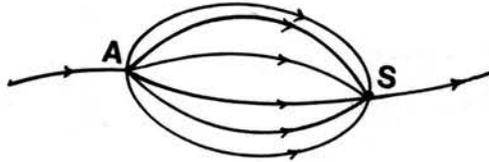


FIG. 5.

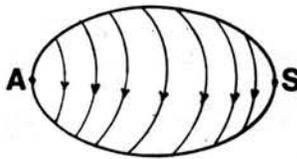


FIG. 6.

This last case would perhaps be verified in a swirling flow at a very low Reynolds number, but from the structural point of view centers are unstable singular points, and we will always consider flows which do not produce centers on the body, but only saddles and nodes or focuses. By applying the principles of structural stability we neglect also singularities of higher indices because they are structurally unstable, so that they split in simpler types for a slight perturbation of the pattern.

It is now important to have a good representation of these patterns and to this end we make a projection of the shear stress lines on a plane whose point at infinity corresponds to a selected node, an attachment or a separation one (Fig. 7), so that in the case of the attached flow we obtain the very simple pattern (Fig. 8) which preserves the peculiar topological features of the shear stress lines.

In order to simplify further the pattern representation, we can also use the symbols described in Fig. 9 where attachment and separation nodes or focuses are respectively reduced to white and black circles, and a saddle is characterized by its peculiar separation lines.

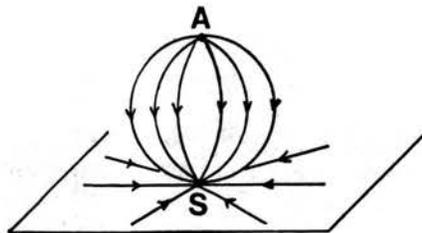


FIG. 7.

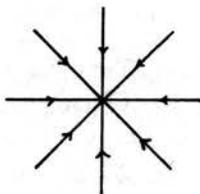


FIG. 8.

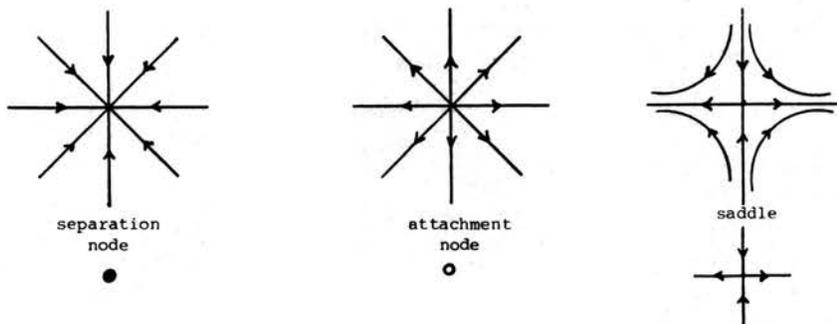


FIG. 9.

These symbols and the projection of shear stress lines a on plane constitute what we call a *standard representation*, and we will always assume that in this representation the point at infinity is a node. One standard representation of the attached flow is given in Fig. 10, where the attachment node is at infinity, and the following patterns, composed of three nodes and one saddle are represented in Fig. 11, where we notice that a separation line is represented by an interrupted line when it goes to infinity, and where the point at infinity is always an attachment node.



FIG. 10.

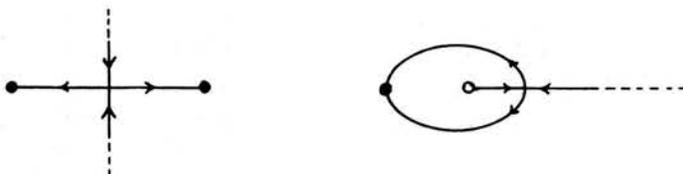
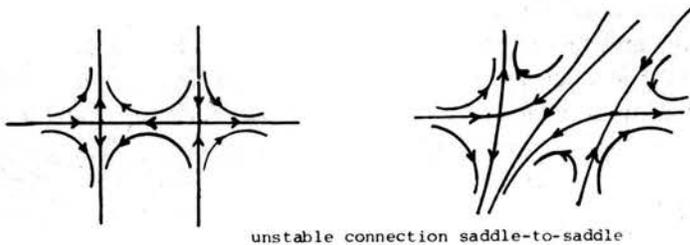


FIG. 11.

