

Displacement field of a rectangular dislocation loop

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THE DISPLACEMENT field of a rectangular tangent dislocation loop is evaluated, using the Green function method. The results are given in an analytic form. In the neighbourhood of the edge type segments of a dislocation line, the component of the displacement field perpendicular to the line and to the Burgers vector has a logarithmic singularity.

Wyliczono pole przemieszczeń prostokątnej pętli dyslokacji stycznej posługując się metodą funkcji Greena. Wyniki dane są w postaci analitycznej. W pobliżu odcinków linii dyslokacji typu krawędziowego składowa pola prostopadła do linii i do wektora Burgersa posiada osobliwości logarytmiczne.

Вычислено поле перемещений прямоугольной петли касательной дислокации, послуживаясь методом функции Грина. Результаты даются в аналитическом виде. Вблизи отрезков линии дислокации краевого типа составляющая поля, перпендикулярная к линии и к вектору Бюргерса, обладает логарифмической особенностью.

1. Introduction

IN THE FRAMES of the linear theory of elastic media, a dislocation can be considered either as a surface defect, to which corresponds a discontinuous displacement field, or the line defect, to which corresponds an unintegrable distortion field; both treatments being equivalent (see for example [5]). The displacement field of a dislocation has the constant jump discontinuity $-b$ across the surface S resting on the dislocation line L . b is the Burgers vector. The dislocation surface S can be chosen in an arbitrary way; the quantities describing the state of the medium, the strain and stress fields do not depend on S , they depend on its boundary L only. In this work, when considering the case of a rectangular plane dislocation loop, we choose for S the segment of the plane cut out by the loop. But our purpose is only to simplify the calculations.

In this work we derive and analyse the expression for the components of the displacement field of a rectangular dislocation loop. It is one of the basic problems of the mathematical dislocation theory. The analysis of the expressions derived indicates that the solutions of the problems of dislocations available in the frames of the linear elasticity theory are highly unsatisfactory. The displacement field of a dislocation goes to infinity in the neighbourhood of a dislocation line, what is an undesired, unphysical effect. The singularities are due to the single force distribution along the edge type segments of a dislocation line, necessary to assure the momentum balance of a dislocation.

2. The calculation of the displacement field of a rectangular dislocation loop

Let us consider the rectangular plane dislocation loop L lying (in the appropriate coordinate system) in the plane (x, z) . The rectangular piece of the plane (x, z) , cut out

by the dislocation line, is chosen as the dislocation surface S . The surface S is given by (see Fig. 1)

$$(2.1) \quad \mathbf{x}' = [x', y', z'] \in S: \quad -M \leq x' \leq M, \\ y' = 0, \\ -L \leq z' \leq L.$$

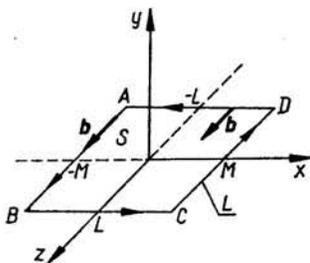


FIG. 1.

The normal vector of the surface S is running along the y -axis. The Burgers vector of the considered dislocation is running along the z -axis, so it is the tangent dislocation loop,

$$(2.2) \quad \mathbf{b} = [0, 0, b].$$

The segments of the dislocation loop parallel to the z -axis have thus the character of a screw dislocation, whereas those parallel to the x -axis have the character of an edge dislocation.

The displacement field \mathbf{u} of a static dislocation in the infinite elastic medium is given by [1, 4]

$$(2.3) \quad u_i(\mathbf{x}) = \int_S ds_b b_n c_{nbrs} \nabla_s G_{ir}(\mathbf{x}' - \mathbf{x}), \quad \mathbf{x}' \in S, \\ \nabla_s \equiv \frac{\partial}{\partial x_s},$$

where S is the dislocation surface, \mathbf{b} is the Burgers vector, \mathbf{c} is the tensor of elastic constants of the medium and \mathbf{G} the Green tensor of the static Lamé equations.

