

Free convective flow around cylinders near horizontal walls

H. JUNGBLUTH and U. MÜLLER (KARLSRUHE)

A STUDY of parameters for free convective flow around cylinders near horizontal plain walls is carried out using numerical methods. Grashof numbers, Prandtl numbers and the distance from the wall are varied. The influence of these parameters on the heat transfer is discussed.

1. The problem

SESTERHENN [1] has studied in a dissertation several phenomena in free convective flow around heated cylinders using finite difference numerical methods. In this investigations he essentially considered the following situations: Cylinders near horizontal and inclined plain walls, positioned above as well as below walls, and fluid flows in horizontal concentric and excentric annuli.

In this work the heat transfer from heated cylinders above a horizontal wall is subjected to a more extensive investigation. The heat transfer rates under these conditions are of basic interest, as this geometry is the simplest generalization of the case of a single cylinder in an infinitely extended environment.

An infinitely long, horizontal cylinder of radius r and surrounded by an incompressible, Newtonian fluid is considered. The cylinder is positioned at a distance d from a horizontal, plain wall. At a time $t = 0$ the wall and cylinder have the same temperature T_0 and the fluid is at rest. For times $t > t_0$, the cylinder shall be at a temperature T_1 while the wall remains at the temperature level T_0 . The history of the convective flow and the heat transfer from the cylinder due to convection is to be calculated. Naturally the rate of heat transfer for the final steady state is of special interest.

It is assumed that the transport coefficients, i.e. kinematic viscosity and thermometric conductivity are constant. The commonly used Boussinesq-approximation is applied to the density.

For a suitable presentation of the results the following dimensionless parameters were used:

$$\text{Gr} = \frac{gr^3\beta(T_1 - T_0)}{\nu^2}, \quad \text{Pr} = \frac{\nu\varrho c_p}{\lambda}.$$

A detailed description of the set of balance equations for the presented problem and the numerical methods for their solution was presented in an earlier issue of this journal (see A. SESTERHENN, U. MÜLLER [2, 1977]). Therefore, only a brief outline of the calculation procedure is given.

The basis from which the numerical calculation starts are the so-called Oberbeck-Boussinesq equations [3]. These partial differential equations are treated by an implicit difference method of the Crank-Nicolson type employing an alternating direction line overrelaxation method (ADLO). The special wall-cylinder geometry is accounted for by transforming the problem into a bipolar coordinate system.

2. Results and discussions

Sesterhenn has performed numerical calculations of free convective flow around cylinders mainly for Grashof numbers $Gr > 100$ and Prandtl number $Pr = 0.7$. In addition to this, the heat transfer rates were calculated for Grashof numbers $Gr = 0.25, 1, 10, 100$

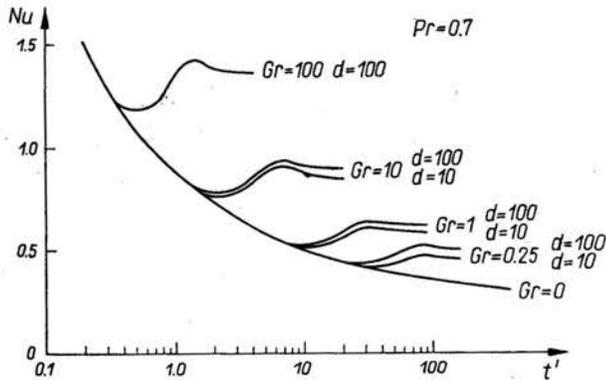


FIG. 1. History of heat transfer.

and wall distances $D = 10, 100$. In Fig. 1 the history of the Nusselt number is shown for different Grashof numbers and wall distances.

From Fig. 1 it can be stated that even for small Grashof numbers and small values of the wall distance, the character of the curves resembles that in the case of large values of Gr and D , i.e.:

a. At the beginning the heat loss of the cylinder is mainly caused by conduction and the curves follow closely that for $Gr = 0$.

b. In the second phase heat transfer by convection sets in and the rate of heat loss culminates at a certain instant of time when a plume separates from the cylinder.

c. Finally, the free convective flow near the cylinder gains a fully-developed steady state.

From Fig. 2 the correlation between the steady heat losses of the cylinder and the wall distances can be seen. For various Grashof numbers the heat transfer curves always maintain their basic shape: a first slight decrease of the Nusselt number with increasing values d until, finally, at larger values d , the Nusselt number becomes independent of d . The minima in the Nusselt number curves are caused physically by a decreasing heat loss by conduction with increasing distances d on one hand and a decreasing inhibition of the convective heat transfer on the other hand.