We notice that in some cases different standard representations could be useful, for example when there are two symmetric attachment points on the body. In such cases it would be better to project at infinity a different node, for example a separation one, and we obtain topologically equivalent representations of the given pattern.

3. Structural stability of the patterns and local gradual merging of new singularities

Taking into account these topological rules and descriptive norms, we can start studying the separation on a three-dimensional body. One way of discussing the problem is very simple: we start from the simplest topological pattern, the attached flow (Fig. 10) and by assuming the gradual merging of new nodes and saddles, we generate and classify more and more complex patterns. It is easy to verify that two simple patterns follow the simplest one of the attached flow, and they are represented in Fig. 11, but it is also easy to realize that the number of patterns increases in an overwhelming way when another couple of saddle and node merges from the patterns of Fig. 11. We can name this set of structures N^4S^2 , because they are composed of four nodes and two saddles, whereas the two patterns of Fig. 11 are the structures N^3S , and the fundamental pattern of the attached flow is the N^2 structure and another important distinction is relative to the nature of the nodes, that can be attachment or separation ones. Obviously the N^2 structures can only be of the form N_aN_s , where the pedices a and s indicate respectively attachment and separation, but internally to the N^3S structures we have $N_aN_s^2S$ and $N_a^2N_sS$ structures, and as subsets of the N^4S^2 structure we have patterns constituted by $N_a^3N_sS^2$, $N_a^2N_s^2S^2$ and $N_aN_s^3S^2$. We notice that it is very important to select those patterns that are structurally stable [8]. As a matter of fact, from the physical point of view we must discard patterns which contain centers, higher order (degenerated), singularities, non-isolated singular points and connections saddle to saddle (Fig. 12), and particular attention must be given to this last source of instability, as we will see in the following.



unstable connection saddle-to-saddle

FIG. 12.

It is interesting also to classify these new structures in accordance with some generating rule, by specifying some mechanism by which the pattern changes its structure. The problem is studied in detail in [3] by PEAKE and TOBAK, and following their suggestions we postulate a simple rule of *local gradual merging* of new structures. We simply define as local gradual merging of new patterns the merging of saddles and nodes such that the topology of the structure is continuously changed from one type of pattern to another, and we illustrate with some examples what we mean. In Fig. 13a P is a regular point, and if we consider the simplest way of local gradual merging of a couple saddle and node such that externally the pattern does not change, we have the new pattern of Fig. 13b.

In Fig. 14a P is a nodal point, and on the same assumption as before we have the new pattern of Fig. 14b or the one sketched in Fig. 14c, and finally in Fig. 15a we consider



FIG. 13.

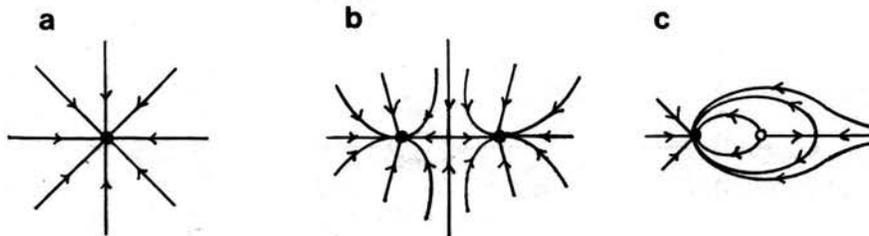


FIG. 14.

a saddle point P , and also in this case the local gradual merging of a couple saddle and node gives place to the structure of Fig. 15b.

A less intuitive way of stating the principle of gradual merging could be based on the value of the index connected to a circle: if we assume that this index can only change when a singular point crosses the boundary of the circle, no matter how little the radius is, we arrive to a more precise definition. Obviously many other types of gradual merging can be supposed. We have analysed the simplest way in which one couple of saddle and node merges from a regular or a singular point, (node or saddle), but more and more complex situations could occur, and as an example in Fig. 16b we have represented the merging of two couples of saddles and nodes from the regular point P of Fig. 16a.