For the isotropic medium

$$(2.4) \quad c_{iklm} = \lambda \delta_{ik} \delta_{lm} + \mu (\delta_{il} \delta_{km} + \delta_{im} \delta_{kl}), \\ \mathbf{G}_{ik}(\mathbf{x}' - \mathbf{x}) = \frac{1}{4\pi\mu} \left\{ \frac{\delta_{ik}}{r} - \frac{1}{2} \frac{\lambda + \mu}{\lambda + 2\mu} \nabla_i \nabla_k r \right\}, \quad r = |\mathbf{x}' - \mathbf{x}|,$$

λ and μ are Lamé constants.

The field \mathbf{u} satisfies the homogeneous Lamé equations outside the surface S , having the jump discontinuity equal to $-\mathbf{b}$ across S :

$$(2.5) \quad |[\mathbf{u}(\mathbf{x})]| = \mathbf{u}^+(\mathbf{x}) - \mathbf{u}^-(\mathbf{x}) = -\mathbf{b}, \quad \mathbf{x} \in S,$$

$\mathbf{u}^+(\mathbf{x})$ and $\mathbf{u}^-(\mathbf{x})$ are the one-sided limits of \mathbf{u} at the point \mathbf{x} of S , when approaching S from its positive or negative side.

In what follows, we consider the dislocation in the isotropic, elastic medium. For the dislocation loop given above, we obtain thus the following expression for the displacement field:

$$(2.6) \quad u_i = b \int_S ds c_{32rs} \nabla_s G_{ir} = b\mu \int_{-M}^M dx' \int_{-L}^L dz' [\nabla_2 G_{i3} + \nabla_3 G_{i2}] \\ = b\mu \int_{-M}^M dx' \int_{-L}^L dz' \nabla_2 G_{i3} - b\mu \int_{-M}^M dx' G_{i2} \Big|_{z'=-L}^{z'=L}.$$

Substituting into the above formulae the expressions for the components of the tensor G given by Eq. (2.4)₂, we obtain the following formulae for the components of the u field:

$$(2.7) \quad u_1 = \frac{b}{4\pi} \frac{\lambda + \mu}{\lambda + 2\mu} \int_{-M}^M dx' \nabla_1 \nabla_2 r \Big|_{z'=-L}^{z'=L}, \\ u_2 = \frac{b}{4\pi} \int_{-M}^M dx' \left[-\frac{1}{r} + \frac{\lambda + \mu}{\lambda + 2\mu} \nabla_2^2 r \right] \Big|_{z'=-L}^{z'=L}, \\ u_3 = \frac{b}{4\pi} \int_{-M}^M dx' \int_{-L}^L dz' \nabla_2 \frac{1}{r} + \frac{b}{4\pi} \frac{\lambda + \mu}{\lambda + 2\mu} \int_{-M}^M dx' \nabla_2 \nabla_3 r \Big|_{z'=-L}^{z'=L}, \\ r = \sqrt{(x' - x)^2 + y^2 + (z' - z)^2}.$$

The first term in Eq. (2.7)₃ is the solid angle at which the surface S is to be seen from the point x .

Let us calculate now the components of the field u . For u_1 we obtain from Eq. (2.7)₁ the following expression:

$$(2.8) \quad u_1 = \frac{b}{4\pi} \frac{\lambda + \mu}{\lambda + 2\mu} (-\nabla_2 r) \Big|_{z'=-L}^{z'=L} \Big|_{x'=-M}^{x'=M} = -\frac{b}{4\pi} \frac{\lambda + \mu}{\lambda + 2\mu} \left(\frac{y}{r} \right) \Big|_{z'=-L}^{z'=L} \Big|_{x'=-M}^{x'=M}.$$

Inserting the integration limits, we thus obtain from here

$$(2.9) \quad u_1 = -\frac{b}{4\pi} \frac{\lambda + \mu}{\lambda + 2\mu} y \left[\frac{1}{\sqrt{(M-x)^2 + y^2 + (L-z)^2}} - \frac{1}{\sqrt{(M+x)^2 + y^2 + (L-z)^2}} \right. \\ \left. - \frac{1}{\sqrt{(M-x)^2 + y^2 + (L+z)^2}} + \frac{1}{\sqrt{(M+x)^2 + y^2 + (L+z)^2}} \right].$$

The field u_1 is equal to zero for $x = 0$, i.e. along the symmetry plane of the dislocation considered. u_1 is the continuous function of the variable y for $y = 0$, that is it does not have the jump discontinuity across S .