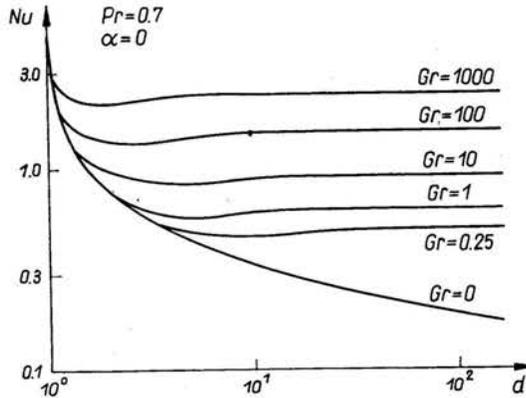


FIG. 2. Heat transfer near walls.

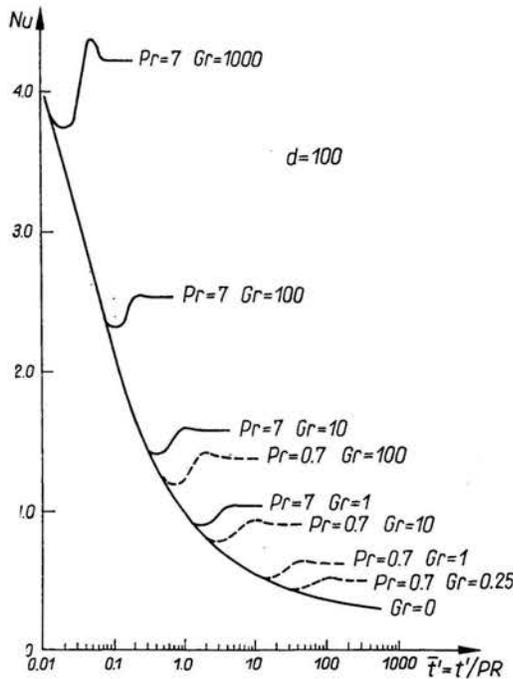


FIG. 3. History of heat transfer for various Prandtl and Grashof numbers.

The influence of the Prandtl number on the heat transfer history is demonstrated in Fig. 3. For comparison, the Nusselt numbers are plotted versus a strained time coordinate $\bar{t} = Pr^{-1}t'$ at a constant wall distance $d = 100$ and for Prandtl numbers $Pr = 0.7$ and $Pr = 7$.

The heat transfer is clearly increased for all times and Grashof numbers, at the higher value of the Prandtl number. This was to be expected due to the structure of the balance equation of momentum and energy.

For the sake of visualizing the flow and temperature conditions even for small wall distances, streamlines and isotherms were calculated for the parameters $Pr = 7$, $Gr = 100$, $d = 2$. In Figs. 4a and 4b some of the steady state streamlines and isotherms are sketched. The isotherms clearly show a hot rising jet above the cylinder having a width of about the

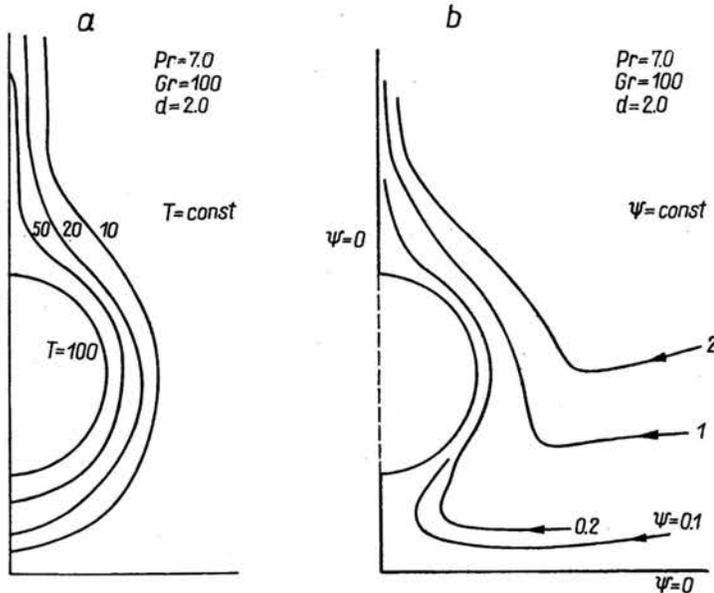


FIG. 4. a) Isotherms, b) Streamlines.

same size as the diameter of the cylinder. The streamlines demonstrate the sidewise flow of cooler fluid near the flat wall.

The calculations were performed on a Univac 1108 computer at the computational Centre of the University of Karlsruhe. For one computer run to obtain a steady state value of the Nusselt Number, an average of 0.75 h computational time was required.

References

1. A. SESTERHENN, *Numerische Berechnung der freien Konvektion um beheizte Zylinder in Wandnähe*, Dissertation Universität Karlsruhe, 1975.
2. A. SESTERHENN, U. MÜLLER, *Numerical calculations of free convective flow around cylinders near rigid walls*, Arch. Mech., **28**, 5-6, 1976.
3. D. D. JOSEPH, *Hydrodynamic stability theory*, Springer 1976.

INSTITUT FÜR REAKTORBAUELEMENTE
KERNFORSCHUNGSZENTRUM KARLSRUHE, BRD.

Received November 26, 1977.