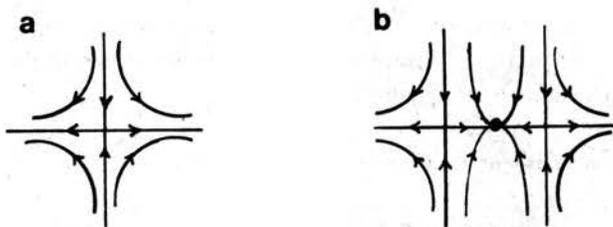


FIG. 15.

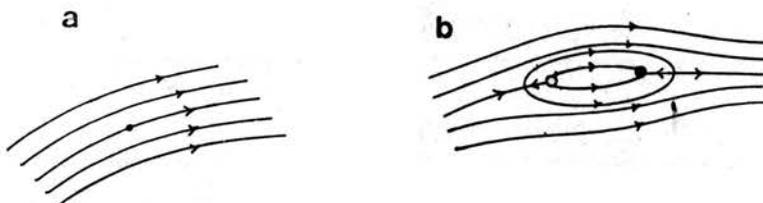


FIG. 16.

Another point must be stressed. In many cases a particular symmetry of the body can induce symmetric merging in the pattern, and in many cases we could have the simultaneous merging of new saddles and nodes from different points. In Fig. 17b we have represented such a situation, where the axis of symmetry, due for example to some symmetry of the body, is the dotted separation line of the primitive pattern 17a.

Finally we notice that in some cases patterns that apparently are very different have the same topological structure. This similarity between patterns is very interesting and introduces a new and fundamental criterion of similitude between flows.

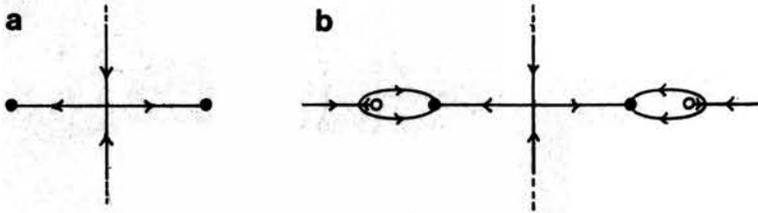


FIG. 17.

4. Topography of shear-stress lines. Selected examples

We will now analyse, with the support of these general considerations, some cases of practical interest. We will consider typical three-dimensional bodies of aerodynamic interest, simply connected, without emphasizing excessively the condition of analyticity of their surface. Pointed protuberances or sharp edges violate effectively analyticity, but the topology of the patterns can be in many cases deduced on the basis of the previous simple assumptions. We have extracted our standard representations from a large number of experimental and computational works on the subject, and particularly interesting and stimulating in this respect is the Agardograph of PEAKE and TOBAK previously cited [3].

After some general remarks on the patterns of shear-stress lines on a body of revolution, we will analyse, as particular examples, shear-stress patterns on spheroids, on cylinder-flare bodies and on slanted afterbodies.

4.1. Shear-stress lines on bodies of revolution. General remarks

Let us first of all consider the characteristic types of separation around a two-dimensional symmetric body without incidence. Starting from the attached flow (Fig. 18a) we have tail separation (Fig. 19a) or bubbles (Fig. 20a).

If we now consider the body of revolution obtained by revolving around its axis this two-dimensional symmetric body, we have, in the standard representation with the attachment point A at infinity, the patterns respectively of Fig. 18b, Fig. 19b, and Fig. 20b.