We calculate now the expression for the field u_2 . Having performed the differentiation in the formula (2.7)₂ $\left(\nabla_2^2 r = \frac{1}{r} - \frac{y^2}{r^3} \right)$, we obtain for u_2 the expression

$$(2.10) \quad u_2 = -\frac{b}{4\pi} \int_{-M}^M dx' \left\{ \frac{\mu}{\lambda + 2\mu} \frac{1}{r} + \frac{\lambda + \mu}{\lambda + 2\mu} \frac{y^2}{r^3} \right\} \Big|_{z'=-L}^{z'=L}.$$

We introduce the denotation

$$(2.11) \quad \varrho^2 = y^2 + (z' - z)^2.$$

Using now the formulae A_1, A_2 for the indefinite integrals, we obtain from Eq. (2.10) the expression

$$(2.12) \quad u_2 = -\frac{b}{4\pi} \left\{ \frac{\mu}{\lambda + 2\mu} \ln(\sqrt{x' - x)^2 + \varrho^2 + x' - x} + \frac{\lambda + \mu}{\lambda + 2\mu} \frac{y^2}{\varrho^2} \frac{x' - x}{r} \right\}_{z' = -L}^{z' = L} \Big|_{x' = -M}^{x' = M}.$$

We examine now the first term of the above expression which has the singularity characteristic of an edge dislocation:

$$(2.13) \quad I = \ln(\sqrt{(x' - x)^2 + \varrho^2} + x' - x) \Big|_{x' = -M}^{z' = L} \Big|_{x' = -M}^{x' = M} \\ = \ln(\sqrt{(M - x)^2 + \varrho^2} + M - x) - \ln(\sqrt{(M + x)^2 + \varrho^2} - M - x), \\ \varrho^2 = y^2 + (L - z)^2.$$

For $\varrho \Rightarrow 0$, i.e. for $z \Rightarrow L$ and for $x < -M$, i.e. outside the surface S , the expression is nonsingular, having the asymptotic form

$$(2.14) \quad I|_{x < -M} \underset{\varrho \rightarrow 0}{\Rightarrow} \ln 2(|x| + M) - \ln 2(|x| - M) = \ln \frac{|x| + M}{|x| - M}.$$

For $-M < x < M$ and for $\varrho \Rightarrow 0$, i.e. at the boundary of the surface S perpendicular to the Burgers vector, the expression (2.13) is singular, we extract its singular part using the formula

$$(2.15) \quad \ln(\sqrt{x^2 + a^2} + x) + \ln(\sqrt{x^2 + a^2} - x) = \ln a^2.$$

Therefore,

$$(2.16) \quad I = -2 \ln \varrho + \ln(\sqrt{(M - x)^2 + \varrho^2} + M - x) + \ln(\sqrt{(M + x)^2 + \varrho^2} + M + x).$$

Thus for $-M < x < M$

$$(2.17) \quad I \underset{\varrho \rightarrow 0}{\Rightarrow} -2 \ln \varrho + \ln 2(M - x) + \ln 2(M + x).$$

For $x > M$, the expression (2.13) is nonsingular, using Eq. (2.15) we can write it down in the form

$$(2.18) \quad I = -\ln(\sqrt{(x - M)^2 + \varrho^2} + x - M) + \ln(\sqrt{(x + M)^2 + \varrho^2} + x + M), \\ I|_{x > M} \underset{\varrho \rightarrow 0}{\Rightarrow} -\ln 2(x - M) + \ln 2(x + M) = \ln \frac{x + M}{x - M}.$$

We are not going to analyse in details the singularities of u_2 at the corners of the surface S .