We notice that the particular symmetry of this body of revolution without incidence introduces in our analysis something very strange and till now not considered in the literature: one or more whole lines of singular points. Non-isolated critical points are highly unstable from the structural point of view. A slight asymmetry or incidence breaks

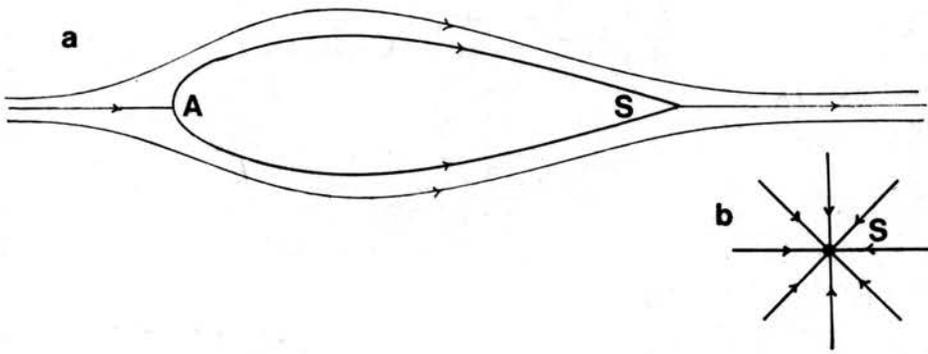


FIG. 18.

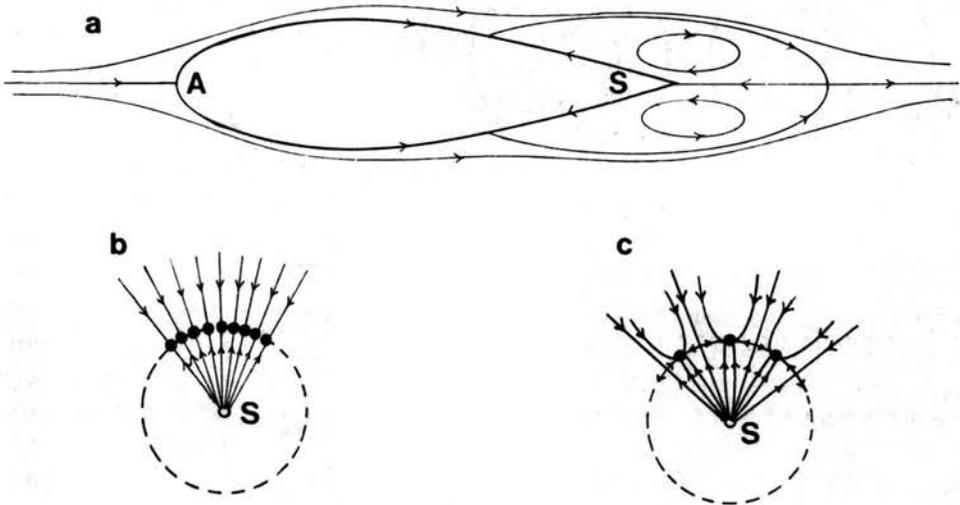


FIG. 19.

this kind of pattern, preserving its peculiar nature, for example by substituting the line of singular points with a sequence of nodes and saddles, as in Fig. 19c.

In the case of the pattern of Fig. 20b, the three-dimensional equivalent of the separation bubbles on the two-dimensional body of Fig. 20a, it is more difficult to conjecture what happens when this unstable structure breaks down. As a matter of fact two different arrangements preserve this peculiar nature, the one sketched in Fig. 20c and the alternating one of Fig. 20d, but the first one must be rejected on the basis of its structural instability, due to the fact that there are connections saddle to saddle. We will return to the stable pattern of Fig. 20d when we will discuss the shear-stress lines on cylinder-flare bodies.

4.2. Shear-stress lines on spheroids

In our considerations of the flow separation on spheroids we have analysed the works of COOKE and BREBNER [9], WANG [10] and HAN and PATEL [11]. In [9] the situation

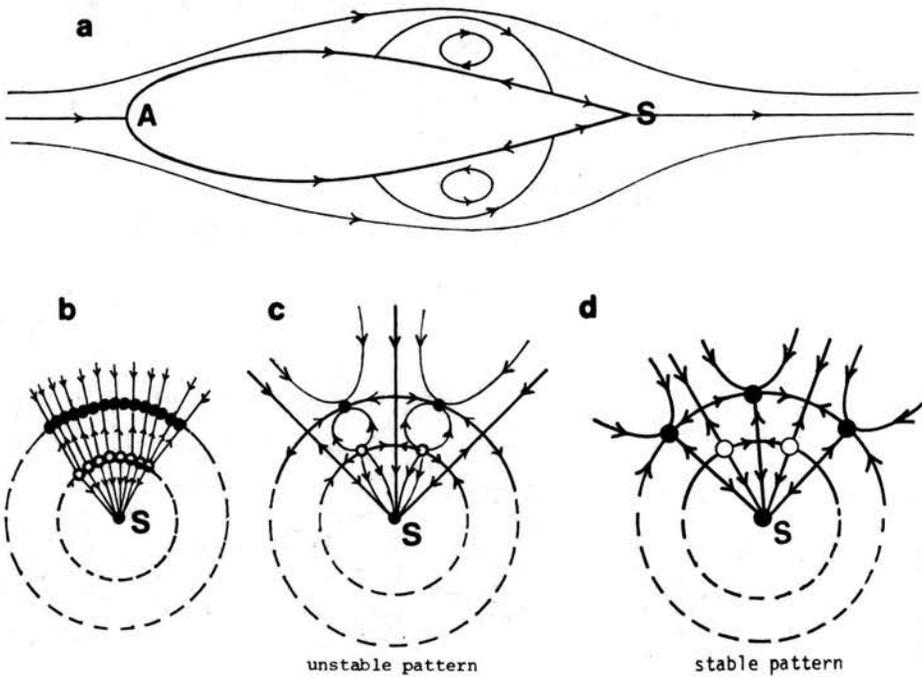


FIG. 20.

is exposed according to EICHELBRENNER [12], and in our standard representation, with the node of attachment at infinity, we have for low incidence, $\alpha < 9^\circ$, the pattern of Fig. 21, and the pattern of Fig. 22 for high incidence, $\alpha > 9^\circ$. Wang apparently postulates the same pattern both at low and at high incidence, $\alpha = 3^\circ$ and $\alpha = 12^\circ$, and this pattern is the same assumed by Eichelbrenner for high incidence, (Fig.22). Finally, in their recent experimental work Han and Patel observe at low and moderate incidences, $\alpha = 5^\circ$ and 10° , the surface flow pattern of Fig. 21, and at high incidence, $\alpha = 30^\circ$, a new flow pattern, the one depicted in Fig. 23.

It is interesting to notice that Lighthill in his classical work about topography of skin-friction lines and vortex lines [1] illustrates a possible pattern of skin-friction lines on a smooth surface, (Fig. 11, 12 at page 79 of Ref. [1]). The relative standard representation

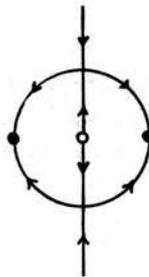


FIG. 21.

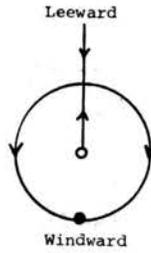


FIG. 22.

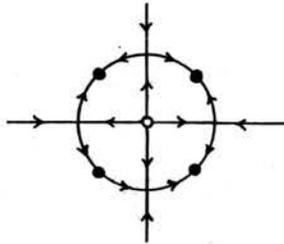


FIG. 23.

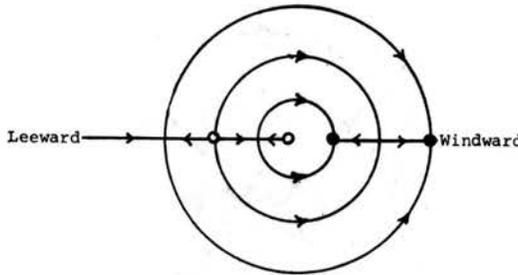


FIG. 24.

with the main attachment node placed at infinity is sketched in Fig. 24, and a possible sequence starting from the fundamental attached flow and based on the gradual merging of new singularities is presented in Fig. 25.