Finally, from Eqs. (2.12), (2.13) and (2.16), we obtain the following expression for u_2 :

$$(2.19) \quad u_2 = -\frac{b}{4\pi} \left\{ \frac{\mu}{\lambda + 2\mu} [-2 \ln \sqrt{y^2 + (L - z)^2} + \ln(\sqrt{(M - x)^2 + y^2 + (L - z)^2} + M - x) \right. \\ \left. + \ln(\sqrt{(M + x)^2 + y^2 + (L - z)^2} + M + x)] \right. \\ \left. + \frac{\lambda + \mu}{\lambda + 2\mu} \frac{y^2}{y^2 + (L - z)^2} \left[\frac{M - x}{\sqrt{(M - x)^2 + y^2 + (L - z)^2}} + \frac{M + x}{\sqrt{(M + x)^2 + y^2 + (L - z)^2}} \right] \right\} \\ + \frac{b}{4\pi} \{ \dots \} \Big|_{L \rightarrow -L}.$$

u_2 is the symmetric function of the variables x and y . In like manner as u_1 , u_2 also does not have the jump discontinuity for $y = 0$, i.e. across the dislocation surface S .

We calculate now the expression for the component u_3 of the field \mathbf{u} . Performing differentiation in the formula (2.7)₃, we obtain for u_3 the expression

$$(2.20) \quad \nabla_2 \frac{1}{r} = -\frac{y}{r^3}, \quad \nabla_2 \nabla_3 r = \frac{y(z'-z)}{r^3},$$

$$(2.21) \quad u_3 = -\frac{b}{4\pi} \int_{-M}^M dx' \int_{-L}^L dz' \frac{y}{r^3} + \frac{b}{4\pi} \frac{\lambda + \mu}{\lambda + 2\mu} \int_{-M}^M dx' \frac{y(z'-z)}{r^3} \Big|_{z'=-L}^{z'=L}.$$

Using A_2 , we obtain therefore

$$(2.22) \quad u_3 = \frac{b}{4\pi} \left\{ - \int_{-M}^M dx' \frac{y(z'-z)}{[r^2 - (z'-z)^2]r} \Big|_{z'=-L}^{z'=L} + \frac{\lambda + \mu}{\lambda + 2\mu} \frac{y(z'-z)}{r^2 - (x'-x)^2} \frac{x'-x}{r} \Big|_{z'=-L}^{z'=L} \Big|_{x'=-M}^{x'=M} \right\}.$$

From A_3 we obtain the following expression for the integral in Eq. (2.22):

$$(2.23) \quad \int \frac{dx'}{[r^2 - (z'-z)^2]r} = \frac{1}{|y(z'-z)|} \operatorname{arc} \operatorname{tg} \frac{|z'-z|}{|y|} \frac{x'-x}{r}.$$

Hence

$$(2.24) \quad u_3 = \frac{b}{4\pi} \left\{ - \operatorname{arc} \operatorname{tg} \frac{x'-x}{y} \frac{z'-z}{r} + \frac{\lambda + \mu}{\lambda + 2\mu} \frac{y(z'-z)}{r^2 - (x'-x)^2} \frac{x'-x}{r} \right\} \Big|_{z'=-L}^{z'=L} \Big|_{x'=-M}^{x'=M}.$$

We transform the above formula to extract in an explicit form the discontinuous part of the function u_3 across the surface S . We replace the function $\operatorname{arc} \operatorname{tg} \frac{x'-x}{y} \frac{z'-z}{r}$, discontinuous for $y = 0$, using the identity

$$(2.25) \quad \operatorname{arc} \operatorname{tg} \frac{x}{y} = \frac{\pi}{2} \operatorname{sgn} x \operatorname{sgn} y - \operatorname{arc} \operatorname{tg} \frac{y}{x},$$

which follows from the multiplication by $\operatorname{sgn} x \operatorname{sgn} y$ the identity ($\operatorname{sgn} x$ is the sign function):

$$(2.26) \quad \operatorname{arc} \operatorname{tg} \frac{|x|}{|y|} = \frac{\pi}{2} - \operatorname{arc} \operatorname{tg} \frac{|y|}{|x|}.$$