4.3. Shear-stress lines on cylinder-flare bodies

Very interesting is the pattern of skin friction lines of cylinder-flare bodies (Fig. 26) at an angle of attack in supersonic flow, conjectured by Peake and Tobak on the basis of experimental and numerical works, (see [3], pp.100–102 for references).

In our standard representation it looks like Fig. 27 for an angle of attack of 4° and a Mach number 2.8. It is probable that for zero incidence the asymmetry between the leeward and the windward sides disappears, giving place to the pattern of Fig. 28, where there is a reabsorption of the saddle *A* and of the two nodes *B*, and it is interesting to notice that this structure is the stable one represented in Fig. 20d and already discussed

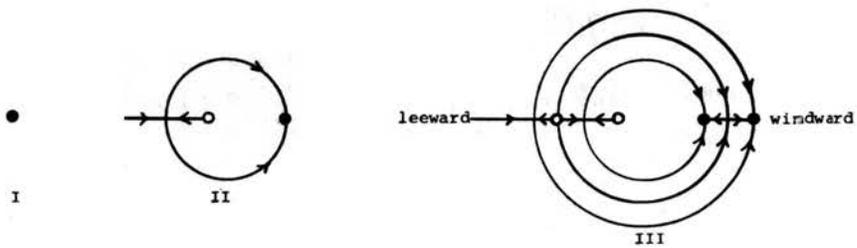


FIG. 25.

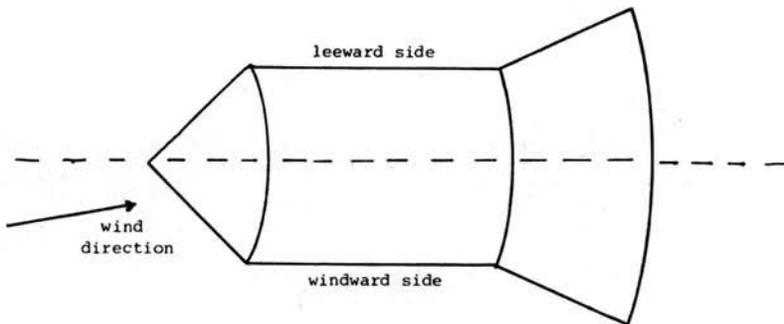


FIG. 26.

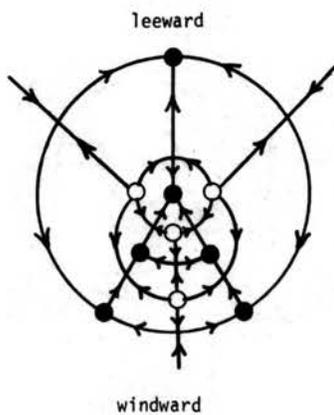


FIG. 27.

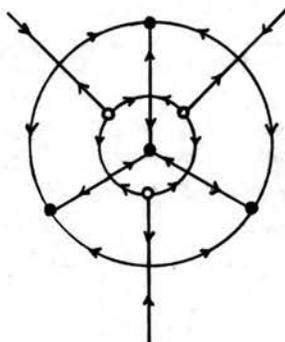


FIG. 28.

when speaking of bodies of revolution. As a matter of fact, if we consider a section of the cylinder-flare body, we have a situation like that presented in Fig. 29, and we can connect this kind of separation with that presented in Fig. 20a.

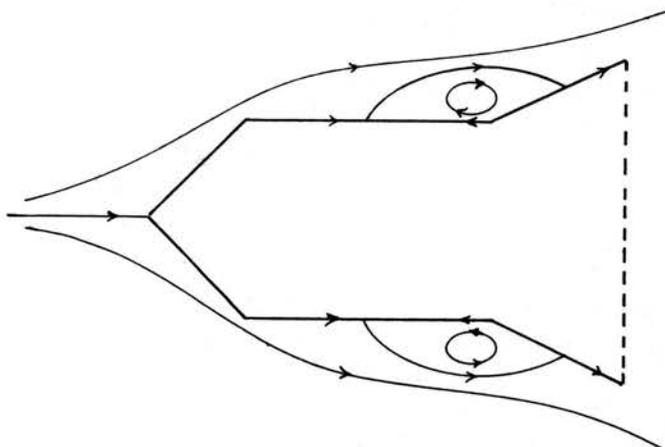


FIG. 29.