Hence

$$(2.27) \quad - \operatorname{arc} \operatorname{tg} \frac{x'-x}{y} \frac{z'-z}{r} = \operatorname{arc} \operatorname{tg} \frac{y}{x'-x} \frac{r}{z'-z} - \frac{\pi}{2} \operatorname{sgn} y \operatorname{sgn}(x'-x) \operatorname{sgn}(z'-z).$$

Inserting into the expression consisting of sign functions the integration limits, we obtain

$$(2.28) \quad \operatorname{sgn} y \operatorname{sgn}(x'-x) \operatorname{sgn}(z'-z) \Big|_{z'=-L}^{z'=L} \Big|_{x'=-M}^{x'=M} \\ = \operatorname{sgn} y [\operatorname{sgn}(M-x) - \operatorname{sgn}(-M-x)] [\operatorname{sgn}(L-z) - \operatorname{sgn}(-L-z)] \\ = 4 \operatorname{sgn} y [\theta(x+M) - \theta(x-M)] [\theta(z+L) - \theta(z-L)],$$

$\theta(x)$ is the Heaviside function.

The product $[\theta(x+M) - \theta(x-M)] [\theta(z+L) - \theta(z-L)]$ is equal to 1 for $x \in [-M, M]$, $z \in [-L, L]$, and is equal to 0 outside this set of points.

We thus obtain the following expression for u_3 :

$$(2.29) \quad u_3 = \frac{b}{4\pi} \left\{ \begin{aligned} & \text{arc tg } \frac{y}{M-x} \frac{\sqrt{(M-x)^2 + y^2 + (L-z)^2}}{L-z} \\ & + \text{arc tg } \frac{y}{M+x} \frac{\sqrt{(M+x)^2 + y^2 + (L-z)^2}}{L-z} + \text{arc tg } \frac{y}{M-x} \frac{\sqrt{(M-x)^2 + y^2 + (L+z)^2}}{L+z} \\ & + \text{arc tg } \frac{y}{M+x} \frac{\sqrt{(M+x)^2 + y^2 + (L+z)^2}}{L+z} \end{aligned} \right\} \\ + \frac{b}{4\pi} \frac{\lambda + \mu}{\lambda + 2\mu} \left\{ \begin{aligned} & \frac{y(L-z)}{y^2 + (L-z)^2} \left[\frac{M-x}{\sqrt{(M-x)^2 + y^2 + (L-z)^2}} + \frac{M+x}{\sqrt{(M+x)^2 + y^2 + (L-z)^2}} \right] \\ & + \frac{y(L+z)}{y^2 + (L+z)^2} \left[\frac{M-x}{\sqrt{(M-x)^2 + y^2 + (L+z)^2}} + \frac{M+x}{\sqrt{(M+x)^2 + y^2 + (L+z)^2}} \right] \end{aligned} \right\} \\ - \frac{b}{2} \text{sgn } y [\theta(x+M) - \theta(x-M)] [\theta(z+L) - \theta(z-L)].$$

Only the last term of the above expression is discontinuous across the surface S , the remaining are continuous and equal to zero on S . For $-M < x < M$, $-L < z < L$,

$$u_3^+ = -\frac{b}{2}, \quad u_3^- = \frac{b}{2}, \quad u_3^+ - u_3^- = -b.$$

3. The displacement field in the neighbourhood of the dislocation line

We examine now the asymptotic behaviour of the field u in the neighbourhood of the boundary of the surface S , where this boundary—the dislocation line—can be approximately considered as the screw or edge dislocation.

First, we examine the behaviour of the displacement field near the segment of the line being parallel to the Burgers vector (the screw dislocation).

We put $x = -M + x''$ and assume that the coordinates x'' , y , z are small when compared with L and M , i.e. we are in the neighbourhood of the central part of the section AB of the dislocation line L . With these assumptions

$$(3.1) \quad \begin{aligned} \theta(z+L) - \theta(z-L) &\Rightarrow 1, \\ \theta(-2M + x'') &\Rightarrow 0, \\ \text{arc tg } \frac{y}{M+x} \frac{\sqrt{(M+x)^2 + y^2 + (L \pm z)^2}}{L \pm z} &\Rightarrow \text{arc tg } \frac{y}{x''}, \\ \text{arc tg } \frac{y}{M-x} \frac{\sqrt{(M-x)^2 + y^2 + (L \pm z)^2}}{L \pm z} &\Rightarrow 0, \\ \frac{y(L \pm z)}{y^2 + (L \pm z)^2} &\Rightarrow 0. \end{aligned}$$

From the above follows

$$(3.2) \quad \begin{aligned} u_1 &\Rightarrow 0, \\ u_2 &\Rightarrow \ln \frac{\sqrt{4M^2 + L^2} + 2M}{L}, \\ u_3 &= \frac{b}{2\pi} \operatorname{arc} \operatorname{tg} \frac{y}{x''} - \frac{b}{2} \operatorname{sgn} y \theta(x''). \end{aligned}$$

If we assume furthermore that $L \gg M$, we obtain the expression for the field of a screw dislocation

$$(3.3) \quad \begin{aligned} u_1 &= 0, \quad u_2 = 0, \\ u_3 &= \frac{b}{2\pi} \operatorname{arc} \operatorname{tg} \frac{y}{x''} - \frac{b}{2} \operatorname{sgn} y \theta(x''). \end{aligned}$$

u_3 is discontinuous for $x'' > 0$, the discontinuity being equal to $-b$. The assumption that z is small is essential because for $z \approx L$, u_2 goes to infinity as $\ln|L-z|$.

We examine now the behaviour of the displacement field near the segment of the dislocation line being perpendicular to the Burgers vector. We put $z = -L+z''$, z'' , y , x are small when compared with L and M , i.e. we are in the neighbourhood of the central part of the segment DA of the dislocation line (see Fig. 1). With these assumptions

$$(3.4) \quad \begin{aligned} u_1 &\Rightarrow 0, \\ \mu_2 &\Rightarrow -\frac{b}{4\pi} \left\{ \frac{\mu}{\lambda+2\mu} \left[-2\ln(2L) + 2\ln(M + \sqrt{M^2 + 4L^2}) + 2\ln \sqrt{y^2 + z''^2} \right. \right. \\ &\quad \left. \left. - 2\ln 2M \right] - \frac{2(\lambda+\mu)}{\lambda+2\mu} \frac{y^2}{y^2 + z''^2} \right\} \\ &= -\frac{b}{2\pi} \left\{ \frac{\mu}{\lambda+2\mu} \left[\ln \sqrt{y^2 + z''^2} - \ln \frac{4ML}{M + \sqrt{M^2 + 4L^2}} \right] - \frac{\lambda+\mu}{\lambda+2\mu} \frac{y^2}{y^2 + z''^2} \right\}. \end{aligned}$$

If we make the additional assumption that $M \gg L$,

$$(3.5) \quad u_2 \Rightarrow -\frac{b}{2\pi} \left\{ \frac{\mu}{\lambda+2\mu} [\ln \sqrt{y^2 + z''^2} - \ln 2L] + \frac{\lambda+\mu}{\lambda+2\mu} \frac{y^2}{y^2 + z''^2} \right\}.$$

Moreover,

$$(3.6) \quad \begin{aligned} \theta(x+M) - \theta(x-M) &\Rightarrow 1, \\ \theta(-2L+z'') &\Rightarrow 0. \end{aligned}$$

Thus

$$(3.7) \quad u_3 \Rightarrow \frac{b}{2\pi} \operatorname{arc} \operatorname{tg} \frac{y}{z''} + \frac{b}{2\pi} \frac{\lambda+\mu}{\lambda+2\mu} \frac{yz''}{y^2 + z''^2} - \frac{b}{2} \operatorname{sgn} y \theta(z'').$$

We obtained the expression for the field of an edge dislocation line.