4.4. Shear-stress lines on slanted afterbodies

Let us now consider the separation around a body with a slanted rear-surface, see Fig. 30. It has been noted that the flow is greatly influenced by changing the angle of the slanted rear surface, (this kind of surface is significant for experiments on vehicle-like bodies), and two regimes have been identified [13], the first one for α ranging from 10° to nearly 45° and the second one for α ranging from 45° to 90° . At 45° there is a sudden decrease in the drag coefficient, revealing a new structure of the flow.

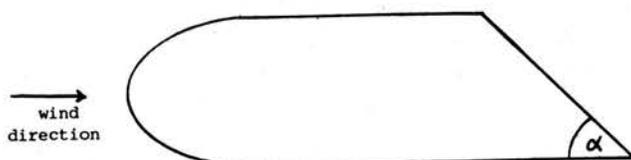


FIG. 30.

We think that in the first case we have a topological structure of the shear-stress lines like the one sketched in Fig. 31a, while in the second case there is a gradual merging of the type described in Fig. 15b, that interests the saddle A , giving place to the pattern of Fig. 31b. Further studies on visualizations would be very useful in this regard.

5. Concluding remarks

Other examples of surface shear-stress line patterns could be given, extracted from the numerous publications on the subject, but it is better now to resume some main points. First of all we stress the importance of the *standard representation* of the pattern in order

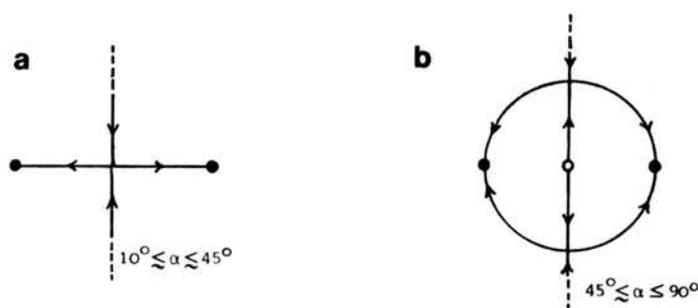


FIG. 31.

to clearly establish what we can call the *topological criteria of similitude* between flows: two patterns can be topologically equivalent notwithstanding apparent big differences provided that they have the same critical points connected in the same way.

A systematic study of these patterns, starting from the simplest one with due attention to particular subsets subjected to some properties of symmetry and selected on the basis of the *structural stability*, would be very rewarding, but little or nothing has been done on this subject that is deeply connected with the mathematical qualitative methods of solving differential equations, (see ARNOLD [8]).

These problems and the index theorems and the criteria of structural stability constitute what we can call the *topological statics* of patterns. By increasing some external parameter like the angle of incidence, or the Reynolds number of the flow, we observe a gradual merging of new patterns. On the basis of the *local gradual merging* principle, and with due attention to particular symmetries of the body, the *topological dynamics* of the patterns can be inferred from experiments. This point is very crucial: we need better and better visualizations obtained on carefully selected bodies in order to feed and test theory.

Finally we must stress the fact that the final aim is the study of the structure of the external flow. What we can deduce in this respect by the analysis of the shear-stress lines is an open problem.

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Note added in proof

Some points of the article deserve a little comment. First of all the author stress the fact that the idea of constructing sequences of topological patterns by using the topological rule of the indices and based on the merging of singular points is a fundamental result contained and developed in detail in the cited Agardograph of Peake and Tobak. The principle of structural stability has been discussed by the same authors in a recent article (M. TOBAK and D. J. PEAKE, *Topology of three-dimensional separated flows*, *Annual Review of Fluid Mech.*, 14, 61-85, 1982), and particularly interesting is the distinction between structural and asymptotic stability, where the asymptotic stability is defined as the stability in time to small perturbations.