4. Comment upon the singularities of the component u_2

The singularities of the component u_2 of the displacement field—perpendicular to the dislocation surface—in the neighbourhood of the edge type segments of the dislocation

line, are an objectionable fact in the model of a dislocation considered as the surface of discontinuity in the continuous, linearly elastic medium. The origin of these singularities is clear if we go back to the force distribution corresponding to a dislocation. It was demonstrated in [7] that the displacement field of a plane dislocation surface, with the tangent Burgers vector, can be interpreted as due to the force distribution:

$$(4.1) \quad X_r(\mathbf{x}) = X_r^0 + X_r^2 = \mu b_r \int_S ds n_k \nabla_k \delta_3(\mathbf{x} - \xi) + \mu n_r \int_L dl n_a \varepsilon_{ams} \tau_m b_s \delta_3(\mathbf{x} - \xi),$$

$\delta_3(\mathbf{x} - \xi)$ is the three-dimensional Dirac delta function, \mathbf{n} the constant normal vector of the plane surface S , $\boldsymbol{\tau}$ the tangent vector of the line L . The first term of the above formula is the surface distribution over the surface S of the force couples, to which corresponds the momentum density $\mu \mathbf{b} \times \mathbf{n}$. The second term is the distribution over the line L of single concentrated forces having the direction of the vector \mathbf{n} and the magnitude $\mathbf{n} \cdot [\boldsymbol{\tau} \times \mathbf{b}]$ (see Fig. 2). For a screw dislocation line $\boldsymbol{\tau} \times \mathbf{b} = 0$, for an edge one $|\boldsymbol{\tau} \times \mathbf{b}| = b$. The dis-

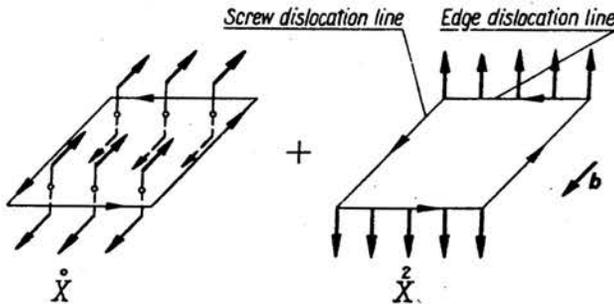


FIG. 2.

tribution of double forces is responsible for the jump discontinuity of the displacement field. The distribution of single forces is necessary to secure the momentum balance of a dislocation; it is exactly the source of logarithmic singularity of a displacement field, corresponding to the singularity of the Green tensor for the two-dimensional problem.

5. Conclusions

The displacement field \mathbf{u} of the rectangular dislocation loop, calculated using the method of Green function, can be presented in the analytic form. The jump discontinuities of \mathbf{u} can be excluded from the whole expression for this field.

The component of the displacement field parallel to the loop plane has the logarithmic singularities, characteristic of an edge dislocation, in the neighbourhood of those segments of the dislocation line where the Burgers vector is perpendicular to the line.

In the neighbourhood of the centers of rectilinear segments of the dislocation line, the asymptotic form of the displacement field is that of an edge or screw straight dislocation line.

Appendix

Here we present the formulae for the indefinite integrals used in the paper. The basic ones are:

$$(A.1) \quad \int \frac{dx}{\sqrt{x^2+a^2}} = \ln(x + \sqrt{x^2+a^2}),$$

$$(A.2) \quad \int \frac{dx}{\sqrt{x^2+a^2}^3} = \frac{x}{a^2 \sqrt{x^2+a^2}},$$

and the less known (see [6], 2.284)

$$(A.3) \quad \int \frac{dx}{(p+a+x^2)\sqrt{a+x^2}} = \frac{1}{\sqrt{-p(a+p)}} \operatorname{arc\,tg} \sqrt{\frac{-p}{a+p}} \frac{x}{\sqrt{a+x^2}},$$

$$-p(a+p) > 0, \quad p < 0.$$

from which follows

$$(A.4) \quad \int \frac{dx'}{[r^2 - (z' - z)^2]r} = \frac{1}{|y(z' - z)|} \operatorname{arc\,tg} \frac{|z' - z|}{|y|} \frac{x' - x}{r},$$

$$r = \sqrt{(x' - x)^2 + y^2 + (z' - z)^2}.$$

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POLISH ACADEMY OF SCIENCES
INSTITUTE OF FUNDAMENTAL TECHNOLOGICAL RESEARCH.

Received October 30, 1979.